p-Adic Particle Massivation: Elementary Particle Masses

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Abstract

The calculation of elementary fermion and boson masses using p-adic thermodynamics is carried out. Leptons and quarks are obey almost identical mass formulas. Charged lepton mass ratios are predicted with relative errors of order one cent and QED renormalization corrections provide a plausible explanation for the discrepancies. Neutrino masses and neutrino mixing matrix can be predicted highly uniquely if the existing experimental inputs are taken seriously: the best fit of the mass squared differences requires $k = 13^2 = 169$ so that extended form of the p-adic length scale hypothesis is needed.

The prediction or quark masses is more difficult since even the deduction of even the p-adic length scale determining the masses of u, d, and s is a non-trivial task. Second difficulty is related to the topological mixing of quarks. Somewhat surprisingly, the model for U and D matrices constructed for a decade ago predicts realistic quark mass spectrum although the new mass formula is based on different assumptions and different identification of p-adic mass scales. Current quark masses and constituent quark masses can be understood if the p-adic length scale of quark is different for free and bound quarks. The analog of Gell-Mann-Okubo type mass formula results if the p-adic length scale depends on hadron. The Higgs contribution to the fermionic mass is of second order and can be even vanishing and there is an argument implying that Higgs field cannot develop vacuum expectation at fermionic space-time sheets. Top quark mass fixes highly uniquely the CP_2 mass scale since second order correction to electron mass must be very small in order to reproduce the top quark mass in the allowed range of values. Also top quark can correspond to several p-adic mass scales and there is direct experimental evidence for this in mass distribution of top quark.

p-Adic thermodynamics cannot explain Z^0 and W boson masses: thermal masses are completely negligible for the p-adic temperature $T = 1/2$ whereas for $T = 1$ they are 20-30 per cent too high. There is a general argument implying that $T = 1/26$ holds true for bosons so that the masses would be completely negligible. TGD allows a candidate for a Higgs field with the same quantum numbers as its standard model counterpart and having wormhole contacts as space-time correlates just as ordinary gauge bosons have. Thus p-adic thermodynamics *resp.* Higgs mechanism would predict in excellent accuracy fermion *resp.* boson masses and allow the Higgs production rate to be about one per cent of the rate predicted by the standard model (the dominating fermionic couplings are now small).

The possibility of exotic states poses a serious problem for the proposed scenario. If elementary particles correspond to CP_2 type extremals, all exotic massless particles can be constructed using colored generators and by color confinement cannot induce macroscopic long range interactions. The essential assumption is that the fermionic quantization for the space-time sheets having CP_2 projection of dimension $D(CP_2) < 4$ is non-conventional. This has also direct relevance for the understanding of the matter antimatter asymmetry.

1 Introduction

In this chapter the detailed predictions of the p-adic description of particle massivation are studied.

1.1 Basic contributions to the particle mass squared

The formula for the p-adic mass squared contains three additive contributions. The first contribution is proportional to the thermal expectation of the Virasoro generator L_0 in in Super Virasoro degrees of freedom. The miracle is that this contribution is small for the particles with the quantum numbers of the observed light particles, when Super Virasoro has $N = 5$ sectors as it does in TGD approach. These sectors correspond to the 5 tensor factors for the $M^4 \times SU(3) \times U(2)_{ew}$ decomposition of the super Kac Moody algebra to gauge symmetries of gravitation, color and electro-weak interactions. These symmetries act on the intersections $X^2 = X^3_l \cap X^7$ of 3-D light like causal determinants (CDs) X^3_l and 7-D light like CDs $X^7 = \delta M^4_+ \times CP_2$. This constraint leaves only the 2 transversal degrees M^4 degrees of freedom since the translations in light like directions associated with X_l^3 and δM_+^4 are eliminated.

Second contribution comes from the modular degrees of freedom associated with the boundary component of the particle like 3-surface and the interpretation as a thermal contribution is possible but not necessary.

The third contribution consists corresponds to the interactions between partonic 2-surfaces and are important inside hadrons. Also corrections from the secondary condensation to the mass squared are expected.

The lowest order contributions to the charged lepton masses are predicted correctly and information about the yet non-calculable second order corrections is obtained by requiring that the mass ratios are reproduced exactly. Since the contribution from modular degrees of freedom dominates for higher generations the successful predictions for fermion masses give strong support for the topological explanation of the family replication phenomenon.

The prediction or quark masses is more difficult since even the deduction of even the p-adic length scale determining the masses of u, d, and s is a non-trivial task. Second difficulty is related to the topological mixing of quarks. Somewhat surprisingly, the model for U and D matrices constructed for a decade ago predicts realistic quark mass spectrum although the new mass formula is based on different assumptions and different identification of p-adic mass scales.

Graviton, photon and gluons are predicted to be exactly massless. Contrary to the long held beliefs, intermediate boson mass scale is predicted to be 20-30 per cent too high if p-adic thermodynamics with temperature $T_p = 1$ is solely responsible for gauge boson masses. $T_p = 1/2$ predicts completely negligible masses.

One can consider two remedies to the situation: either Higgs type particle exists or the lack of covariant constancy of electro-weak charge matrices can be mimicked by the generation of vacuum expectation value of Higgs.

1.2 The identification of Higgs as a weakly charged wormhole contact

Quantum classical correspondence suggests that electro-weak massivation should have simple space-time description allowing also to identify Higgs boson if it exists. This description indeed exists and allows also to understand the precise relationship between gravitational and inertial masses and how Equivalence Principle is weakened in TGD framework.

The basic observation is that gauge and gravitational fluxes flow to larger space-time sheets through # (wormhole) contacts. If gravitational energy can be regarded in the Newtonian limit as a gauge charge, the contacts feed the gravitational energy regarded as a gauge flux to the lower condensate levels. The non-conservation of gravitational gauge flux means that # contacts can carry gravitational four-momentum. Since CP_2 type vacuum extremals are the natural candidates for # contacts, the natural hypothesis is that the non-vanishing light-like gravitational four-momentum of # contacts is responsible for the non-conservation of gravitational four-momentum flux. The non-conservation of the light-like gravitational four-momentum of CP_2 type extremals is in turn responsible for the non-conservation of the net gravitational four-momentum.

contacts can be also carriers of inertial four-momentum which must be conserved in absence of four-momentum exchange between environment and wormhole contact. Therefore Equivalence Principle cannot hold true in strict sense. Equivalence Principle is satisfied in a weak sense if the inertial four-momentum is equal to the average four-momentum associated with the zitterbewegung motion and corresponds to the center of mass motion for the # contact.

The non-conservation of weak gauge currents for CP_2 type extremals implies a non-conservation of weak charges and the finite range of weak forces. If wormhole contacts correspond to pieces of CP_2 type vacuum extremal, electro-weak gauge currents are not conserved classically unlike color and Kähler current. The non-conservation of weak isospin corresponds to the presence of pairs of right/left handed fermion and left/right handed antifermion at wormhole contacts. These wormhole contacts are excellent candidates for the TGD counterpart of Higgs boson providing the most natural mechanism for the massivation of weak bosons. The finding that that CP_2 parts of the induced gamma matrices connect different M^4 chiralities of induced spinor fields provided the original motivation for the belief that Higgs mechanism is realized in some manner in TGD Universe. This coupling must be crucial for the formation of weakly charged wormhole contacts.

There are two contributions to the mass of elementary particle corresponding to the primary and secondary topological condensation.

1. The dominant contribution to the fermion masses would be due to p-adic thermodynamics describing primary topological condensation. If weak form of Equivalence Principle holds true, inertial mass would result simply as the average of non-conserved light-like gravitational four-momentum. This contribution to the inertial mass is generated in the topological condensation of CP_2 type extremal representing elementary particle involving only single light like elementary particle horizon, say fermion, and by randomness of the zitterbewegung corresponds naturally to the contribution given by p-adic thermodynamics.

2. For gauge bosons the contribution from primary condensation should be very small or vanishing if the radius of zitterbewegung orbit is larger than the size of the space-time sheet containing the topologically condensed boson so that the motion is along a light-like geodesic in a good approximation. The space-time sheet representing massless state suffered secondary topologically condensation at a larger space-time sheet and viewed as a particle can develop mass via Higgs mechanism since wormhole contacts cannot be regarded as moving along light like geodesics in the length and time scale involved. # contacts carrying net left handed weak isospin have interpretation as TGD counterparts of neutral Higgs bosons and the formation of a coherent state involving superposition of states with varying number of wormhole contacts corresponds to the generation of a vacuum expectation value of Higgs field.

1.3 Could also gauge bosons correspond to wormhole contacts?

The developments in the formulation of quantum TGD which have taken place during the period 2005-2007 suggest dramatic simplifications of the general picture about elementary particle spectrum. p-Adic mass calculations leave a lot of freedom concerning the detailed identification of elementary particles. The basic open question is whether the *theory is free at parton level* as suggested by the recent view about the construction of S-matrix and by the almost topological QFT property of quantum TGD at parton level [C3]. Or more concretely: do partonic 2-surfaces carry only free many-fermion states or can they carry also bound states of fermions and anti-fermions identifiable as bosons?

What is known that Higgs boson corresponds naturally to a wormhole contact [C5]. The wormhole contact connects two space-time sheets with induced metric having Minkowski signature. Wormhole contact itself has an Euclidian metric signature so that there are two wormhole throats which are light-like 3-surfaces and would carry fermion and anti-fermion number in the case of Higgs. Irrespective of the identification of the remaining elementary particles MEs (massless extremals, topological light rays) would serve as space-time correlates for elementary bosons. Higgs type wormhole contacts would connect MEs to the larger space-time sheet and the coherent state of neutral Higgs would generate gauge boson mass and could contribute also to fermion mass.

The basic question is whether this identification applies also to gauge bosons (certainly not to graviton). This identification would imply quite a dramatic simplification since the theory would be free at single parton level and the only stable parton states would be fermions and anti-fermions. As will be found this identification allows to understand the dramatic difference between graviton and other gauge bosons and the weakness of gravitational coupling, gives a connection with the string picture of gravitons, and predicts that stringy states are directly relevant for nuclear and condensed matter physics as has been proposed already earlier [F8, J1, J2].

The 1-throat character of fermions is consistent with the generation-genus correspondence. The 2-throat character of bosons predicts that bosons are characterized by the genera (g_1, g_2) of the wormhole throats. Note that the interpretation of fundamental fermions as wormhole contacts with second throat identified as a Fock vacuum is excluded.

The general bosonic wave-function would be expressible as a matrix M_{g_1,g_2} and ordinary gauge bosons would correspond to a diagonal matrix $M_{g_1,g_2} = \delta_{g_1,g_2}$ as required by the absence of neutral flavor changing currents (say gluons transforming quark genera to each other). 8 new gauge bosons are predicted if one allows all 3×3 matrices with complex entries orthonormalized with respect to trace meaning additional dynamical $SU(3)$ symmetry. Ordinary gauge bosons would be $SU(3)$ singlets in this sense. The existing bounds on flavor changing neutral currents give bounds on the masses of the boson octet. The 2-throat character of bosons should relate to the low value $T = 1/n \ll 1$ for the p-adic temperature of gauge bosons as contrasted to $T = 1$ for fermions and provide thus explanation for why Higgs contribution dominates in the mass of gauge bosons.

In this framework p-adic calculations become extremely simple since only the calculation of fermionic masses is necessary. In this chapter however also p-adic thermodynamics calculations for bosons are carried out.

1.4 Exotic states

The physical consequences of the exotic light leptons, quarks, and bosons are considered in the chapter devoted to the New Physics [F5]. Here it only suffices to make a short summary.

2 Exotic states

The possibility of exotic states poses a serious problem. The assumption that only free many fermion states are possible eliminates a huge number of exotics and only the degrees of freedom associated with ground states remain. Coset construction implying duality between SCV and $SKMV$ algebras removes a huge number of exotic states. The strongest form of the duality holds is that one can use either SC or SKM to construct states. In this case the situation reduces more or less to that in super string models in algebraic sense. Also other kinds of exotic states are predicted.

2.1 What kind of exotic states one expects

The physical consequences of the exotic light leptons, quarks, and bosons are considered in the chapter devoted to the New Physics [F5]. Here it only suffices to make a short summary. Consider first what kind of exotic particles extended conformal symmetries predict.

1. Massless states are expected to become massive by p-adic thermodynamics meaning that one has superposition of states with Super Kac-Moody conformal weight equal to Super Virasoro conformal weight and annihilated by SKMV and SCV generators G_n,L_n, $n > 0$. This condition allows degeneracy since there are many manners to create a ground state with a given angular momentum and color quantum numbers and conformal weight n and annihilated by L_n, $n < 0$, by using super-canonical generators. The combinations of super-canonical generators which do not belong to super Kac-Moody algebra and create singlets in color and rotational degrees of freedom would be responsible for this degeneracy. The condition that the states in the superposition are annihilated by G_n, L_n, $n > 0$, reduces the number of the massless states.

2. The original expectation that the spectrum has $N = 1$ space-time super-symmetry seems to be wrong. The understanding of the super-conformal symmetries as at parton level allowed to identify partonic super-conformal symmetries in terms of a generalization of large $N = 4$ SCA with Kac-Moody group extended to contain also canonical transformations of $\delta H_{\pm}$. Thus an immense generalization of string model conformal symmetries is in question. This allows to conclude that sparticles in the sense of super Poincare symmetry are certainly absent. This does not affect the mass calculations in any manner and dramatically reduces the number of exotic states.

3. If elementary particles correspond to CP_2 type extremals, one can argue that all massless exotic massless particles can be constructed using colored generators and by color confinement cannot induce macroscopic long range interactions.

4. The possibility that conformal weights have imaginary part expressible as linear combination of imaginary parts of zeros of ζ function associated with the modified Dirac operator satisfying Riemann hypothesis brings in additional richness of structure. A possible interpretation is that the non-vanishing imaginary part allows to distinguish between positive energy particle propagating into geometric future and negative energy propagating to the geometric past. Phase conjugate photons for which dissipation occurs in time reversed direction would be basic examples of this. Dissipation would be visible already in the mathematical description of partons. The imaginary part of the conformal weight might relate directly to the decay rate of the particle or to the length of the time interval separating positive energy particle and corresponding negative energy particle in zero energy ontology where all physical states have vanishing net quantum numbers [C3].

These exotic particles relate to the extended conformal symmetries. There are also other kinds of exotic particles.

1. The existence of fermionic families suggests the existence of higher bosonic families too. If gauge bosons correspond to wormhole contacts, three families would mean that bosons are labelled by pairs (g_i, g_j) of genera associated with wormhole contacts and $U(3)$ dynamical gauge symmetry emerges naturally. The observed gauge bosons would correspond to SU(3) singlets which do not induced genus changing transitions. The new view about particle decay as a branching of partonic 2-surface is consistent with this picture but not the earlier stringy view. Only three fermion families are predicted if $g > 2$ topologies for partonic 2-surfaces correspond to free many-handle states rather than bound states as for $g < 3$ topologies: who this could happen is discussed in [F1].

2. Also p-adically scaled up copies of various particles are possible as well as scaled-up/scaled-down versions of QCD associated with both quarks [F8] and colored leptons [F7]. There is now quite a lot of evidence that neutrino masses depend on environment [23]: this dependence could have an explanation in terms of topological condensation occurring in several p-adic length scales.

3. Dark matter hierarchy based on the spectrum of Planck constants [A9] infinite number of zoomed up copies of ordinary elementary particles with same mass spectrum.

4. Electro-weak doublet Higgs particle would be present in the spectrum and be identifiable as wormhole contact, contrary to the long held beliefs. Also $q - \overline{q}$ bound states of M_{89} hadron physics such that quark and anti-quark have parallel spins and relative angular momentum $L = 1$ could mimic scalar mesons. The effective couplings of these states to leptons and quarks could mimic the couplings of Higgs boson to some degree. Scalar bound states of heavy quarks are also present in ordinary hadron physics.

To sum up, the results of the calculations provide a considerable support for TGD and the notion p-adicization. What remains still to be understood are various sources of the second order corrections to the masses and the values of some integer valued small parameters fixed completely by the empirical constraints. The detailed analysis and application of the results to derive information about hadron masses is left to the next chapter.

3 Various contributions to the particle masses

In the sequel various contributions to the mass squared are discussed.

3.1 General mass squared formula

The thermal independence of Super Virasoro and modular degrees of freedom implies that mass squared for elementary particle is the sum of Super Virasoro, modular and renormalization correction contributions:

$$M^2 = M^2(color) + M^2(SV) + M^2(mod) + M^2(ren) \ . \tag{1}$$

At this stage the small second order renormalization correction is not yet calculable but can be deduced from the known particle masses by comparing them to the predictions of the theory.

3.2 Color contribution to the mass squared

The mass squared contains a non-thermal color contribution to the ground state conformal weight coming from the mass squared of CP_2 spinor harmonic. The color contribution is an integer multiple of $m_0^2/3$, where $m_0^2 = 2\Lambda$ denotes the 'cosmological constant' of CP_2 (CP_2 satisfies Einstein equations $G^{\alpha\beta} = \Lambda g^{\alpha\beta}$).

The color contribution to the p-adic mass squared is integer valued only if $m_0^2/3$ is taken as a fundamental p-adic unit of mass squared. This choice has an obvious relevance for p-adic mass calculations since the simplest form of the canonical identification does not commute with a division by integer. More precisely, the image of number xp in canonical identification has a value of order 1 when x is a non-trivial rational number whereas for $x = np$ the value is n/p and extremely is small for physically interesting primes.

The choice of the p-adic mass squared unit are no effects on zeroth order contribution which must vanish for light states: this requirement eliminates quark and lepton states for which the CP_2 contribution to the mass squared is not integer valued using m_0^2 as a unit. There can be a dramatic effect on the first order contribution. The mass squared $m^2 = p/3$ using $m_0^2/3$ means that the particle is light. The mass squared becomes $m^2 = p/3$ when m_0^2 is used as a unit and the particle has mass of order 10^{-4} Planck masses. In the case of W and Z^0 bosons this problem is actually encountered. For light states using $m_0^2/3$ as a unit only the second order contribution to the mass squared is affected by this choice.

3.3 Modular contribution to the mass of elementary particle

The general form of the modular contribution is derivable from p-adic partition function for conformally invariant degrees of freedom associated with the boundary components. The general form of the vacuum functionals as modular invariant functions of Teichmuller parameters was derived in [F1] and the square of the elementary particle vacuum functional can be identified as a partition function. Even theta functions serve as basic building blocks and the functionals are proportional to the product of all even theta functions and their complex conjugates. The number of theta functions for genus $g>0$ is given by

$$N(g) \quad = \quad 2^{g-1}(2^g+1) \ . \tag{2}$$

One has $N(1)=3$ for muon and $N(2)=10$ for τ.

1. Single theta function is analogous to a partition function. This implies that the modular contribution to the mass squared must be proportional to $2N(g)$. The factor two follows from the presence of both theta functions and their conjugates in the partition function.

2. The factorization properties of the vacuum functionals imply that handles behave effectively as particles. For example, at the limit, when the surface splits into two pieces with g_1 and $g-g_1$ handles, the partition function reduces to a product of g_1 and $g-g_1$ partition functions. This implies that the contribution to the mass squared is proportional to the genus of the surface. Altogether one has

$$\begin{aligned} M^2(mod,g) &= 2k(mod)N(g)g\frac{m_0^2}{p} \ , \\ k(mod) &= 1 \ . \end{aligned} \tag{3}$$

   Here $k(mod)$ is some integer valued constant (in order to avoid ultra heavy mass) to be determined. $k(mod)=1$ turns out to be the correct choice for this parameter.

Summarizing, the real counterpart of the modular contribution to the mass of a particle belonging to $g+1$:th generation reads as

$$\begin{aligned} M^2(mod) &= 0 \ for \ e,\nu_e,u,d \ , \\ M^2(mod) &= 9\frac{m_0^2}{p(X))} \ for \ X=\mu,\nu_\mu,c,s \ , \\ M^2(mod) &= 60\frac{m_0^2}{p(X)} \ for \ X=\tau,\nu_\tau,t,b \ . \end{aligned} \tag{4}$$

The requirement that hadronic mass spectrum and CKM matrix are sensible however forces the modular contribution to be the same for quarks, leptons and bosons. The higher order modular contributions to the mass squared are completely negligible if the degeneracy of massless state is $D(0,mod,g)=1$ in the modular degrees of freedom as is in fact required by $k(mod)=1$.

3.4 Thermal contribution to the mass squared

One can deduce the value of the thermal mass squared in order $O(p^2)$ (an excellent approximation) using the general mass formula given by p-adic thermodynamics. Assuming maximal p-adic temperature $T_p=1$ one has

$$\begin{aligned} M^2 &= k(sp+Xp^2+O(p^3)) \ , \\ s_\Delta &= \frac{D(\Delta+1)}{D(\Delta)} \ , \\ X_\Delta &= 2\frac{D(\Delta+2)}{D(\Delta)}-\frac{D^2(\Delta+1)}{D^2(\Delta)} \ , \\ k &= 1 \ . \end{aligned} \tag{5}$$

Δ is the conformal weight of the operator creating massless state from the ground state.

The ratios $r_n = D(n+1)/D(n)$ allowing to deduce the values of s and X have been deduced from p-adic thermodynamics in [F2]. Light state is obtained only provided $r(\Delta)$ is an integer. The remarkable result is that for lowest lying states this is the case. For instance, for Ramond representations the values of r_n are given by

$$(r_0, r_1, r_2, r_3) = (8, 5, 4, \frac{55}{16}) \ . \tag{6}$$

The values of s and X are

$$\begin{aligned} (s_0, s_1, s_2) &= (8, 5, 4) \ , \\ (X_0, X_1, X_2) &= (16, 15, 11 + 1/2)) \ . \end{aligned} \tag{7}$$

The result means that second order contribution is extremely small for quarks and charged leptons having $\Delta < 2$. For neutrinos having $\Delta = 2$ the second order contribution is non-vanishing.

3.5 Second order renormalization contribution

Second order renormalization contribution comes from the loops contributing to the propagators and cannot be calculated at this stage. This contribution is expected to be same for all particle families with given electro-weak quantum numbers and information about its value can be deduced from the experimental values of the particle masses. Since the value of the second order contribution Xp^2 is bounded from above $(Xp^2)_R \leq 1/p$ it is not possible to reproduce particle masses by a suitable choice of Y.

As far as the comparison of the predictions to the experimental numbers is considered, the basic problem is whether the measured mass for a particle corresponds to the primarily condensed particle and hence to the predicted mass or whether it corresponds to the mass of the secondarily condensed particle. One could argue that one must compare the mass predictions in some common p-adic length scale determined by the experimental arrangement. For instance, in the case of muon and electron this could mean considering both masses in the p-adic length scale associated with the electron. One might hope that in a good approximation the mass change generated by the secondary condensation corresponds to the mass renormalization predicted by QED. It turns out that in the proposed scenario mass renormalization correction must be taken into account to achieve better than few percent accuracy for the mass ratios and that the sign and order of magnitude for the corrections are correct.

3.6 General mass formula for Ramond representations

By taking the modular contribution from the boundaries into account the general p-adic mass formulas for the Ramond type states read for states for which the color contribution to the conformal weight is integer valued as

$$\begin{aligned} \frac{m^2(\Delta=0)}{m_0^2} &= (8 + n(g))p + Yp^2 \ , \\ \frac{m^2(\Delta=1)}{m_0^2} &= (5 + n(g)p + Yp^2 \ , \\ \frac{m^2(\Delta=2)}{m_0^2} &= (4 + n(g))p + (Y + \frac{23}{2})p^2 \ , \\ n(g) &= 3g \cdot 2^{g-1}(2^g + 1) \ . \end{aligned} \tag{8}$$

Here Δ denotes the conformal weight of the operators creating massless states from the ground state and g denotes the genus of the boundary component. The values of $n(g)$ for the three lowest generations are $n(0) = 0$, $n(1) = 9$ and $n(2) = 60$. The value of second order thermal contribution is nontrivial for neutrinos only. The value of the rational number Y can, which corresponds to the renormalization correction to the mass, can be determined using experimental inputs.

Using m_0^2 as a unit, the expression for the mass of a Ramond type state reads in terms of the electron mass as

$$
\begin{aligned}
M(\Delta, g, p)_R &= K(\Delta, g, p)\sqrt{\frac{M_{127}}{p}} m_e \\
K(0, g, p) &= \sqrt{\frac{n(g) + 8 + Y_R}{X}} \\
K(1, g, p) &= \sqrt{\frac{n(g) + 5 + Y_R}{X}} \\
K(2, g, p) &= \sqrt{\frac{n(g) + 4 + Y_R}{X}} \ , \\
X &= \sqrt{5 + Y(e)_R} \ .
\end{aligned} \tag{9}
$$

Y can be assumed to depend on the electromagnetic charge and color representation of the state and is therefore same for all fermion families. Mathematica provides modules for calculating the real counterpart of the second order contribution and for finding realistic values of Y.

3.7 General mass formulas for NS representations

Using $m_0^2/3$ as a unit, the expression for the mass of a light NS type state for $T_p = 1$ ad $k_B = 1$ reads in terms of the electron mass as

$$
\begin{aligned}
M(\Delta, g, p, N)_R &= K(\Delta, g, p, N)\sqrt{\frac{M_{127}}{p}} m_e \\
K(0, g, p, 1) &= \sqrt{\frac{n(g) + Y_R}{X}} \ , \\
K(0, g, p, 2) &= \sqrt{\frac{n(g) + 1 + Y_R}{X}} \ , \\
K(1, g, p, 3) &= \sqrt{\frac{n(g) + 3 + Y_R}{X}} \ , \\
K(2, g, p, 4) &= \sqrt{\frac{n(g) + 5 + Y_R}{X}} \ , \\
K(2, g, p, 5) &= \sqrt{\frac{n(g) + 10 + Y_R}{X}} \ , \\
X &= \sqrt{5 + Y(e)_R} \ .
\end{aligned} \tag{10}
$$

Here N is the number of the 'active' NS sectors (sectors for which the conformal weight of the massless state is non-vanishing). Y denotes the renormalization correction to the boson mass and in general depends on the electro-weak and color quantum numbers of the boson.

The thermal contribution to the mass of W boson is too large by roughly a factor $\sqrt{3}$ for $T_p = 1$. Hence $T_p = 1/2$ must hold true for gauge bosons and their masses must have a non-thermal origin perhaps analogous to Higgs mechanism. Alternatively, the non-covariant constancy of charge matrices could induce the boson mass [F2].

It is interesting to notice that the minimum mass squared for gauge boson corresponds to the p-adic mass unit $M^2 = m_0^2 p/3$ and this just what is needed in the case of W boson. This forces to ask whether $m_0^2/3$ is the correct choice for the mass squared unit so that non-thermally induced W mass would be the minimal $m_W^2 = p$ in the lowest order. This choice would mean the replacement

$$Y_R \rightarrow \frac{(3Y)_R}{3}$$

in the preceding formulas and would affect only neutrino mass in the fermionic sector. $m_0^2/3$ option is excluded by charged lepton mass calculation. This point will be discussed later.

3.8 Primary condensation levels from p-adic length scale hypothesis

p-Adic length scale hypothesis states that the primary condensation levels correspond to primes near prime powers of two $p \simeq 2^k$, k integer with prime values preferred. Black hole-elementary particle analogy [E5] suggests a generalization of this hypothesis by allowing k to be a power of prime. The general number theoretical vision discussed in [E1] provides a first principle justification for p-adic length scale hypothesis in its most general form. The best fit for the neutrino mass squared differences is obtained for $k = 13^2 = 169$ so that the generalization of the hypothesis might be necessary.

A particle primarily condensed on the level k can suffer secondary condensation on a level with the same value of k: for instance, electron ($k = 127$) suffers secondary condensation on $k = 127$ level. u, d, s quarks ($k = 107$) suffer secondary condensation on nuclear space-time sheet having $k = 113$). All quarks feed their color gauge fluxes at $k = 107$ space-time sheet. There is no deep reason forbidding the condensation of p on p. Primary and secondary condensation levels could also correspond to different but nearly identical values of p with the same value of k.

4 Fermion masses

In the earlier model the coefficient of $M^2 = kL_0$ had to be assumed to be different for various particle states. $k = 1$ was assumed for bosons and leptons and $k = 2/3$ for quarks. The fact that $k = 1$ holds true for all particles in the model including also super-canonical invariance forces to modify the earlier construction of quark states. This turns out to be possible without affecting the earlier p-adic mass calculations whose outcome depend in an essential manner on the ground state conformal weights h_{gr} of the fermions (h_{gr} can be negative). The structure of lepton and quark states in color degrees of freedom was discussed in [F2].

4.1 Charged lepton mass ratios

The overall mass scale for lepton and quark masses is determined by the condensation level given by prime $p \simeq 2^k$, k prime by length scale hypothesis. For charged leptons k must correspond to $k = 127$ for electron, $k = 113$ for muon and $k = 107$ for τ. For muon $p = 2^{113} - 1 - 4 * 378$ is assumed (smallest prime below 2^{113} allowing $\sqrt{2}$ but not $\sqrt{3}$). So called Gaussian primes are to complex integers what primes are for the ordinary integers and the Gaussian counterparts of the Mersenne primes are Gaussian primes of form $(1 \pm i)^k - 1$. Rather interestingly, $k = 113$ corresponds to a Gaussian Mersenne so that all charged leptons correspond to generalized Mersenne primes.

For $k = 1$ the leptonic mass squared is integer valued in units of m_0^2 only for the states satisfying

$$p \; mod \; 3 \neq 2 \ \ .$$

Only these representations can give rise to massless states. Neutrinos correspond to (p,p) representations with $p \geq 1$ whereas charged leptons correspond to $(p, p+3)$ representations. The earlier mass calculations demonstrate that leptonic masses can be understood if the ground state conformal weight is $h_{gr} = -1$ for charged leptons and $h_{gr} = -2$ for neutrinos.

The contribution of color partial wave to conformal weight is $h_c = (p^2+2p)/3$, $p \geq 1$, for neutrinos and $p=1$ gives $h_c = 1$ (octet). For charged leptons $h_c = (p^2+5p+6)/3$ gives $h_c = 2$ for $p=0$ (decuplet). In both cases super-canonical operator O must have a net conformal weight $h_{sc} = -3$ to produce a correct conformal weight for the ground state. p-adic considerations suggests the use of operators O with super-canonical conformal weight $z = -1/2 - i\sum n_k y_k$, where $s_k = 1/2 + iy_k$ corresponds to zero of Riemann ζ. If the operators in question are color Hamiltonians in octet representation net super-canonical conformal weight $h_{sc} = -3$ results. The tensor product of two octets with conjugate super-canonical conformal weights contains both octet and decuplet so that singlets are obtained. What strengthens the hopes that the construction is not adhoc is that the same operator appears in the construction of quark states too.

Using CP_2 mass scale m_0^2 [F2] as a p-adic unit, the mass formulas for the charged leptons read as

$$\begin{aligned}
M^2(L) &= A(\nu)\frac{m_0^2}{p(L)} \ , \\
A(e) &= 5 + X(p(e)) \ , \\
A(\mu) &= 14 + X(p(\mu)) \ , \\
A(\tau) &= 65 + X(p(\tau)) \ .
\end{aligned} \tag{11}$$

$X(\cdot)$ corresponds to the yet unknown second order corrections to the mass squared.

The following table gives the basic parameters as determined from the mass of electron for some values of Y_e. The mass of top quark favors as maximal value of CP_2 mass which corresponds to $Y_e = 0$.

Y_e	0	.5	.7798
$(m_0/m_{Pl}) \times 10^3$	.2437	.2323	.2266
$K \times 10^{-7}$	2.5262	2.7788	2.9202
$(L_R/\sqrt{G}) \times 10^{-4}$	3.1580	3.3122	3.3954

Table 1. Table gives the values of CP_2 mass m_0 using Planck mass $m_{Pl} = 1/\sqrt{G}$ as unit, the ratio $K = R^2/G$ and CP_2 geodesic length $L = 2\pi R$ for $Y_e \in \{0, 0.5, 0.7798\}$.

The following table lists the lower and upper bounds for the charged lepton mass ratios obtained by taking second order contribution to zero or allowing it to have maximum possible value. The values of lepton masses are $m_e = .510999$ MeV, $m_\mu = 105.76583$ MeV, $m_\tau = 1775$ MeV.

$$\begin{aligned}
\frac{m(\mu)_+}{m(\mu)} &= \sqrt{\frac{15}{5}} 2^7 \frac{m_e}{(\mu)} \simeq 1.0722 \ , \\
\frac{m(\mu)_-}{m(\mu)} &= \sqrt{\frac{14}{6}} 2^7 \frac{m_e}{m_(\mu)} \simeq 0.9456 \ ,
\end{aligned}$$

$$\begin{aligned}
\frac{m(\tau)_+}{m(\tau)} &= \sqrt{\frac{66}{5}2^{10}\frac{m_e}{m(\tau)}} \simeq 1.0710 \ , \\
\frac{m(\tau)_-}{m(\tau)} &= \sqrt{\frac{65}{6}2^{10}\frac{m_e}{m(\tau)}} \simeq .9703 \ .
\end{aligned} \tag{12}$$

For the maximal value of CP_2 mass the predictions for the mass ratio are systematically too large by a few per cent. From the formulas above it is clear that the second order corrections to mass squared can be such that correct masses result.

τ mass is least sensitive to $X(p(e)) \equiv Y_e$ and the maximum value of $Y_e \equiv Y_{e,max}$ consistent with τ mass corresponds to $Y_{e,max} = .7357$ and $Y_\tau = 1$. This means that the CP_2 mass is at least a fraction .9337 of its maximal value. If Y_L is same for all charged leptons and has the maximal value $Y_{e,max} = .7357$, the predictions for the mass ratios are

$$\begin{aligned}
\frac{m(\mu)_{pr}}{m(\mu)} &= \sqrt{\frac{14+Y_{e,max}}{5+Y_{e,max}}} \times 2^7 \frac{m_e}{m(\mu)} \simeq .9922 \ , \\
\frac{m(\tau)_{pr}}{m(\tau)} &= \sqrt{\frac{65+Y_{e,max}}{5+Y_e(max}} \times 2^{10} \frac{m_e}{m(\tau)} \simeq .9980 \ .
\end{aligned} \tag{13}$$

The error is .8 per cent *resp.* .2 per cent for muon *resp.* τ.

The argument leading to estimate for the modular contribution to the mass squared [F2] leaves two options for the coefficient of the modular contribution for $g = 2$ fermions: the value of coefficient is either $X = g$ for $g \leq 1$, $X = 3g - 3$ for $g \geq 2$ or $X = g$ always. For $g = 2$ the predictions are $X = 2$ and $X = 3$ in the two cases. The option $X = 3$ allows slightly larger maximal value of Y_e equal to $Y^{1)}_{e,max} = Y_{e,max} + (5 + Y_{e,max})/66$.

4.2 Neutrino masses

The estimation of neutrino masses is difficult at this stage since the prediction of the primary condensation level is not yet possible and neutrino mixing cannot yet be predicted from the basic principles. The cosmological bounds for neutrino masses however help to put upper bounds on the masses. If one takes seriously the LSND data on neutrino mass measurement of [16, 17] and the explanation of the atmospheric ν-deficit in terms of $\nu_\mu - \nu_\tau$ mixing [20, 21] one can deduce that the most plausible condensation level of μ and τ neutrinos is $k = 167$ or $k = 13^2 = 169$ allowed by the more general form of the p-adic length scale hypothesis suggested by the blackhole-elementary particle analogy. One can also deduce information about the mixing matrix associated with the neutrinos so that mass predictions become rather precise. In particular, the mass splitting of μ and τ neutrinos is predicted correctly if one assumes that the mixing matrix is a rational unitary matrix.

4.2.1 Super Virasoro contribution

Using $m_0^2/3$ as a p-adic unit, the expression for the Super Virasoro contribution to the mass squared of neutrinos is given by the formula

$$M^2(SV) = (s + (3Yp)_R/3)\frac{m_0^2}{p} \ ,$$

$$
\begin{aligned}
s &= 4 \text{ or } 5 \ , \\
Y &= \frac{23}{2}+Y_1 \ ,
\end{aligned} \tag{14}
$$

where m_0^2 is universal mass scale. One can consider two possible identifications of neutrinos corresponding to $s(\nu)=4$ with $\Delta=2$ and $s(\nu)=5$ with $\Delta=1$. The requirement that CKM matrix is sensible forces the asymmetric scenario in which quarks and, by symmetry, also leptons correspond to lowest possible excitation so that one must have $s(\nu)=4$. Y_1 represents second order contribution to the neutrino mass coming from renormalization effects coming from self energy diagrams involving intermediate gauge bosons. Physical intuition suggest that this contribution is very small so that the precise measurement of the neutrino masses should give an excellent test for the theory.

With the above described assumptions and for $s=4$, one has the following mass formula for neutrinos

$$
\begin{aligned}
M^2(\nu) &= A(\nu)\frac{m_0^2}{p(\nu))} \ , \\
A(\nu_e) &= 4+\frac{(3Y(p(\nu_e)))_R}{3} \ , \\
A(\nu_\mu) &= 13+\frac{(3Y(p(\nu_\mu)))_R}{3} \ , \\
A(\nu_\tau) &= 64+\frac{(3Y(p(\nu_\tau)))_R}{3} \ , \\
3Y &\simeq \frac{1}{2} \ .
\end{aligned} \tag{15}
$$

The predictions must be consistent with the recent upper bounds [22] of order 10 eV, 270 keV and 0.3 MeV for ν_e, ν_μ and ν_τ respectively. The recently reported results of LSND measurement [17] for $\nu_e->\nu_\mu$ mixing gives string limits for $\Delta m^2(\nu_e,\nu_\mu)$ and the parameter $sin^2(2\theta)$ characterizing the mixing: the limits are given in the figure 30 of [17]. The results suggests that the masses of both electron and muon neutrinos are below 5 eV and that mass squared difference $\Delta m^2=m^2(\nu_\mu)-m^2(\nu_e)$ is between $.25-25\ eV^2$. The simplest possibility is that ν_μ and ν_e have common condensation level (in analogy with d and s quarks). There are three candidates for the primary condensation level: namely $k=163,167$ and $k=169$. The p-adic prime associated with the primary condensation level is assumed to be the nearest prime below 2^k allowing p-adic $\sqrt{2}$ but not $\sqrt{3}$ and satisfying $p\ mod\ 4=3$. The following table gives the values of various parameters and unmixed neutrino masses in various cases of interest.

k	p	$(3Y)_R/3$	$m(\nu_e)/eV$	$m(\nu_\mu)/eV$	$m(\nu_\tau)/eV$
163	$2^{163}-4*144-1$	1.36	1.78	3.16	6.98
167	$2^{167}-4*144-1$	.34	.45	.79	1.75
169	$2^{169}-4*210-1$	.17	.22	.40	.87

4.2.2 Could neutrino topologically condense also in other p-adic length scales than $k=169$?

One must keep mind open for the possibility that there are several p-adic length scales at which neutrinos can condense topologically. In fact, the quantum model for hearing [M6] requires that both $k=169$ and $k=151$ correspond to p-adic length scales at which neutrinos can condense topologically. Rather interestingly, the ratio for the mass scales of $k=151$ and $k=169$ neutrinos equals to 512 and is same as the ratio of the mass scales of the ordinary $k = 107$ hadron physics and $k = 89$ hadronic physics predicted by TGD.

In fact, all intermediate p-adic length scales $k = 151, 157, 163, 167$ could correspond to metastable neutrino states. The point is that these p-adic lengths scales are number theoretically completely exceptional in the sense that there exist Gaussian Mersenne $2^k \pm i$ (prime in the ring of complex integers) for all these values of k. Since charged leptons, atomic nuclei ($k = 113$) , hadrons and intermediate gauge bosons correspond to ordinary or Gaussian Mersennes, it would not be surprising if the biologically important Gaussian Mersennes would correspond to length scales giving rise to metastable neutrino states. Of course, one can keep mind open for the possibility that $k = 167$ rather than $k = 13^2 = 169$ is the length scale defining the stable neutrino physics.

4.2.3 Neutrino mixing

Consider next the neutrino mixing. A quite general form of the neutrino mixing matrix D given by the table below will be considered.

	ν_e	ν_μ	ν_τ
ν_e	c_1	s_1c_3	s_1s_3
ν_μ	$-s_1c_2$	$c_1c_2c_3 - s_2s_3exp(i\delta)$	$c_1c_2s_3 + s_2c_3exp(i\delta)$
ν_τ	$-s_1s_2$	$c_1s_2c_3 + c_2s_3exp(i\delta)$	$c_1s_2s_3 - c_2c_3exp(i\delta)$

Physical intuition suggests that the angle δ related to CP breaking is small and will be assumed to be vanishing. Topological mixing is active only in modular degrees of freedom and one obtains for the first order terms of mixed masses the expressions

$$\begin{aligned} s(\nu_e) &= 4 + 9|U_{12}|^2 + 60|U_{13}|^2 = 4 + n_1 \ , \\ s(\nu_\mu) &= 4 + 9|U_{22}|^2 + 60|U_{23}|^2 = 4 + n_2 \ , \\ s(\nu_\tau) &= 4 + 9|U_{32}|^2 + 60|U_{33}|^2 = 4 + n_3 \ . \end{aligned} \tag{16}$$

The requirement that resulting masses are not ultraheavy implies that $s(\nu)$ must be small integers. The condition $n_1 + n_2 + n_3 = 69$ follows from unitarity. The simplest possibility is that the mixing matrix is a rational unitary matrix. The same ansatz was used successfully to deduce information about the mixing matrices of quarks. If neutrinos are condensed on the same condensation level, rationality implies that $\nu_\mu - \nu_\tau$ mass squared difference must come from the first order contribution to the mass squared and is therefore quantized and bounded from below.

The first piece of information is the atmospheric ν_μ/ν_e ratio, which is roughly by a factor 2 smaller than predicted by standard model [20]. A possible explanation is the CKM mixing of muon neutrino with τ-neutrino, whereas the mixing with electron neutrino is excluded as an explanation. The latest results from Kamiokande [20] are in accordance with the mixing $m^2(\nu_\tau) - m^2(\nu_\mu) \simeq 1.6 \cdot 10^{-2}\ eV^2$ and mixing angle $sin^2(2\theta) = 1.0$: also the zenith angle dependence of the ratio is in accordance with the mixing interpretation. If mixing matrix is assumed to be rational then only $k = 169$ condensation level is allowed for ν_μ and ν_τ. For this level $\nu_\mu - \nu_\tau$ mass squared difference turns out to be $\Delta m^2 \simeq 10^{-2}\ eV^2$ for $\Delta s \equiv s(\nu_\tau) - s(\nu_\mu) = 1$, which is the only acceptable possibility and predicts $\nu_\mu - \nu_\tau$ mass squared difference correctly within experimental uncertainties! The fact that the predictions for mass squared differences are practically exact, provides a precision test for the rationality assumption.

What is measured in LSND experiment is the probability $P(t, E)$ that ν_μ transforms to ν_e in time t after its production in muon decay as a function of energy E of ν_μ. In the limit that ν_τ and ν_μ masses are identical, the expression of $P(t, E)$ is given by

$$
\begin{aligned}
P(t,E) &= sin^2(2\theta)sin^2(\frac{\Delta Et}{2}) \ , \\
sin^2(2\theta) &= 4c_1^2s_1^2c_2^2 \ ,
\end{aligned} \tag{17}
$$

where ΔE is energy difference of ν_μ and ν_e neutrinos and t denotes time. LSND experiment gives stringent conditions on the value of $\sin^2(2\theta)$ as the figure 30 of [17] shows. In particular, it seems that $sin^2(2\theta)$ must be considerably below 10^{-1} and this implies that s_1^2 must be small enough.

The study of the mass formulas shows that the only possibility to satisfy the constraints for the mass squared and $sin^2(2\theta)$ given by LSND experiment is to assume that the mixing of the electron neutrino with the tau neutrino is much larger than its mixing with the muon neutrino. This means that s_3 is quite near to unity. At the limit $s_3 = 1$ one obtains the following (nonrational) solution of the mass squared conditions for $n_3 = n_2 + 1$ (forced by the atmospheric neutrino data)

$$
\begin{aligned}
s_1^2 &= \frac{69 - 2n_2 - 1}{60} \ , \\
c_2^2 &= \frac{n_2 - 9}{2n_2 - 17} \ , \\
sin^2(2\theta) &= \frac{4(n_2 - 9)}{51}\frac{(34 - n_2)(n_2 - 4)}{30^2} \ , \\
s(\nu_\mu) - s(\nu_e) &= 3n_2 - 68 \ .
\end{aligned} \tag{18}
$$

The study of the LSND data shows that there is only one acceptable solution to the conditions obtained by assuming maximal mass squared difference for ν_e and ν_μ

$$
\begin{aligned}
n_1 &= 2 \quad n_2 = 33 \quad n_3 = 34 \ , \\
s_1^2 &= \frac{1}{30} \quad c_2^2 = \frac{24}{49} \ , \\
sin^2(2\theta) &= \frac{24}{49}\frac{2}{15}\frac{29}{30} \simeq .0631 \ , \\
s(\nu_\mu) - s(\nu_e)) &= 31 \leftrightarrow .32 \ eV^2 \ .
\end{aligned} \tag{19}
$$

That c_2^2 is near 1/2 is not surprise taking into account the almost mass degeneracy of ν_{mu} and ν_τ. From the figure 30 of [17] it is clear that this solution belongs to 90 per cent likelihood region of LSND experiment but $sin^2(2\theta)$ is about two times larger than the value allowed by Bugey reactor experiment. The study of various constraints given in [17] shows that the solution is consistent with bounds from all other experiments. If one assumes that $k > 169$ for ν_e $\nu_\mu - \nu_e$ mass difference increases, implying slightly poorer consistency with LSND data.

There are reasons to hope that the actual rational solution can be regarded as a small deformation of this solution obtained by assuming that c_3 is non-vanishing. $s_1^2 = \frac{69-2n_2-1}{60-51c_3^2}$ increases in the deformation by $O(c_3^2)$ term but if c_3 is positive the value of $c_2^2 \simeq \frac{24-102c_1^0c_2^0s_2^0c_3}{49} \sim \frac{24-61c_3}{49}$ decreases by $O(c_3)$ term so that it should be possible to reduce the value of $sin^2(2\theta)$. Consistency with Bugey reactor experiment requires $.030 \leq sin^2(2\theta) < .033$. $sin^2(2\theta) = .032$ is achieved for $s_1^2 \simeq .035$,$s_2^2 \simeq .51$ and $c_3^2 \simeq .068$. The construction of U and D matrices for quarks shows that very stringent number theoretic conditions are obtained and as in case of quarks it might be necessary to allow complex CP breaking phase in the mixing matrix. One might even hope that the solution to the conditions is unique.

For the minimal rational mixing one has $s(\nu_e) = 5$, $s(\nu_\mu) = 36$ and $s(\nu_\tau) = 37$ if unmixed ν_e corresponds to $s = 4$. For $s = 5$ first order contributions are shifted by one unit. The masses ($s = 4$ case) and mass squared differences are given by the following table.

k	$m(\nu_e)$	$m(\nu_\mu)$	$m(\nu_\tau)$	$\Delta m^2(\nu_\mu-\nu_e)$	$\Delta m^2(\nu_\tau-\nu_\mu)$
169	.27 eV	.66 eV	.67 eV	.32 eV^2	.01 eV^2

Predictions for neutrino masses and mass squared splittings for $k=169$ case.

4.2.4 Evidence for the dynamical mass scale of neutrinos

In recent years (I am writing this towards the end of year 2004 and much later than previous lines) a great progress has been made in the understanding of neutrino masses and neutrino mixing. The pleasant news from TGD perspective is that there is a strong evidence that neutrino masses depend on environment [23]. In TGD framework this translates to the statement that neutrinos can suffer topological condensation in several p-adic length scales. Not only in the p-adic length scales suggested by the number theoretical considerations but also in longer length scales, as will be found.

The experiments giving information about mass squared differences can be divided into three categories [23].

i) There along baseline experiments, which include solar neutrino experiments [24], KamLAND [27], K2K [26], and SuperK [25] as well as earlier studies of solar neutrinos. These experiments see evidence for the neutrino mixing and involve significant propagation through dense matter. For the solar neutrinos and KamLAND the mass splittings are estimated to be of order $O(8\times 10^{-5})$ eV^2 or more cautiously 8×10^{-5} eV$^2 < \delta m^2 < 2\times 10^{-3}$ eV^2. For K2K and atmospheric neutrinos the mass splittings are of order $O(2\times 10^{-3})eV^2$ or more cautiously $\delta m^2 > 10^{-3}$eV^2. Thus the scale of mass splitting seems to be smaller for neutrinos in matter than in air, which would suggest that neutrinos able to propagate through a dense matter travel at space-time sheets corresponding to a larger p-adic length scale than in air.

ii) There are null short baseline experiments including CHOOZ, Bugey, and Palo Verde reactor experiments, and the higher energy CDHS, JARME, CHORUS, and NOMAD experiments, which involve muonic neutrinos (for references see [23]. No evidence for neutrino oscillations have been seen in these experiments.

iii) The results of LSND experiment[17] are consistent with oscillations with a mass splitting greater than $3\times 10^{-2}eV^2$. LSND has been generally been interpreted as necessitating a mixing with sterile neutrino. If neutrino mass scale is dynamical, situation however changes.

If one assumes that the p-adic length scale for the space-time sheets at which neutrinos can propagate is different for matter and air, the situation changes. According to [23] a mass 3×10^{-2} eV in air could explain the atmospheric results whereas mass of of order .1 eV and $.07eV^2 < \delta m^2 < .26eV^2$ would explain the LSND result. These limits are of the same order as the order of magnitude predicted by $k=169$ topological condensation.

Assuming that the scale of the mass splitting is proportional to the p-adic mass scale squared, one can consider candidates for the topological condensation levels involved.

1. Suppose that $k=169=13^2$ is indeed the condensation level for LSND neutrinos. $k=173$ would predict $m_{\nu_e}\sim 7\times 10^{-2}$ eV and $\delta m^2 \sim .02$ eV^2. This could correspond to the masses of neutrinos propagating through air. For $k=179$ one has $m_{\nu_e}\sim .8\times 10^{-2}$ eV and $\delta m^2\sim 3\times 10^{-4}$ eV^2 which could be associated with solar neutrinos and KamLAND neutrinos.

2. The primes $k=157,163,167$ associated with Gaussian Mersennes would give $\delta m^2(157) = 2^6\delta m^2(163) = 2^{10}\delta m^2(167) = 2^{12}\delta m^2(169)$ and mass scales $m(157)\sim 22.8$ eV, $m(163)\sim 3.6$ eV, $m(167)\sim .54$ eV. These mass scales are unrealistic or propagating neutrinos. The interpretation consistent with TGD inspired model of condensed matter in which neutrinos screen the classical Z^0 force generated by nucleons would be that condensed matter neutrinos

are confined inside these space-time sheets whereas the neutrinos able to propagate through condensed matter travel along $k > 167$ space-time sheets.

4.2.5 The results of MiniBooNE group as a support for the energy dependence of p-adic mass scale of neutrino

The basic prediction of TGD is that neutrino mass scale can depend on neutrino energy and the experimental determinations of neutrino mixing parameters support this prediction. The newest results (11 April 2007) about neutrino oscillations come from MiniBooNE group which has published its first findings [19] concerning neutrino oscillations in the mass range studied in LSND experiments [18].

1. The motivation for MiniBooNE

Neutrino oscillations are not well-understood. Three experiments LSND, atmospheric neutrinos, and solar neutrinos show oscillations but in widely different mass regions (1 eV^2 , 3×10^{-3} eV^2, and 8×10^{-5} eV^2).

In TGD framework the explanation would be that neutrinos can appear in several p-adically scaled up variants with different mass scales and therefore different scales for the differences Δm^2 for neutrino masses so that one should not try to try to explain the results of these experiments using single neutrino mass scale. In single-sheeted space-time it is very difficult to imagine that neutrino mass scale would depend on neutrino energy since neutrinos interact so extremely weakly with matter. The best known attempt to assign single mass to all neutrinos has been based on the use of so called sterile neutrinos which do not have electro-weak couplings. This approach is an ad hoc trick and rather ugly mathematically and excluded by the results of MiniBooNE experiments.

2. The result of MiniBooNE experiment

The purpose of the MiniBooNE experiment was to check whether LSND result $\Delta m^2 = 1eV^2$ is genuine. The group used muon neutrino beam and looked whether the transformations of muonic neutrinos to electron neutrinos occur in the mass squared region $\Delta m^2 \simeq 1$ eV^2. No such transitions were found but there was evidence for transformations at low neutrino energies.

What looks first as an over-diplomatic formulation of the result was *MiniBooNE researchers showed conclusively that the LSND results could not be due to simple neutrino oscillation, a phenomenon in which one type of neutrino transforms into another type and back again.* rather than direct refutation of LSND results.

3. LSND and MiniBooNE are consistent in TGD Universe

The habitant of the many-sheeted space-time would not regard the previous statement as a mere diplomatic use of language. It is quite possible that neutrinos studied in MiniBooNE have suffered topological condensation at different space-time sheet than those in LSND if they are in different energy range (the preferred rest system fixed by the space-time sheet of the laboratory or Earth). To see whether this is the case let us look more carefully the experimental arrangements.

1. In LSND experiment 800 MeV proton beam entering in water target and the muon neutrinos resulted in the decay of produced pions. Muonic neutrinos had energies in 60-200 MeV range [18].

2. In MiniBooNE experiment [19] 8 GeV muon beam entered Beryllium target and muon neutrinos resulted in the decay of resulting pions and kaons. The resulting muonic neutrinos had energies the range 300-1500 GeV to be compared with 60-200 MeV.

Let us try to make this more explicit.

1. Neutrino energy ranges are quite different so that the experiments need not be directly comparable. The mixing obeys the analog of Schrödinger equation for free particle with energy replaced with $\Delta m^2/E$, where E is neutrino energy. The mixing probability as a function of distance L from the source of muon neutrinos is in 2-component model given by

$$P = sin^2(\theta) sin^2(1.27\Delta m^2 L/E) \ .$$

The characteristic length scale for mixing is $L = E/\Delta m^2$. If L is sufficiently small, the mixing is fifty-fifty already before the muon neutrinos enter the system, where the measurement is carried out and no mixing is detected. If L is considerably longer than the size of the measuring system, no mixing is observed either. Therefore the result can be understood if Δm^2 is much larger or much smaller than E/L, where L is the size of the measuring system and E is the typical neutrino energy.

2. MiniBooNE experiment found evidence for the appearance of electron neutrinos at low neutrino energies (below 500 MeV) which means direct support for the LSND findings and for the dependence of neutron mass scale on its energy relative to the rest system defined by the space-time sheet of laboratory.

3. Uncertainty Principle inspires the guess $L_p \propto 1/E$ implying $m_p \propto E$. Here E is the energy of the neutrino with respect to the rest system defined by the space-time sheet of the laboratory. Solar neutrinos indeed have the lowest energy (below 20 MeV) and the lowest value of Δm^2. However, atmospheric neutrinos have energies starting from few hundreds of MeV and $\Delta; m^2$ is by a factor of order 10 higher. This suggests that the the growth of Δm^2 with E^2 is slower than linear. It is perhaps not the energy alone which matters but the space-time sheet at which neutrinos topologically condense. For instance, MiniBooNE neutrinos above 500 MeV would topologically condense at space-time sheets for which the p-adic mass scale is higher than in LSND experiments and one would have $\Delta m^2 >> 1$ eV^2 implying maximal mixing in length scale much shorter than the size of experimental apparatus.

4. One could also argue that topological condensation occurs in condensed matter and that no topological condensation occurs for high enough neutrino energies so that neutrinos remain massless. One can even consider the possibility that the p-adic length scale L_p is proportional to E/m_0^2, where m_0 is proportional to the mass scale associated with non-relativistic neutrinos. The p-adic mass scale would obey $m_p \propto m_0^2/E$ so that the characteristic mixing length would be by a factor of order 100 longer in MiniBooNE experiment than in LSND.

4.2.6 Comments

Some comments on the proposed scenario are in order: some of the are written much later than the previous text.

1. Mass predictions are consistent with the bound $\Delta m(\nu_\mu, \nu_e) < 2\ eV^2$ coming from the requirement that neutrino mixing does not spoil the so called r-process producing heavy elements in Super Novae [28].

2. TGD neutrinos cannot solve the dark matter problem: the total neutrino mass required by the cold+hot dark matter models would be about 5 eV. In [D4] a model of galaxies based on string like objects of galaxy size and providing a more exotic source of dark matter, is discussed.

3. One could also consider the explanation of LSND data in terms of the interaction of ν_μ and nucleon via the exchange of $g=1$ W boson. The fraction of the reactions $\bar{\nu}_\mu + p \rightarrow e^+ + n$ is at low neutrino energies $P \sim \frac{m_W^4(g=0)}{m_W^4(g=1)} sin^2(\theta_c)$, where θ_c denotes Cabibbo angle. Even if the condensation level of $W(g=1)$ is $k=89$, the ratio is by a factor of order .05 too small to explain the average $\nu_\mu \rightarrow \nu_e$ transformation probability $P \simeq .003$ extracted from LSND data.

4. The predicted masses exclude MSW and vacuum oscillation solutions to the solar neutrino problem unless one assumes that several condensation levels and thus mass scales are possible for neutrinos. This is indeed suggested by the previous considerations.

4.3 Quark masses

The prediction or quark masses is more difficult due the facts that the deduction of even the p-adic length scale determining the masses of these quarks is a non-trivial task, and the original identification was indeed wrong. Second difficulty is related to the topological mixing of quarks. The new scenario leads to a unique identification of masses with top quark mass as an empirical input and the thermodynamical model of topological mixing as a new theoretical input. Also CKM matrix is predicted highly uniquely.

4.3.1 Basic mass formulas

By the earlier mass calculations and construction of CKM matrix the ground state conformal weights of U and D type quarks must be $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$. The formulas for the eigenvalues of CP_2 spinor Laplacian imply that if m_0^2 is used as a unit, color conformal weight $h_c \equiv m_{CP_2}^2$ is integer for $p \ mod \ = \pm 1$ for U type quark belonging to $(p+1,p)$ type representation and obeying $h_c(U) = (p^2+3p+2)/3$ and for $p \ mod \ 3 = 1$ for D type quark belonging $(p,p+2)$ type representation and obeying $h_c(D) = (p^2+4p+4)/3$. Only these states can be massless since color Hamiltonians have integer valued conformal weights.

In the recent case the minimal $p=1$ states correspond to $h_c(U)=2$ and $h_c(D)=3$. $h_{gr}(U) = -1$ and $h_{gr}(D)=0$ reproduce the previous results for quark masses required by the construction of CKM matrix. This requires super-canonical operators O with a net conformal weight $h_{sc} = -3$ just as in the leptonic case. The facts that the values of p are minimal for spinor harmonics and the super-canonical operator is same for both quarks and leptons suggest that the construction is not had hoc. The real justification would come from the demonstration that $h_{sc} = -3$ defines null state for SCV: this would also explain why h_{sc} would be same for all fermions.

Consider now the mass squared values for quarks. For $h(D)=0$ and $h(U)=-1$ and using $m_0^2/3$ as a unit the expression for the thermal contribution to the mass squared of quark is given by the formula

$$\begin{aligned} M^2 &= (s+X)\frac{m_0^2}{p} \ , \\ s(U) &= 5 \ , \ s(D) = 8 \ , \\ X &\equiv \frac{(3Yp)_R}{3} \ , \end{aligned} \tag{20}$$

where the second order contribution Y corresponds to renormalization effects coming and depending on the isospin of the quark. When m_0^2 is used as a unit X is replaced by $X = (Y_p)_R$.

With the above described assumptions one has the following mass formula for quarks

$$M^2(q) = A(q)\frac{m_0^2}{p(q)} \ ,$$

$$\begin{array}{lll} A(u) = 5 + X_U(p(u) \ , & A(c) = 14 + X_U(p(c)) \ , & A(t) = 65 + X_U(p(t)) \ , \\ A(d) = 8 + X_D(p(d)) \ , & A(s) = 17 + X_D(p(s)) \ , & A(b) = 68 + X_D(p(b)) \ . \end{array} \tag{21}$$

p-Adic length scale hypothesis allows to identify the p-adic primes labelling quarks whereas topological mixing of U and D quarks allows to deduce topological mixing matrices U and D and CKM matrix V and precise values of the masses apart from effects like color magnetic spin orbit splitting, color Coulombic energy, etc..

Integers n_{q_i} satisfying $\sum_i n(U_i) = \sum_i n(D_i) = 69$ characterize the masses of the quarks and also the topological mixing to high degree. The reason that modular contributions remain integers is that in the p-adic context non-trivial rationals would give CP_2 mass scale for the real counterpart of the mass squared. In the absence of mixing the values of integers are $n_d = n_u = 0$, $n_s = n_c = 9$, $n_b = n_t = 60$.

The fact that CKM matrix V expressible as a product $V = U^\dagger D$ of topological mixing matrices is near to a direct sum of 2×2 unit matrix and 1×1 unit matrix motivates the approximation $n_b \simeq n_t$. The large masses of top quark and of $t\bar{t}$ meson encourage to consider a scenario in which $n_t = n_b = n \leq 60$ holds true.

The model for topological mixing matrices and CKM matrix predicts U and D matrices highly uniquely and allows to understand quark and hadron masses in surprisingly detailed level.

1. $n_d = n_u = 60$ is not allowed by number theoretical conditions for U and D matrices and by the basic facts about CKM matrix but $n_t = n_b = 59$ allows almost maximal masses for b and t. This is not yet a complete hit. The unitarity of the mixing matrices and the construction of CKM matrix to be discussed in the next section forces the assignments

$$(n_d, n_s, n_b) = (5, 5, 59) \ , \quad (n_u, n_c, n_t) = (5, 6, 58) \ . \tag{22}$$

   fixing completely the quark masses apart possible Higgs contribution [F4]. Note that top quark mass is still rather near to its maximal value.

2. The constraint that valence quark contribution to pion mass does not exceed pion mass implies the constraint $n(d) \leq 6$ and $n(u) \leq 6$ in accordance with the predictions of the model of topological mixing. $u - d$ mass difference does not affect $\pi^+ - \pi^0$ mass difference and the quark contribution to $m(\pi)$ is predicted to be $\sqrt{(n_d + n_u + 13)/24} \times 136.9$ MeV for the maximal value of CP_2 mass (second order p-adic contribution to electron mass squared vanishes).

4.3.2 The p-adic length scales associated with quarks and quark masses

The identification of p-adic length scales associated with the quarks has turned to be a highly non-trivial problem. The reasons are that for light quarks it is difficult to deduce information about quark masses for hadron masses and that the unknown details of the topological mixing (unknown until the advent of the thermodynamical model [F4]) made possible several p-adic length scales for quarks. It has also become clear that the p-adic length scale can be different form free quark and bound quark and that bound quark p-adic scale can depend on hadron.

Two natural constraints have however emerged from the recent work.

1. Quark contribution to the hadron mass cannot be larger than color contribution and for quarks having $k_q \neq 107$ quark contribution to mass is added to color contribution to the mass. For quarks with same value of k conformal weight rather than mass is additive whereas for quarks with different value of k masses are additive. An important implication is that for diagonal mesons $M = q\overline{q}$ having $k(q) \neq 107$ the condition $m(M) \geq \sqrt{2}m_q$ must hold true. This gives strong constraints on quark masses.

2. The realization that scaled up variants of quarks explain elegantly the masses of light hadrons allows to understand large mass splittings of light hadrons without the introduction of strong isospin-isospin interaction.

The new model for quark masses is based on the following identifications of the p-adic length scales.

1. The nuclear p-adic length scale $L(k)$, $k = 113$, corresponds to the p-adic length scale determining the masses of u, d, and s quarks. Note that $k = 113$ corresponds to a so called Gaussian Mersenne. The interpretation is that quark massivation occurs at nuclear space-time sheet at which quarks feed their em fluxes. At $k = 107$ space-time sheet, where quarks feed their color gauge fluxes, the quark masses are vanishing in the first p-adic order. This could be due to the fact that the p-adic temperature is $T_p = 1/2$ at this space-time sheet so that the thermal contribution to the mass squared is negligible. This would reflect the fact that color interactions do not involve any counterpart of Higgs mechanism.

   p-Adic mass calculations turn out to work remarkably well for massive quarks. The reason could be that M_{107} hadron physics means that *all* quarks feed their color gauge fluxes to $k = 107$ space-time sheets so that color contribution to the masses becomes negligible for heavy quarks as compared to Super-Kac Moody and modular contributions corresponding to em gauge flux feeded to $k > 107$ space-time sheets in case of heavy quarks. Note that Z^0 gauge flux is feeded to space-time sheets at which neutrinos reside and screen the flux and their size corresponds to the neutrino mass scale. This picture might throw some light to the question of whether and how it might be possible to demonstrate the existence of M_{89} hadron physics.

   One might argue that $k = 107$ is not allowed as a condensation level in accordance with the idea that color and electro-weak gauge fluxes cannot be feeded at the space-time space time sheet since the classical color and electro-weak fields are functionally independent. The identification of η' meson as a bound state of scaled up $k = 107$ quarks is not however consistent with this idea unless one assumes that $k = 107$ space-time sheets in question are separate.

2. The requirement that the masses of diagonal pseudoscalar mesons of type $M = q\overline{q}$ are larger but as near as possible to the quark contribution $\sqrt{2}m_q$ to the valence quark mass, fixes the p-adic primes $p \simeq 2^k$ associated with c, b quarks but not t since toponium does not exist. These values of k are "nominal" since k seems to be dynamical. c quark corresponds to the p-adic length scale $k(c) = 104 = 2^3 \times 13$. b quark corresponds to $k(b) = 103$ for $n(b) = 5$. Direct determination of p-adic scale from top quark mass gives $k(t) = 94 = 2 \times 47$ so that secondary p-adic length scale is in question*[1].

   Top quark mass tends to be slightly too low as compared to the most recent experimental value of $m(t) = 169.1$ GeV with the allowed range being [164.7, 175.5] GeV [50]. The optimal situation corresponds to $Y_e = 0$ and $Y_t = 1$ and happens to give top mass exactly equal to the

[1] Earlier calculation contained a stupid error: only modular contribution to $m(t)$ had been taken into account, which led to the erratic identification $k(t) = 93$ and difficulties in understanding top quark mass

most probable experimental value. It must be emphasized that top quark is experimentally in a unique position since toponium does not exist and top quark mass is that of free top.

In the case of light quarks there are good reasons to believe that the p-adic mass scale of quark is different for free quark and bound state quark and that in case of bound quark it can also depend on hadron. This would explain the notions of valence (constituent) quark and current quark mass as masses of bound state quark and free quark and leads also to a TGD counterpart of Gell-Mann-Okubo mass formula [F4].

1. Constituent quark masses

Constituent quark masses correspond to masses derived assuming that they are bound to hadrons. If the value of k is assumed to depend on hadron one obtains nice mass formula for light hadrons as will be found later. The following table summarizes constituent quark masses as predicted by this model.

q	d	u	s	c	b	t
n_q	4	5	6	6	59	58
s_q	12	10	14	11	67	63
$k(q)$	113	113	113	104	103	94
$m(q)/GeV$	.105	.092	.105	2.191	7.647	167.8

Table 2. Constituent quark masses predicted for diagonal mesons assuming $(n_d, n_s, n_b) = (5, 5, 59)$ and $(n_u, n_c, n_t) = (5, 6, 58)$, maximal CP_2 mass scale($Y_e = 0$), and vanishing of second order contributions.

2. Current quark masses

Current quark masses would correspond to masses of free quarks which tend to be lower than valence quark masses. Hence k could be larger in the case of light quarks. The table of quark masses in Wikipedia [51] gives the value ranges for current quark masses depicted in the table below together with TGD predictions for the spectrum of current quark masses.

q	d	u	s
$m(q)_{exp}/MeV$	4-8	1.5-4	80-130
$k(q)$	(122,121,120)	(125,124,123,122)	(114,113,112)
$m(q)/MeV$	(4.5,6.6,9.3)	(1.4,2.0,2.9,4.1)	(74,105,149)
q	c	b	t
$m(q)_{exp}/MeV$	1150-1350	4100-4400	1691
$k(q)$	(106,105)	(105,104)	92
$m(q)/MeV$	(1045,1477)	(3823,5407)	167.8

Table 3. The experimental value ranges for current quark masses [51] and TGD predictions for their values assuming $(n_d, n_s, n_b) = (5, 5, 59)$, $(n_u, n_c, n_t) = (5, 6, 58)$, and $Y_e = 0$.

Some comments are in order.

1. The long p-adic length associated with light quarks seem to be in conflict with the idea that quarks have sizes smaller than hadron size. The paradox disappears when one realized that $k(q)$ characterizes the electromagnetic "field body" of quark having much larger size than hadron.
2. u and d current quarks correspond to a mass scale not much higher than that of electron and the ranges for mass estimates suggest that u could correspond to scales $k(u) \in$

$(125, 124, 123, 122) = (5^3, 4 \times 31, 3 \times 41, 2 \times 61)$, whereas d would correspond to $k(d) \in (122, 121, 120) = (2 \times 61, 11^2, 3 \times 5 \times 8)$.

3. The TGD based model for nuclei based on the notion of nuclear string leads to the conclusion that exotic copies of $k = 113$ quarks having $k = 127$ are present in nuclei and are responsible for the color binding of nuclei [F8, F9].

4. The predicted values for c and b masses are slightly too low for $(k(c), k(b)) = (106, 105) = (2 \times 53, 3 \times 5 \times 7)$. Second order Higgs contribution could increase the c mass into the range given in [51] but not that of b.

4.3.3 Can Higgs field develop a vacuum expectation in fermionic sector at all?

An important conclusion following from the calculation of lepton and quark masses is that if Higgs contribution is present, it can be of second order p-adically and even negligible, perhaps even vanishing. There is indeed an argument forcing to consider this possibility seriously. The recent view about elementary particles is following.

1. Fermions correspond to CP_2 type vacuum extremals topologically condensed at positive/negative energy space-time sheets carrying quantum numbers at light-like wormhole throat. Higgs and gauge bosons correspond to wormhole contacts connecting positive and negative energy space-time sheets and carrying fermion and anti-fermion quantum numbers at the two light-like wormhole throats.

2. If the values of p-adic temperature are $T_p = 1$ and $T_p = 1/26$ for fermions and bosons, one can understand relate both the value of the Kähler coupling strength and p-adic temperature to the integer valued parameter k characterizing Chern-Simons action defining partonic dynamics as almost topological QFT [C3]. The basic implication is that the thermodynamical contribution to the gauge boson mass is completely negligible.

3. Different p-adic temperatures and Kähler coupling strengths for fermions and bosons make sense if bosonic and fermionic partonic 3-surfaces meet only along their ends at the vertices of generalized Feynman diagrams but have no other common points [C3]. This forces to consider the possibility that fermions cannot develop Higgs vacuum expectation value although they can couple to Higgs. This is not in contradiction with the modification of sigma model of hadrons based on the assumption that vacuum expectation of σ field gives a small contribution to hadron mass [F5] since this field can be assigned to some bosonic space-time sheet pair associated with hadron.

4.4 Photon, graviton and gluon

The only possibility to get massless states is to have $\Delta = 0$ state with one active sector so that NS thermodynamics becomes trivial due to the absence of the thermodynamical excitations satisfying the gauge conditions. The model for the Weinberg mixing suggests that a second order non-thermal contribution to the mass squared is present in case of photon. Also in case of the gluon this contribution is expected to be present. For graviton the contribution should be extremely small or even higher order in p.

4.5 Higgs mechanism for electro-weak gauge bosons

Contrary to the original beliefs, it seems that p-adic thermodynamics alone cannot explain boson masses. The problem is that the lower limit for W mass is 20-30 per cent higher than the experimental value of the W mass squared for $T_p = 1$. For $T_p = 1/2$ thermal masses are completely negligible. Rather ironically, bosonic masses are the Achilles's heel of p-adic thermodynamics whereas fermionic masses are the Achilles's heel of standard model. One can consider two solutions to the problem.

1. The first option is based on the combination of p-adic thermodynamics and Higgs mechanism. The identification of Higgs boson as a weakly charged wormhole contact provides a strong support for this approach. The vacuum expectation value of Higgs field would correspond to the generation of a coherent state of wormhole contacts. $T_p = 1/26$ for gauge bosons [C5] implies that thermodynamical contribution is completely negligible so that Higgs coupling would determined completely boson mass whereas fermion masses would be determined completely by thermodynamics.

2. Second option is based on the idea that the non-covariant constancy of electro-weak charge matrices except that of photon is responsible for the Higgs mechanism.

4.5.1 p-Adic thermodynamics does not explain intermediate gauge boson masses

For photon, gluon and graviton the conformal weight of the $p = 0$ ground state is $h_{gr} = h_{vac}$. The crucial condition is that $h = 0$ ground state is non-degenerate: otherwise one would obtain several physically more or less identical photons and this would be seen in the spectrum of black body radiation. This occurs if one can construct several ground states not expressible in terms of the action of the Super Virasoro generators.

The requirement that the electron-intermediate gauge boson mass ratios are sensible, serves as a stringent test for the hypothesis that intermediate gauge boson masses result from the p-adic thermodynamics. Contrary to the original beliefs, it seems however impossible to predict W/e and Z/e mass ratios correctly in p-adic thermodynamics scenario. Although the errors are of order ten percent, they seem to be enough to exclude p-adic thermodynamics explanation for the massivation of gauge bosons.

1. The thermal mass squared for a boson state with N active sectors (non-vanishing vacuum weight) is determined by the partition function for the tensor product of N NS type Super Virasoro algebras. The degeneracies of the excited states as a function of N and the weight Δ of the operator creating the massless state are given in the table below.

2. Both W and Z must correspond to $N = 2$ active sectors for which $D(1) = 1$ and $D(2) = 3$ so that (using the formulas of p-adic thermodynamics) the thermal mass squared is $m^2 = (p + 5p^2)$ for $T_p = 1$. The second order contribution to the thermal mass squared is extremely small so that Weinberg angle vanishes in the thermal approximation. Z/e mass-ratio is predicted to be about 22 per cent too high. The thermal prediction for W-boson mass is the same as for Z^0 mass and thus even worse since the two masses should be related $M_W^2 = M_Z^2 cos^2(\theta_W)$. For $T_p = 1/2$ thermal masses are completely negligible. Of course, thermodynamics does not provide any obvious counterpart for the group theory based explanation of Z/W mass ratio.

3. An important question concerns the mass squared unit. $m_0^3/3$ seems to correspond to m_W^2 but lepton masses favor m_0^2.

4. It seems that the Achilles's heel of the p-adic thermodynamics is bosonic sector whereas the weak point of the standard model is fermionic sector. This suggests that these two approaches should be combined.

N, Δ	0	1/2	1	3/2	2	5/2	3
2	1	1	1	3	3	4	4
3	1	2	3	9	11		
4	1	3	5	19	26		
5	1	4	10	24	150		

Table 4. Degeneracies $d(\Delta, N)$ of the operators satisfying NS type gauge conditions as a function of the number N of the active sectors and of the conformal weight Δ of the operator. Only those degeneracies, which are needed in the mass calculation of light bosons are listed.

4.5.2 p-Adic thermodynamics is not consistent with the group theoretical understanding of m_Z/m_W mass ratio

p-Adic thermodynamics approach should be consistent with the group theoretical description of m_Z/m_W ratio. There is no obvious reason why this ratio should be group theoretical if p-adic thermodynamics contribution dominates. Even if it were, the mapping of p-adic mass squared to its real counterpart by the standard form of the canonical identification does not respect algebraic structures, which forms the core of group theoretical thinking. The problem is much more general. For instance, the unitarity of CKM matrix is not respected by canonical identification in its standard form. Also the sensible correspondence between real and p-adic coupling constants requires that algebraic structures should be preserved by the canonical identification. One could also argue that the map of p-adic probabilities to their real counterparts should respect probability conservation.

The relationship between p-adic and real scattering amplitudes discussed in [F5] encourages the conclusion that canonical identification Id at the level of the scattering amplitudes should be replaced with its variant in which rational r/s is mapped to $Id(r)/Id(p)$ rather than $Id(r/p)$, where $r(p$ is expanded in powers of $1/p$. This map is unique, when r and p are chosen to be mutually prime. The basic reason is that this correspondence respects the unitarity of CKM matrix appearing in S-matrix elements under some additional rather natural conditions.

This form of canonical identification can be modified further in p-adic thermodynamics where p-adic probabilities as a function of conformal weight n are of form $P(n) = g(n)p^n/Z$ by defining the identification as $I(P(n)) = I(g(n)p^n)/I(Z)$ so that the sum of real counterparts of p-adic probabilities equals to one without further normalization for $g(n) < p$. This number theoretic condition is extremely restrictive since the p-adic Boltzmann weights for independent events cannot have common pinary digits in their pinary expansion. In the recent case the condition $g(n) < p$ fails for sufficiently high values of n since $g(n)$ increases exponentially. The assumption about a physical cutoff in n has no physical implications since the values of p are so large. The cutoff is of order $n \sim k$ for $p \simeq 2^k$ and has interpretation in terms of the p-adic length scale L_k assignable to the elementary particle horizon.

If this variant of canonical identification is applied to map the mass squared values to their real counterparts some essential changes occur although the predictions in the fermionic sector are not changed in any essential manner [F5]. The crucial implication is that the algebraic relationship between W and Z masses might be described in this framework if $sin^2(\theta_w)$ corresponds to a ratio of small integers. The basic implications of the modified view about canonical identification are following.

1. Second order contribution to the mass squared is always negligible unless it is of form $(p-k)p^2$, where k is a reasonable fraction of p or $k << p$ holds true. The large second order contributions to charged lepton masses must be of this form. Second order thermodynamical contribution to the neutrino mass squared is negligible. Also intermediate gauge boson masses could correspond to the second order contribution

$$m_Z^2 = (p-k_Z)p^2 \ , \qquad m_W^2 = (p-k_W)p \ .$$
$$p-k_Z = \left[cos^2(\theta_W)p\right] \ , \quad p-k_W = \left[cos^4(\theta_W)p\right] \ . \tag{23}$$

Here $[x]$ denotes integer part of x. The formulas are tailored to explain also why Z/e ratio is by a factor $cos^2(\theta_W)$ smaller than for $m_Z^2 = p$. What does not look nice is that $sin^2(\theta_W)$ does not appear as a genuine rational number in the formula as its coupling constant character would suggest.

2. First order contribution to the mass squared can be of form $(m/n) \times p$ since the real counterpart is $(m/n) \times (1/p)$ rather than being of order CP_2 mass squared for the ordinary form of canonical identification. The fact that intermediate gauge boson mass squared is smaller than the natural unit requires a ground state degeneracy or that the source of mass is not thermodynamical. The simplest manner to understand gauge boson masses is to assume

$$m_Z^2 = cos^2(\theta_W)p \ , \quad m_W^2 = cos^4(\theta_W)p \ . \tag{24}$$

Now $sin^2(\theta_W)$ is a genuine rational number. The basic assumption of the successful model for topological mixing of quarks [F4] is that the modular contribution to the masses is of form np. This assumption loses its original justification for this option and some other justification for its should be found. The first guess is that the conditions on mass squared plus probability conservation might not be consistent with unitarity unless the modular contribution to the mass squared remains integer valued in the mixing (note that all integer values are not possible [F4]). Direct numerical experimentation however shows that that this is not the case. The conclusion is that p-adic thermodynamics cannot explain the group theoretic character of the mass ratio for intermediate gauge bosons.

4.5.3 Group theoretical evidence for the existence of Higgs boson

The hint for the existence of Higgs boson comes from the identification of electro-weak super-Kac Moody algebra. The identification relies on the observation that for the known solutions of field equations, field equations in CP_2 degrees of freedom separate into two separate equations corresponding to the variations with respect to induced metric and induced Kähler form. This implies that $SU(3)$ isometry charges decompose to sums of charges which are conserved separately. Color charges correspond to the conserved current associated with the variations of the induced metric. Since $U(2)_{ew}$ can be identified as the $U(2)$ subgroup of $SU(3)$ one can identify the $U(2)$ charges associated with the variations of the induced Kähler form. Thus one has beautiful identification of the basic Kac-Moody algebra structures associated with color and electro-weak sectors.

One should however find interpretation also for the $SU(3)$ charges Q_J associated with the complement of $U(2)$ in $SU(3)$ algebra associated with the variations with respect to the induced Kähler form. These Lie-algebra generators provide standard coordinatization of CP_2 in symmetric space decomposition $su(3) = h + t = u(2) + t$. The interpretation of $h = u(2)$ as electro-weak charges means symmetry breaking so that the charges in t do not form usual Kac Moody algebra acting as pure gauge symmetries but create genuine physical states. This is completely analogous to what happens at the level of CP_2 geometry: h corresponds to gauge degrees of freedom and t to CP_2 coordinates and thus to genuine dynamical degrees of freedom. The charges in t transform as two electro-weak isospin doublets with a non-vanishing color hypercharge. Thus the couplings to electro-weak gauge bosons and fermions are the same as associated with the ordinary electro-weak Higgs field. The natural conclusion is that the particles described by Kac Moody generators in t are nothing but the Higgs particles.

This suggests that Higgs mechanism could correspond to the generation of coherent states in the neutral degrees of freedom associated with t. In fact, the direction of electromagnetic charge can be defined by requiring that the vacuum expectation value of t is annihilated by em charge. What is interesting is that the components of the vacuum expectation value associated with the coherent state could be proportional to the values of the CP_2 coordinates of a space-time sheet at which particles are topologically condensed. Thus the coherent state associated might provide a representation for slowly varying CP_2 coordinates of the space-time surface (in p-adic context this representation would be cognitive representation!).

This mechanism is not in conflict with the thermal massivation of fermions if the vacuum expectation is of order $O(p^2)$: the coupling to the Higgs would only induce relatively small shifts of the fermionic masses. As matter fact, the argument of [C5] predicting $T_p = 1/26$ for gauge bosons would mean thermal contribution to boson mass completely negligible. This model predicts a strict upper bound for the induced mass consistent with the observed masses which are below the maximum mass resulting from p-adic thermodynamics with temperature $T_p = 1$.

4.5.4 Higgs boson as a wormhole contact

Quantum classical correspondence requires that Higgs boson should have a concrete space-time correlate. This correlate can be identified as a wormhole contact carrying lefthanded weak charge. The quantum of Higgs field could be visualized in terms of a wormhole contact connecting the positive and negative energy space-time sheets connected also by gauge boson wormhole contact. For fermions this kind situation would not be possible so that Higgs vacuum expectation would not be possible. In zero energy ontology [C3] the coherent state of Higgs field would be a zero energy state identifiable as a superposition of different numbers of Higgs quanta.

4.5.5 Why Higgs is not detected?

Both options could explain naturally the failure to detect Higgs. Various contributions to Higgs production are discussed in [29, 30]. The basic contributions to the production of Higgs bosons in p-p collisions at LHC corresponds to gluon fusion, associated production, and vector boson fusion. Various production cross sections for $p-p$ collisions at cm energy of $\sqrt{s} = 14$ TeV are given in [29], see also the figures of [30]. The dominating contribution corresponds to the triangle diagram $gg \rightarrow q\overline{q} \rightarrow H$. Since the coupling of quarks to Higgs in TGD can be much smaller than in standard model, this contribution can be very small in TGD framework. Also the rates for direct annihilations $q\overline{q} \rightarrow H$ are small for the same reason.

The rates for the vector boson fusion and associated production are in the lowest order same in TGD as in the standard model. Vector boson fusion corresponds to the scattering of quarks via the exchange of W/Z boson coupling to Higgs ($q\overline{q} \rightarrow q\overline{q}H$). The rate for this process is roughly one hundred times lower than for the associated production. Associated production corresponds to the diagram $q\overline{q} \rightarrow W \rightarrow W + H$. The rate is below the rate of the vector boson fusion if Higgs mass is above ~ 100 GeV: on basis recent searches the Higgs mass is known to be in the range $114.4 - 237$ GeV [30].

It seems safe to conclude that TGD predicts Higgs particle. The fact that the rate of Higgs production can be about 100 times lower than in standard model and even this could easily explain the unsuccessful search for Higgs.

4.5.6 Higgs mass determination from high precision electro-weak observables

Higgs mass can be estimated from the measured values of electro-weak high precision electro-weak observables. The values of these observables can be deduced from fermion-antifermions scattering at Z^0 resonance [45]. The dependence on Higgs mass comes from radiative corrections involving the coupling of Higgs to the fundamental fermions and gauge bosons. The radiative corrections affect the couplings of gauge bosons to fundamental fermions and introduce renormalization corrections to gauge boson masses and decay widths. Hence one can deduce Higgs mass in several independent manners and at the same time test the internal consistency of the theory. The variation of the values of observables is surprisingly wide: roughly an order of magnitude variation appears [46].

The dependence of the loop corrections on Higgs mass is logarithmic and this together with experimental uncertainties could explain the great variation. One could also ask whether this finding could be seen as an evidence for small couplings of fundamental fermions to Higgs so that $h - f - \overline{f}$ contributions to radiative corrections would effectively vanish and only boson-Higgs couplings contribute significantly. This is indeed allowed by TGD where fermionic masses come from p-adic thermodynamics rather than coupling to Higgs vacuum expectation.

Unfortunately this idea does not work as the detailed discussion of high precision electro-weak observables in fermion-antifermion scattering at Z^0 resonance pole can be found in [45] shows. The point is that already in standard model fermion-Higgs type contributions to radiative corrections are very small except for top quark since the contribution of $hf\overline{f}$ vertex in the loop is proportional to the fermion mass. Hence the radiative corrections from the couplings of gauge bosons to Higgs appearing in the boson propagators dominate. For $t\bar{t}$ scattering left-right asymmetry due to $\gamma - Z^0$ interference and forward-backward asymmetry involve sizable contributions from Higgs exchange and in principle could be used to distinguish between TGD and standard model. In practice this is not possible.

4.5.7 Constraints on Higgs mass from the evolution of Higgs self coupling

The constraints on the coupling constant evolution of λ give constraints on the Higgs mass. There are two competing effects. Quartic self coupling tends to increase λ and if it dominates it gives rise to a logarithmic behavior leading to large values of λ in the ultraviolet [43]. This situation prevails provided some critical value of λ can be reached since other couplings tend to slow down the growth of λ. An alterative option is that the Yukawa coupling to top quark wins and λ becomes very small and even changes sign. Coupling constant evolution can also induce the change of minimum of Higgs potential to a maximum.

1. Upper bound on Higgs mass from perturbative unitarity

An upper bound on Higgs mass comes from the requirement that perturbative unitarity is not lost in the energy range considered characterized by the value Λ of UV cutoff. The loss of perturbative unitary would have interpretation in terms of new physics above Λ. This requires that the initial value of λ cannot be too high.

1. The upper bound for $\lambda(t_Z)$ at intermediate gauge boson mass using the basic formula in terms of vacuum expectation value and λ: $m_H^2 = 2\lambda v^2$. Here $v = \sqrt{\sqrt{2}G_F} = 247$ GeV is fixed from the intermediate boson mass scale and therefore genuine upper bound results. $\lambda = .2098$ for $m_H = 160$ GeV makes sense at this energy.

2. If the initial value of λ, or equivalently Higgs boson mass, is too large, λ starts to grow leading to a loss of perturbative unitarity at some energy. The requirement that this does not occur below Λ defining the mass scale scale for the new physics gives an upper bound on Higgs mass. For instance, if the new physics is not allowed below GUT mass scale 10^{16} GeV, one obtains the upper bound $m_H < 153$ GeV [43]. The counterpart for GUT length scale is CP_2 size and corresponds to energy of $M \sim 10^{-4} M_{Planck} \sim 10^{15}$ GeV.

2. Standard model lower bound on Higgs mass from coupling constant evolution of λ

The natural conditions are that λ stays positive and that the extremum of the effective potential $V(H(t))$ does not transform from minimum to a maximum. The large coupling to top quark tends to reduce λ and the latter condition gives a lower bound on the low energy value of λ and thus to Higgs mass. For instance, according to some estimates $\Lambda_{GUT} \simeq 10^{16}$ GeV restricts the Higgs mass in the range 130-190 GeV, which does not have overlap with the mass range allowed by the range allowed by the best fit using high precision estimates for electro-weak parameters.

4.5.8 An estimate for Higgs mass

In standard model Higgs and W boson masses are given by

$$\begin{aligned} m_H^2 &= 2v^2\lambda = \mu^2\lambda^3 \ , \\ m_W^2 &= \frac{g^2v^2}{4} = \frac{e^2}{8sin^2(\theta_W)}\mu^2\lambda^2 \ . \end{aligned} \tag{25}$$

This gives

$$\lambda = \frac{4\pi}{8\alpha_{em}sin^2(\theta_W)}(\frac{m_H}{m_W})^2 \ . \tag{26}$$

In standard model one cannot predict the value of m_H.

In TGD framework one can try to understand Higgs mass from p-adic thermodynamics as resulting via the same mechanism as fermion masses so that the value of the parameter λ would follow as a prediction.

One must assume that p-adic temperature equals to $T_p = 1$. The natural assumption is that Higgs can be regarded as superposition of pairs of fermion and anti-fermion at opposite throats of wormhole contact. With these assumptions the thermal expectation of the Higgs conformal weight is just the sum of contributions from both throats and *two* times the average of the conformal weight over that for quarks and leptons (one might question the presence of factor 2):

$$\begin{aligned} s_H &= 2\times\langle s\rangle = 2\times\frac{\left[\sum_q s_q + \sum_L s_L\right]}{N_q+N_L} \\ &= 2\frac{\sum_{g=0}^2 s_{mod}(g)}{3} + \frac{(s_L+s_{\nu_L}+s_U+s_D)}{2} \\ &= 26 + \frac{5+4+5+8}{2} = 37 \ . \end{aligned} \tag{27}$$

A couple of comments about the formula are in order.

1. The first term - two times the average of the genus dependent modular contribution to the conformal weight - equals to 26, and comes from modular degrees of freedom and does not depend on the charge of fermion.

2. The contribution of p-adic thermodynamics for super-conformal generators gives same contribution for all fermion families and depends on the em charge of fermion. The values of thermal conformal weights deduced earlier have been used. Note that only the value $s_{\nu_L} = 4$ (also $s_{\nu_L} = 5$ could be considered. This is possible if one requires that the conformal weight is integer. If the standard form of the canonical identification mapping p-adics to reals is used, this must be the case since otherwise real mass would be super-heavy.

The first guess would be that the p-adic length scale associated with Higgs boson is M_{89}. Second option is $p \simeq 2^k$, $k = 97$ (restricting k to be prime). If one allows k to be non-prime (these values of k are also realized. One can consider also $k = 91 = 7 \times 13$. By scaling from the expression for the electron mass, one obtains the estimates

$$\begin{aligned} m_H(89) &\simeq \sqrt{\frac{37}{5}} \times 2^{21} m_e \simeq 727.3 \ \ GeV \ \ , \\ m_H(91) &\simeq \sqrt{\frac{37}{5}} \times 2^{18} m_e \simeq 363.5 GeV \ \ , \\ m_H(97) &\simeq \sqrt{\frac{37}{5}} \times 2^{15} m_e \simeq 45.5 \ \ GeV \ \ . \end{aligned} \quad (28)$$

A couple of comments are in order.

1. From [46] one learns that the latest estimates for Higgs mass give two widely different values, namely $m_H = 31^{33}_{-19}$ GeV and $m_H = 420^{+420}_{-190}$ GeV. Since the p-adic mass scale of both neutrinos and quarks and possibly even electron can vary in TGD framework, one cannot avoid the question whether - depending on experimental situation- Higgs could appear in two different mass scales corresponding to $k = 91$ and 97.

2. The low value of $m_H(97)$ might be consistent with experimental facts since the couplings of fermions to Higgs can in TGD framework be weaker than in standard model.

The value of λ is given in the three cases given by

$$\lambda(89) \simeq 4.41 \ , \ \lambda(91) = 1.10 \ \ , \ \lambda(97) = .2757 \ . \quad (29)$$

Unitarity would thus favor $k = 97$ and $k = 91$ also favored by the high precision data and $k = 91$ is just at the unitarity bound $\lambda = 1$ (here I am perhaps naive!). A possible interpretation is that for M_{89} Higgs mass forces λ to break unitarity bound and that this corresponds to the emergence of M_{89} copy of hadron physics.

In August 2008 some fresh information about Higgs mass emerged.

1. A press release from Tevatron [52] excluded the possibility that the mass is in a narrow interval around 170 GeV, roughly the average of the above mentioned mass values. Ironically, this mass value corresponds exactly to the Higgs mass predicted by the non-commutative variant of standard model of Alain Connes [53].

2. The second piece of information [54] discussed in detail in Tommaso Dorigo's blog [55] gives much stronger limits on Higgs mass. The first plot discussed in Tommaso's blog is obtained by combining enormous amount of information except that coming from LEPII and Tevatron and at 1 sigma limit bounds Higgs mass to the interval 57-100 GeV with favored value around 80 GeV. At 2 sigma the interval is 39-156 GeV. In TGD framework $k = 96$ would predict mass 91 GeV which is near the upper bound of the 1 sigma range 57-100 GeV. $k = 97$ would predict mass 45.5 GeV belonging to the lower boundary of the 2 sigma range.

3. If one includes also the information from LEPII and Tevatron the mass range 115-135 GeV [54]. TGD would predict mass 129 GeV for k=94 which is near the upper end of the allowed interval 115-135 GeV. If these limits are taken absolutely seriously, one can say that TGD is able to predict correctly also Higgs mass. Recalling that the prediction is exponentially

sensitive to the value of the integer k, this could be regarded as as triumph of TGD. The reported results are however consistent with the proposal that Higgs appears with at least two different mass values. All these mass values and even others could be there depending on experimental conditions.

4.6 Has Higgs been detected?

Un the beginning of year 2007 there were cautious claims [37, 47, 49] about the possible detection of first Higgs events. Before the end of the year the indications about Higgs events had suffered the usual fate of a statistical fluke.

These speculations however inspired more precise considerations of the experimental signatures of TGD counterpart of Higgs. This kind of theorizing is of course speculative and remains on general qualitative level only since no calculational formalism exists and one must assume that gauge field theory provides an approximate description of the situation. I leave it for the reader to decided whether to skip over this subsection.

4.6.1 Indications for Higgs

The indications for Higgs comes from two sources [37, 47, 49]. In both cases Higgs would have been produced as gluons decay to two $b\bar{b}$ pairs and virtual $b\bar{b}$ pair fuses to Higgs, which then decays either to τ lepton pair or b quark pair.

John Conway, the leader of CDF team analyzing data from Tevatron, has reported about a slight indication for Higgs with mass $m_H = 160$ GeV as a small excess of events in the large bump produced by the decays of Z^0 bosons with mass of $m_Z \simeq 94$ GeV to $\tau\overline{\tau}$ pairs in the blog Cosmic Variance [47]. These events have 2σ significance level meaning that the probability that they are statistical fluctuations is about 2 per cent.

The interpretation suggested by Conway is as Higgs of minimal super-symmetric extension of standard model (MSSM) [42]. In MSSM there are two complex Higgs doublets and this predicts three neutral Higgs particles denoted by h, H, and A. If A is light then the rate for the production of Higgs bosons is proportional to the parameter $tan(\beta)$ define as the ratio of vacuum expectation values of the two doublets. The rate for Higgs production is by a factor $tan(\beta)^2$ higher than in standard model and this has been taken as a justification for the identification as MSSM Higgs [47] (the proposed value is $tan(\beta) \sim 50$ [49]). If the identification is correct, about recorded 100 Higgs candidates should already exist [37] so that this interpretation can be checked.

Also Tommaso Dorigo, the blogging member of second team analyzing CDF results, has reported in his blog [49] a slight evidence for an excess of $b\bar{b}$ pairs in $Z^0 \to b\bar{b}$ decays at the same mass $m_H = 160$ GeV [49]. The confidence level is around 2 sigma. The excess could result from the decays of Higgs to $b\bar{b}$ pair associated with $b\bar{b}$ production.

What forces to take these reports with some seriousness is that the value of m_H is same in both cases. John Conway has however noticed [48] that if both signals correspond to Higgs then it is possible to deduce estimate for the number of excess events in $Z^0 \to b\bar{b}$ peak from the excess in $\tau\overline{\tau}$ peak. The predicted excess is considerably larger than the real excess. Therefore a statistical fluke could be in question, or staying in an optimistic mood, there is some new particle there but it is not Higgs.

$m_H = 160$ GeV is not consistent with the standard model estimate by D0 collaboration for the mass of standard model Higgs boson mass based on high precision measurement of electro-weak parameters $sin(\theta_W)$, α, α_s , m_t and m_Z depending on $log(m_H)$ via the radiative corrections. The best fit is in the range $96-117$ GeV [38]. The upper bound from the same analysis for Higgs mass is 251 GeV with 95 per cent confidence level. The estimate $m_t = 178.0 \pm 4.3$ GeV for the mass of top quark is used. The range for the best estimate is not consistent with the lower bound of 114 GeV on m_H coming from the consistency conditions on the renormalization group evolution of the effective potential $V(H)$ for Higgs [37], see Fig. 4.6.1. Here one must of course remember that the estimates vary considerably.

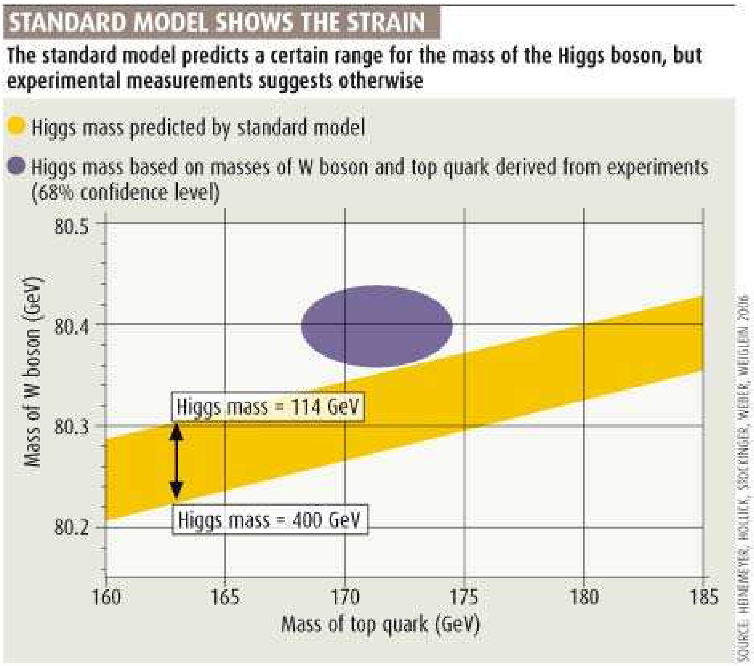

Figure 1: The regions of parameter space allowed by high precision measurements of top and W boson masses and the bounds on Higgs mass coming from the evolution of Higgs self coupling λ do not overlap.

4.6.2 TGD picture about Higgs briefly

Since TGD cannot yet be coded to precise Feynman rules, the comparison of TGD to standard model is not possible without some additional assumptions. It is assumed that p-adic coupling constant evolution reduces in a reasonable approximation to the coupling constant evolution predicted by a gauge theory so that one can apply at qualitative level the basic wisdom about the effects of various couplings of Higgs to the coupling constant evolution of the self coupling λ of Higgs giving upper and lower bounds for the Higgs mass. This makes also possible to judge the determinations of Higgs mass from high precision measurements of electro-weak parameters in TGD framework.

In TGD framework the Yukawa coupling of Higgs to fermions can be much weaker than in standard model. This has several implications.

1. The rate for the production of Higgs via channels involving fermions is much lower. This could explain why Higgs has not been observed even if it had mass around 100 GeV.

2. In standard model the large Yukawa coupling of Higgs to top, call it h, tends to reduce the quartic self coupling constant λ for Higgs in ultraviolet. The condition that the minimum for Higgs potential is not transformed to a maximum gives a lower bound on the initial value of λ and thus to the value of m_H. In TGD framework the weakness of fermionic couplings implies that there is no lower bound to Higgs mass.

3. The weakness of Yukawa couplings means that self coupling of Higgs tends to increase λ faster than in standard model. Note also that when Yukawa coupling h_t to top is small ($h_t^2 < \lambda$, see [39]), its contribution tends to increase the value of β_λ. Thus the upper bound from perturbative unitarity to the scalar coupling λ (and m_H) is reduced. This would force the value of Higgs mass to be even lower than in standard model. The coupling constant evolution using $\beta_\lambda = (3/4\pi^2)\lambda^2$ obtained taking into account only the contribution of Higgs would give $\lambda(t) = \lambda_0/(1 - k\lambda_0 log(t/t_0))$, $k = 3/4\pi^2$. For $\lambda(M_Z^2) = .2$ the value $\lambda(t) = 1$ would be achieved for $M/M_Z \simeq 5 \times 10^5$.

   In TGD framework new physics can however emerge in the length scales corresponding to Mersenne primes $M_n = 2^n - 1$. Ordinary QCD corresponds to M_{107} and one cannot exclude even M_{89} copy of QCD corresponding to mass scale $M \sim 128$ GeV. M_{61} corresponding to the mass scale $M \sim 2 \times 10^6$ GeV would define the next candidate. The quarks of M_{89} QCD would give to the beta function β_λ a negative contribution tending to reduce the value λ so that unitary bound would not be violated. If this new physics is accepted $m_H = 160$ GeV can be considered.

Can one then identify the Higgs candidate with $m_H = 160$ with the TGD variant of standard model Higgs? This is far from clear.

1. Even in standard model the rate for the production of Higgs is low. In TGD the rate for the production of the counterpart of standard model Higgs is reduced since the coupling of quarks to Higgs is expected to be much smaller than in standard model. This might exclude the interpretation as Higgs.

2. The slow rate for the production of Higgs could also allow the presence of Higgs at much lower mass and explain why Higgs has not been detected in the mass range $m_H < 114$ GeV. Interestingly, around 1990 a 2σ evidence for Higgs with mass about 100 GeV was reported and one might wonder whether there might be genuine Higgs there after all.

3. M_{89} hadron physics might be required in TGD framework by the requirement of perturbative unitarity. By a very naive scaling by factor $2^{(107-89)/2} = 2^9$ the mass of the pion of M_{89} physics would be about 70 GeV. This estimate is not reliable since color spin-spin splittings distinguishing between pion and ρ mass do not scale naively. For M_{89} mesons this splitting should be very small since color magnetic moments are very small. The mass of pion in absence of splitting would be around 297 MeV and 512-fold scaling gives $M(\pi_{89}) = 152$ GeV which is not too far from 160 GeV. Could the decays of this exotic pion give rise to the excess of fermion pairs? This interpretation might also allow to understand why b-pair and t-pair excesses are not consistent. Monochromatic photon pairs with photon energy around 76 GeV would be the probably easily testable experimental signature of this option.

4.6.3 Could the claimed inconsistency of $Z^0 \rightarrow \tau\overline{\tau}$ and $Z^0 \rightarrow b\overline{b}$ excesses be understood in TGD framework?

According to simple argument of John Conway [48] based on branching ratios of Z^0 and standard model Higgs to $\tau\overline{\tau}$ and $b\overline{b}$, $Z^0 \rightarrow \tau\overline{\tau}$ excess predicts that the ratio of Higgs events to Z^0 events $Z^0 \rightarrow b\overline{b}$ is related by a scaling factor

$$\frac{B(H \rightarrow b\overline{b})}{B(H \rightarrow \tau\overline{\tau})} / \frac{B(Z^0 \rightarrow b\overline{b})}{B(Z^0 \rightarrow \tau\overline{\tau})} \simeq \frac{10}{5.6} = 1.8$$

to that in $Z^0 \rightarrow \tau\overline{\tau}$ case. The prediction seems to be too high.

In a shamelessly optimistic mood and forgetting that mere statistical fluctuations might be in question, one might ask whether the inconsistency of $\tau\overline{\tau}$ and $b\overline{b}$ excesses could be understood in TGD framework.

1. The couplings of Higgs to fermions need not scale as mass in TGD framework. Rather, the simplest guess is that the Yukawa couplings scale like p-adic mass scale $m(k) = 1/L(k)$, where $L(k)$ is the p-adic length scale of fermion. Fermionic masses can be written as $m(F) = x(F)/L(k)$, where the numerical factor $x(F) > 1$ depends on electro-weak quantum numbers and is different for quarks and leptons. If the leading contribution to the fermion mass comes from p-adic thermodynamics, Yukawa couplings in TGD framework can be written as $h(F) = \varepsilon(F)m(F)/x(F)$, $\varepsilon << 1$. The parameter ε should be same for all quarks *resp.* leptons but need not be same for leptons and quarks so that that one can write $\varepsilon(quark) = \varepsilon_Q$ and $\varepsilon(lepton) = \varepsilon_L$. This is obviously an important feature distinguishing between Higgs decays in TGD and standard model.

2. The dominating contribution to the mass highest generation fermion which in absence of topological mixing correspond to genus $g = 2$ partonic 2-surface comes from the modular degrees of freedom and is same for quarks and leptons and does not depend on electro-weak quantum numbers at all (p-adic length scale is what matters only). Topological mixing inducing CKM mixing affects $x(F)$ and tends to reduce $x(\tau), x(b)$, and $x(t)$ but the reduction is very small [F4].

3. In TGD framework the details of the dynamics leading to the final states involving Z^0 bosons and Higgs bosons are different since one expects that it fermion-Higgs vertices suppressed to the degree that weak-boson-Higgs vertices could dominate in the production of Higgs. Since these details should not be relevant for the experimental determination of $Z^0 \rightarrow \tau\overline{\tau}$ and $Z^0 \rightarrow b\overline{b}$ distributions, then the above argument can be modified in a straightforward manner by looking how the branching ratio $R(b\overline{b})/R(\tau\overline{\tau})$ is affected by the modification of Yukawa couplings for b and τ. What happens is following:

$$\frac{B(H \rightarrow b\overline{b})}{B(H \rightarrow \tau\overline{\tau})} = \frac{m_b^2}{m_\tau^2} \rightarrow \frac{B(H \rightarrow b\overline{b})}{B(H \rightarrow \tau\overline{\tau})} X \ \ , \ \ X = \frac{\varepsilon^2(q)}{\varepsilon^2(L)} \frac{x_\tau^2}{x_b^2} \ \ .$$

   Generalizing the simple argument of Conway one therefore has

$$\frac{H}{Z^0}(b\overline{b}) = 1.8 \frac{\varepsilon_Q^2}{\varepsilon_L^2} \frac{x_\tau^2}{x_b^2} \times \frac{H}{Z^0}(\tau\overline{\tau}) \ \ .$$

   Since the topological mixing of both charged leptons and quarks of genus 2 with lower genera is predicted to be very small, $x_\tau/x_b \simeq 1$ is expected to hold true. Hence the situation is not improved unless one has $r = \frac{\varepsilon_Q}{\varepsilon_L} < 1$ meaning that the coupling of Higgs to the p-adic mass scale would be weaker for quarks than for leptons.

Can one then guess then value of r and perhaps even Yukawa coupling from general arguments?

1. The actual value of r should relate to electro-weak physics at very fundamental level. The ratio $r = 1/3$ of Kähler couplings of quarks and leptons is certainly this kind of number. This would reduce the prediction for $\frac{H}{Z^0}(b\overline{b})$ by a factor of 1/9.

2. Kähler charge Q_K equals electro-weak $U(1)$ charge $Q_{U(1)}$. Furthermore, Kähler coupling strength which is RG invariant equals to $U(1)$ coupling strength at the p-adic length scale of electron but not generally [C5]. This observation encourages the guess that, apart from

a numerical factor of order unity, ε^2 itself is given by either $\alpha_K Q_K^2$ and RG invariant or by $\alpha_{U(1)} Q_{U(1)}^2$. The contribution of Higgs vacuum expectation to fermionic mass would be roughly a fraction $10^{-2} - 10^{-3}$ about fermion mass in consistency with p-adic mass calculations.

Of course, it might turn out that fake Higgs is in question. What is however important is that the deviation of the Yukawa coupling allowed by TGD for Higgs from those predicted by standard model could manifest itself in the ratio of $Z_0 \to b\bar{b}$ and $Z^0 \to \tau\overline{\tau}$ excesses.

5 Appendix

The appendix has become somewhat obsolete because of the dramatic simplifications in the construction of states. I have however decided to still keep it.

5.1 Gauge invariant states in color sector

The construction of states satisfying Super Virasoro and Kac Moody gauge conditions for various values of conformal weight is essential ingredient in the calculation of degeneracies for various values of mass squared operator in order to estimate thermal mass expectation value. If one has obtained the multiplicities of various representations with weight n then it is easy to calculate the multiplicities for the states satisfying Super Virasoro conditions and Kac Moody conditions. Kac Moody conditions are implied by Super Virasoro conditions since $T^{a,n} \propto [L^n, T^{a,0}]$ holds true so that only Super Virasoro conditions need to be taken into account. If the gauge conditions associated with $G^{1/2}$ and $G^{3/2}$ in N-S representation induce surjective maps to the levels $n-1/2$,$n-1$ and $n-2$ then the multiplicity of gauge invariant representation is given by $m = m(n) - m(n-1/2) - m(n-3/2)$. In Ramond sector the gauge conditions for L^1 and G^1 guarantee the remaining gauge conditions and one has $m = m(n) - 2m(n-1)$ under similar assumptions.

The construction of the gauge invariant states relies on the following observations.

1. The states at each level n (conformal weight) of color Kac Moody algebra can be classified into irreducible representations of color group. The states are created by the monomials $O(F)$ of the 'fermionic' generators $F^{A,k}$, which can be regarded as an element of Grassmann algebra generated by $F^{A,k}$. The monomials of $F^{A,1/2}$ satisfy the gauge conditions of the bosonic Kac Moody identically.

2. The operators $O(F)$ creating nonzero norm sates can be classified into irreducible representations of the color group. The basic building blocks are the representations defined by N:th order monomials of generators F^{Ak} with k fixed. These representations are completely antisymmetrized tensor products of $N = 0, 1, ..., 8$ octets and representation content is same for all values of k. The representation content can be coded into multiplicity vector $m(N; k)$, $k = 1, 8, 10, ...$

3. Once the representation contents for antisymmetrized tensor products are known in terms of multiplicity vectors, the representation contents for tensor products of N_1, k_1 and N_2, k_2 can be determined by standard tensor product construction since anticommutativity does not produce no effects for $k_1 \neq k_2$. One can express the multiplicity vector for the tensor product $(N_1, k_1) \otimes (N_2, k_2)$ in terms of the multiplicity vector $D(k_1, k_2, k_3)$ for the tensor product of irreducible representations $k_1, k_2 = 1, 8, 10, ...$

$$m((N_1, k_1) \otimes (N_2, k_2; k) = m(N_1; k_1) D(k_1, k_2, k_3) m(N_2; k_2) \ . \quad (30)$$

4. It is useful to calculate total multiplicity vector $m(n;k)$ for each conformal weight n by considering all possible states having this conformal weight. The multiplicity vector is just the sum of multiplicity vectors of various tensor products satisfying $\sum N_i k_i = N$:

$$\begin{aligned} m(n;k) &= \sum_{S=N} m((N_1,k_1)\otimes ....\otimes (N_r,k_r;k)) \ , \\ S &\equiv \sum N_i k_i \ . \end{aligned} \tag{31}$$

The multiplicity vectors $m(n;k)$ are basic objects in the systematic construction of tensor products of several Super Virasoro algebras.

5.1.1 Multiplicity vectors for antisymmetric tensor products

Consider first the construction of N-fold antisymmetric tensor products of octets F^{Ak}, k fixed. The tensor products are obviously analogous to the antisymmetric tensors of 8-dimensional space. The completely antisymmetric 8-dimensional permutation symbol $\epsilon_{A_1,....,A_8}$ transforms as color singlet and induces duality operation in the set of antisymmetric representations: the antisymmetric representations N are mapped to representations $8-N$. This implies that the representation contents are same for $N=0$ and 8, $N=1$ and 7, $N=2$ and $N=6$, $N=3$ and $N=5$ respectively. $N=4$ is self dual. It is relatively easy to determine the representation content of the lowest completely antisymmetric representations and the results can be summarized conveniently as multiplicity vectors defined as

$$\begin{aligned} \bar{m} \equiv\ &(m(1),m(8),m(10),m(\bar{10}),m(27),m(28),m(\bar{28}), \\ &m(64),m(81),m(\bar{81}),m(125),...) \end{aligned} \tag{32}$$

The multiplicity vectors are given by the following formulas

$$\begin{aligned} \bar{m}(F)=\bar{m}(F^7) &= (0,1) \ , \\ \bar{m}(F^2)=\bar{m}(F^6) &= (0,1,1,1) \ , \\ \bar{m}(F^3)=\bar{m}(F^5) &= (1,1,1,1,1) \ , \\ \bar{m}(F^4) &= (0,2,0,0,2) \ , \end{aligned} \tag{33}$$

where F^N denotes N:th tensor power of F^{Ak}.

5.1.2 Multiplicity vectors for general states

The next task is to calculate multiplicity vectors for various conformal weights. The task is straightforward application of Young Tableaux. The representation contents for various conformal weights for N-S algebra are given by

$$\begin{aligned} n &= 0:1 \\ n &= 1/2:1/2 \\ n &= 1:(1/2)^2) \\ n &= 3/2:3/2\oplus(1/2)^3 \end{aligned}$$

$$
\begin{aligned}
n &= 2 : (3/2) \otimes (1/2) \oplus (1/2)^4 \\
n &= 5/2 : 5/2 \oplus (3/2) \otimes (1/2)^2 \oplus (1/2)^5 \\
n &= 3 : (5/2) \otimes (1/2) \oplus (3/2)^2 \oplus (3/2) \otimes (1/2)^3 \oplus (1/2)^6 \\
n &= 7/2 : 7/2 \oplus (5/2) \otimes (1/2)^2 \oplus (3/2)^2 \otimes (1/2) \oplus 3/2 \otimes (1/2)^4 \oplus (1/2)^7 \\
n &= 4 : (7/2) \otimes (1/2) \oplus (5/2) \otimes (3/2) \oplus (5/2) \otimes (1/2)^3 \oplus (3/2)^2 \otimes (1/2)^2 ... \\
&\oplus (3/2) \otimes (1/2)^5 \oplus (1/2)^8 \\
n &= 9/2 : 9/2 \oplus (7/2) \otimes (1/2)^2 ... \\
&\oplus (5/2) \otimes (3/2) \otimes (1/2) \oplus 5/2 \otimes (1/2)^4 \oplus (3/2)^3 .... \\
&\oplus (3/2)^2 \otimes (1/2)^3 \oplus (3/2) \otimes (1/2)^6
\end{aligned}
\tag{34}
$$

Multiplicity vectors obtained as sums of multiplicity vectors associated with summands in the direct sum composition and are given by the following table

n	1	8	10	$\overline{10}$	27	28	$\overline{28}$	35	$\overline{35}$	64	81	$\overline{81}$
0	1											
1/2		1										
1		1	1	1								
3/2	1	2	1	1	1							
2	1	4	1	1	3							
5/2	2	6	3	3	4							
3	2	10	6	6	6			2	2	1		
7/2	4	16	8	8	12			4	4	2		
4	8	24	12	12	21	1	1	7	7	4		
9/2	10	36	21	21	32	1	1	12	12	8	1	1

Table 5. Multiplicity vectors for various conformal weights for N-S type Super Virasoro algebra.

Similar arguments can be used to deduce the multiplicity vectors in case of Ramond type Super Virasoro algebra.

n	1	8	10	$\overline{10}$	27	28	$\overline{28}$	35	$\overline{35}$	64	80	$\overline{80}$	81	$\overline{81}$	125
0	1														
1		1													
2		1	1	1											
3	2	4	2	2	2										
4	2	10	4	4	6			1	1						
5	6	20	10	10	14			4	4	1					
6	12	40	22	22	32	1	1	10	10	6					
7	17	68	36	36	55	1	1	20	20	11			1	1	
8	33	124	70	70	113	5	5	44	44	29			5	5	1
9	70	276	170	170	276	16	16	122	122	94	1	1	22	22	6

Table 6. Multiplicity vectors for various conformal weights for Ramond type Super Virasoro algebra.

5.1.3 Multiplicity vectors for conformally invariant states

Multiplicity vectors for gauge invariant states are obtained from the formulas $m(n) \rightarrow m(n) - m(n-1/2) - m(n-3/2)$ and $m(n) \rightarrow m(n) - 2m(n-1)$. The inspection of the above tables gives following tables for the multiplicity vectors of gauge invariant states needed in the mass calculations to find the possible ground states.

n	1	8	10	$\overline{10}$	27	28	$\overline{28}$	35	$\overline{35}$	64	81	$\overline{81}$
0	1											
1/2		1										
1		1	1	1								
3/2		1		2	1							
2	1	1		1	2							
5/2	1	1	1	1	1							
3	1	2	2	1	1			2	2	1		
7/2		2	1	3	2			2	2	1		
4	2	2	1	3	2	1	1	3	3	2		
9/2		2	3	5	1			3	3	3	1	1

Table 7. Multiplicity vectors for the conformal weights of gauge invariant states for N-S type Super Virasoro algebra.

n	1	8	10	$\overline{10}$	27	28	$\overline{28}$	35	$\overline{35}$	64	80	$\overline{80}$	81	$\overline{81}$	125
0	1														
1		1													
2			1	1											
3	2	2			2										
4		2			2			1	1						
5	2		2	2	2			2	2	1					
6			2	2	4	1	1	2	2	4					
7													1	1	
8					3	3	3	4	4	7			3	3	1
9	4	28	30	30	50	6	6	34	34	36	1	1	12	12	4

Table 8. Multiplicity vectors for various conformal weights of gauge invariant states of Ramond type Super Virasoro algebra.

5.2 Number theoretic auxiliary results

The ground state degeneracies for fermions and bosons need not to be identical to their ideal values $D = 64$ and $D = 16$ and it is of interest to find under what conditions the degeneracy can be said to be near to its ideal value. This amounts to calculating the p-adic inverse of the D in general case. Mathematica provides means for calculating modular inverses as well as modular powers (also fractional assuming that they exists). Despite this it is useful show how the real counterpart of a fractional p-adic number can be deduced.

The calculation of the modular inverse goes as follows.

1. The problem is to find the lowest order term in p-adic expansion of the inverse y of p-adic number $x \in 1, ...p-1$. The remaining terms in expansion in powers of p can be found iteratively. The equation to be solved is

$$yx = 1\ mod\ p\ , \quad (35)$$

for a given value of x, which gives $y = mp + 1$.

2. One can express p in the form

$$p = Nx + r\ . \quad (36)$$

The evaluation of N and $r \in \{1, .., x-1\}$ is a straightforward exercise in modulo arithmetics. The defining equation for y can be written as

$$yx = m(Nx + r) + 1 = mNx + mr + 1\ . \quad (37)$$

From this one must have

$$mr + 1 = kx\ , \quad (38)$$

and any pair (m, k) satisfying this condition gives solution to y:

$$y = mN + k\ . \quad (39)$$

y must be chosen to be the smallest possible one.

Consider as examples two practical cases.

1. $p = M_n = 2^n - 1$ and $x = 15 = 2^4 - 1$. One obtains r by substituting repeatedly $2^4 = 1\ mod\ x$ to the expression of M_n. M_n can be written in the form $M_n = 15(2^{n-4} + 2^{n-8} + ...) + r$ and the previous condition reads $mr + 1 = 15k$.
   i) For M_{89} one has $r = 1$ and $(m, k) = (14, 1)$ giving $y = 14(2^{n-4} + 2^{n-8} + ...) + 1$. For the real counterpart of $Xp^2/2D$ one has the approximate expression $(7X\ mod\ 16)/15$ and approximately N-S mass formula for small quantum numbers results.
   ii) For M_{127} and M_{107} one has $r = 7$ and $7m + 1 = 15k$ gives $(m, k) = 2, 1)$ and $y = 2(2^{n-4} + ...) + 1$. For $Xp^2/2D$ one has $Xmod16/15$: the factor 7 is absent.

2. $p = M^n$ and $x = 63 = 64 - 1$. One obtains r by substituting repeatedly $2^6 - 1\ mod\ x$ to the expression of M_n. One has $r = 1$ for $n = 127$, $r = 31$ for $n = 107$ and $n = 89$. For the real counterpart R of Xp^2/D one has $R = (62X\ mod\ 64)/(63M_n)$ and $y = (60X\ mod\ 64)/(63M_n)$ for $n = 127$ and $107, 89$ respectively so that mass formulas change somewhat and in n-dependent manner if one has $D = 63$ instead of $D = 64$.

3. 1/5 factor appears in mass formulas for leptons and the previous argument leads to the expression $p^2/5 = (2^{126} - 2^{124} + 2^{122} - ..)p^2$. From this formula the real counterpart of, say 1/5, is in a good approximation 4/5. It must be emphasized that Mathematica provides the number theoretical modules for calculating the real counterparts for numbers of form rp, r rational number.

Acknowledgements

I am grateful for Tony Smith for pointing me the puzzling aspects related to the determination of the top quark mass.

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