Earlier Attempts to Construct S-matrix

M. Pitkänen[1], February 1, 2006

[1] Department of Physical Sciences, High Energy Physics Division,
PL 64, FIN-00014, University of Helsinki, Finland.
matpitka@rock.helsinki.fi, http://www.physics.helsinki.fi/∼matpitka/.
Recent address: Puutarhurinkatu 10,10960, Hanko, Finland.

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Abstract

The dream of finding master formula for S-matrix of TGD is 27 year old as I write this. The realization that configuration space spinors correspond to von Neumann algebras known as hyper-finite factors of type II_1 was the decisive discovery and stimulated a rapid progress in the understanding of the mathematical structure of TGD and also the long waited master formula for S-matrix finally saw the daylight.

This master formula made however a lot of previous work obsolete. These earlier attempts might seem rather childish from the recent point of view which could of course look equally childish from the perspective I perhaps have around 2010. The germs of recent vision are however in these older attempts although strongly suppressed by cognitive noise. These older visions contain also a lot of real stuff which complements the recent vision. For these reasons I thought that it would not be wise to throw away this chapter. I have not added the recent overall view about construction of S-matrix which is contained in the chapter "Construction of Quantum Theory" and is warmly recommended to the critical reader as a background.

The gigantic symmetries of quantum TGD are bound to lead to a highly unique S-matrix (actually a hierarchy of S-matrices) but the practical construction of U-matrix remains still a formidable challenge. Despite this one can write Feynman rules for the S-matrix in the approximation that the consideration is restricted to elementary particles modelled as CP_2 type extremals. Two developments have changed the situation dramatically.

a) The first development was inspired by the idea that generalized Feynman diagrams might allow a generalization of duality symmetry of string models meaning that the diagrams are always equivalent with tree diagrams. The diagrams could be seen as generalization of braid diagrams and this notion can be formulated axiomatically in terms of Hopf algebras.

b) The second development was inspired by the better understanding of the role of the classical non-determinism of Kähler action and led to the discovery of 7–3 duality and effective 2-dimensionality meaning that all relevant physics can be coded into 2-dimensional intersections of 3-D and 7-D light like causal determinants. This gives concrete realization for the equivalence of generalized diagrams with tree diagrams.

1. Basic philosophy behind the construction of S-matrix

In TGD framework quantum transitions correspond to a quantum jump between two different quantum histories (configuration space spinor field) rather than to a non-deterministic behavior of a single quantum history. Therefore S-matrix relates to each other two quantum histories rather than the initial and final states of a single quantum history.

To understand the philosophy behind the construction of S-matrix it is useful to notice that in TGD framework there is actually a 'holy trinity' of time developments instead of single time development encountered in ordinary quantum field theories.

a) The classical time development determined by the absolute minimization of Kähler action.

b) The unitary "time development" defined by U associated with each quantum jump and defining U-matrix. One cannot however assign to the U-matrix an interpretation as a unitary time-translation operator and this means that one must leave open the identification of U-matrix with S-matrix.

c) The time development of subjective experiences by quantum jumps identified as moments of consciousness. The value of psychological time associated with a given quantum jump is determined by the contents of consciousness of the observer. The understanding of psychological time and its arrow and of the dynamics of subjective time development requires the construction of theory of consciousness. A crucial role is played by the classical nondeterminism of Kähler action implying that the nondeterminism of quantum jump and hence also the contents of conscious experience can be concentrated into a finite volume of the imbedding space.

Quantum classical correspondence states that not only quantum states but also quantum jump sequences and even the complex anatomy of quantum jump must have representation at space-time level. This has far reaching implications.

a) Configuration space S-matrix or U-matrix is induced from space-time S-matrix acting in fermionic degrees of freedom (configuration space "spin degrees of freedom").

b) The fact that there is an infinite number of anatomies of quantum jump connecting given quantum states means, predicts that there is a infinite number of space-time surfaces giving rise to the same space-time S-matrix. This is nothing but the equivalence of generalized Feynman diagrams to tree diagrams.

c) Single quantum jump and thus a particular space-time S-matrix must correspond to a finite space-time region, perhaps single space-time sheet with the maximal deterministic regions of the space-time sheet correlating with with the anatomy of quantum jump.

2. 7–3 duality as a key to the construction of S-matrix

The notion of 7–3 duality emerged from the interaction between TGD and M-theory. The attempts to construct quantum TGD have gradually led to the conclusion that the geometry of the configuration space ("world of classical worlds") involves both 7-D and 3-D light like surfaces as causal determinants. 7-D light like surfaces X^7 are unions of future and past light cone boundaries and play a role somewhat resembling that of branes. 3-D light like surfaces X_l^3 can correspond to boundaries of space-time sheets, regions separating two maximally deterministic space-time regions, and elementary particle horizons at which the signature of the induced metric changes.

7–3 duality states that it is possible to formulate the theory using either the data at 3-D space-like 3-surfaces resulting as intersections of the space-time surface with 7-D CDs or the data at 3-D light like CDs. This results if the data needed is actually contained by 2-D intersections $X^2 = X_l^3 \cap X^7$. This effective 2-dimensionality has far-reaching implications. It simplifies dramatically the basic formulas related to the configuration space geometry and spinor structure, it leads to the explicit identification of the generalized Feynman diagrams at space-time level as light like 3-D CDs. The basic philosophy is that quantum-classical correspondence stating that space-time sheets provide a description for the physics associated with the configuration space spin degrees of freedom (fermionic degrees of freedom).

The generalized Feynman diagrammatics is simple. The fermions do not carry four-momenta but are on mass shell particles characterized by the eigenvalues of the modified Dirac operator D. There is no propagator associated with 3-D CDs: only a unitary transformation U_λ representing braiding in spin and electroweak spin degrees of freedom can be present. Vertices are the inner products at X^2 for the positive energy states and negative energy states entering to the vertex, finite, and in principle computable. The equivalence of generalized Feynman diagrams with tree diagrams is expected on basis of the effective 2-dimensionality, and indeed follows from on mass shell property directly. Unitarity follows trivially. No loop summations are thus involved.

The counterparts of loop sums are absent in TGD framework and p-adic number fields and their extensions defining an infinite hierarchy of fixed point values of Kähler coupling strength and thus of gauge coupling constants. The question is whether this discrete coupling constant evolution can mimic a QFT type coupling constant evolution (or vice versa). Is it possible to have renormalization without renormalization? The construction of quantum state using generalization of coset construction for super-canonical and super Kac-Moody algebra allows to answer this question. The counterparts of bare states are non-orthogonal and have a natural multi-grading. Gram-Schmidt orthogonalization procedure makes the bare states dressed and brings in TGD counterpart of loop corrections to the S-matrix. The counterparts of renormalization group equations result by formally regarding p-adic prime p as a continuous variable. Quantum field theory approximation results when the inner products defining simplest particle decays are described as coupling constants.

3. Quantum criticality and Hopf algebra approach to S-matrix

Quantum criticality leads to a generalization of duality symmetry of string models stating that the generalized Feynman diagrams with loops are equivalent with diagrams having no loops. This means that each S-matrix element correspond to a unique tree diagram. The conditions for this equivalence can be formulated as algebraic conditions characterizing a Hopf algebra like structure, and, using the language of ordinary Feynman diagrams, correspond to the vanishing of the loop corrections in the configuration space integral crucial for the p-adicization. This symmetry is expected to be of crucial importance for practical evaluation of S-matrix elements as should be also the reduction of the matrix elements of generators of the enveloping algebra of super-canonical algebra to n-point functions of super-conformal field theory in the complex plane of super-canonical conformal weights.

4. S-matrix as Glebsch-Gordan coefficients

U-matrix relates 'free' and 'interacting' representations of the super-canonical and Super Kac-Moody algebras acting as symmetries of quantum TGD. The construction is based on the association of 3-surfaces Y_i^3 and corresponding absolute minima $X^4(Y_i^3)$ to incoming states as well as the interacting four-surface $X^4(\cup_i Y_i^3)$ describing the interactions classically. The generators for various super-algebras associated with $X^4(\cup_i Y_i^3)$ are modified by interactions so that the generator basis is not just a union of the generator basis associated with $X^4(Y_i^3)$. U-matrix relates the tensor product for the representations associated with the incoming 'free' space-time surfaces $X^3(Y_i^3)$ and the interaction representation associated with $X^4(\cup Y_i^3)$: generalized Glebch-Gordan coefficients are clearly in question and unitarity is obvious.

5. Perturbation theoretic approach to U-matrix

This formal approach starts from the identification of U-matrix elements as Glebsch-Gordan coefficients relating free and interacting states and tries to construct U-matrix perturbatively by reducing it to stringy perturbation theory. The starting point is that U-matrix must follow from Super Virasoro invariance alone and that the condition $L_0(tot)\Psi = 0$ (plus the corresponding conditions for other super-Virasoro generators) must determine U-matrix. Here $L_0(tot)$ corresponds to the Virasoro generators associated with the interacting space-time surface $X^4(\cup_i Y_i^3)$ whereas $L_0(free, i)$ correspond to the free generators associated with $X^3(Y_i^3)$. It is however not at all obvious whether the generators $L_0(tot)$ are perturbatively related to the the generators $L_0(free, i)$ and whether U-matrix allows perturbative expansion.

6. Number theoretic approach to U-matrix

The task of assigning to the surfaces Y_i^3 the free space-time surfaces $X^4(Y_i^3)$ and interacting space-time surface $X^4(\cup_i Y_i^3)$ is the basic stumbling block for the construction of U-matrix. The super-algebra generators creating the excitations of the incoming ground states are super-algebra generators associated with $\cup X^4(Y_i^3)$ whereas the outgoing states are created by the super-algebra generators associated with $X^4(\cup_i Y_i^3)$. The surfaces $X^4(Y_i^3)$ correspond to the space-time surfaces associated with infinite primes P_i representing ground states of super-conformal representations whereas $X^4(\cup_i Y_i^3)$ corresponds to the space-time surface associated with the infinite integer $N = \prod_i P_i^{k_i}$. This means that the worst part of the problem is solved. The remaining challenge is to relate the super-algebra basis to each other.

7. Construction of the S-matrix at high energy limit

It is possible to write Feynman rules for the S-matrix in the approximation that only CP_2 type extremals appear as virtual and real particles. All CP_2 type extremals are locally isometric with CP_2 itself and only the random lightlike curve is dynamical. The classical dynamics is actually isomorphic with stringy dynamics since classical Virasoro conditions are satisfied. Fermions belong to the representations of Super-Kac-Moody algebra of $M^4 \times SO(3,1) \times SU(3) \times U(2)_{ew}$. The classical nondeterminism of the dynamics implies that Feynman graph expansion is topologized. This saves from the troubles caused by fermionic divergences since the exponent of the momentum generator effecting translation along the line of the Feynman graph corresponds to that associated with the modified Dirac action and thus to a free quantum theory for fermions.

Vertex operators $V(a, b, c)$ are generalizations of the vertex operators of string theory: instead of strings 3-surface inside CP_2 type extremal fuse together. Propagator factors are products of the exponent of the Kähler action for CP_2 type extremal proportional to the volume of the CP_2 type extremal; the 'stringy' $1/(L_0 + i\epsilon)$ factor, which comes from the vertices;

and a unitary translation operator (counterpart of S-matrix as time translation operator) along the geodesic representing average cm motion.

The theory has some features which are characteristic for quantum TGD.

a) One can assume that each quantum jump involves localization in zitterbewegung degrees of freedom. The resulting S-matrix is independent of the choice of the representative for the zitterbewegung orbit as long as the cm motion connects the lines of the vertices. The predictions depend however on an arbitrary function of U of CP_2 coordinates giving rise to a decomposition of CP_2 to 'time slices'. The dependence of the propagator is only through the volume of CP_2 type extremal determined by U whereas coupling constants have more complicated, but presumably very mild dependence on U. The dependence on the function U means that one must average the scattering rates over the allowed spectrum of functions U. This dependence of the fundamental coupling constants on U is in accordance with spin glass analogy and means that fundamental coupling constants are not strictly speaking constants.

b) The volume of the internal line, which is a fraction of CP_2 volume determines the value of the exponent of Kähler action and provides thus a suppression factor serving as an infrared cutoff. A constraint to the allowed functions U results from the topological condensation of CP_2 in particle like space-time sheet (for instance, massless extremal), which implies that CP_2 type extremals cannot extend outside the region with size of order p-adic length scale L_p. The only plausible interpretation seems to be that the information about the infrared cutoff length scale is coded into the structure of particle: particle in the box is quite not the same as free particle. This suggests new view about color confinement: quarks and gluons correspond to CP_2 type extremals which cannot exist too long time as free particles and therefore cannot leave hadron.

1 Introduction

The dream of finding master formula for S-matrix of TGD is 27 year old as I write this. The realization that configuration space spinors correspond to von Neumann algebras known as hyper-finite factors of type II_1 was the decisive discovery and stimulated a rapid progress in the understanding of the mathematical structure of TGD and also the long waited master formula for S-matrix finally saw the daylight.

This master formula made however a lot of previous work obsolete. These earlier attempts might seem rather childish from the recent point of view which could of course look equally childish from the perspective I perhaps have around 2010. The germs of recent vision are however in these older attempts although strongly suppressed by cognitive noise. These older visions contain also a lot of real stuff which complements the recent vision. In particular, the section "Approximate construction of S-matrix" is to a surprisingly high degree consistent with master formula". For these reasons I thought that it would not be wise to throw away this chapter. I have not added the recent overall view about construction of S-matrix which is contained in the chapter "Construction of Quantum Theory" [C1] and is warmly recommended to the critical reader as a background.

1.1 The problem

The enormous symmetries of quantum are bound to lead to a highly unique S-matrix but the practical construction of S-matrix is a formidable challenge and necessitates deep grasp about the physics involved so that one can make the needed approximations. The evolution of the ideas related to S-matrix involves several side-tracks and strange twists characteristic for a mathematical problem solving when a direct contact with the experimental reality is lacking. The work with S-matrix has taught me that principles are more important than formulas and that the only manner to proceed is from top to bottom by gradually solving the philosophical problems, identifying all the relevant symmetries and understanding the horribly nonlinear dynamics defined by the absolute minimization of Kähler action.

The poor understanding of the philosophical issues has led to frustratingly many candidates for S-matrix. TGD inspired theory of consciousness has however gradually led to a clarification of various issues and it seems that it is safer to distinguish between two matrices: U-matrix and S-matrix. Moreover, it seems that one must talk in plural: there is entire hierarchy of U-matrices and S-matrices correspond to higher levels of the hierarchy.

1. U-matrix is much more fundamental object than S-matrix conventionally defined as time-translation operator and characterizes what happens in single quantum jump $\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f$. A good candidate for U-matrix is as Glebsch-Gordan coefficients relating free and interacting Super Virasoro representations.

2. Quantum measurement theory requires 1-1 correlation between the values of zero modes representing macroscopic classical variables and quantum fluctuating degrees of freedom. This implies the required localization in zero modes without which one would encounter problems with infinities since the Gaussian exponent making possible sensible definition of the functional integral over zero modes is lacking as is lacking. On the other hand, Gaussian would give Gaussian determinant introducing also infinite factor. The U-matrix associated with the fiber degrees of freedom in turn decomposes into a tensor product of the local U-matrices associated with various space-time sheets. The tensor factor of U describing dynamics in super Kac-Moody conformal degrees of freedom for a given space-time sheet could correspond to the TGD counterpart of the stringy S-matrix.

3. The new view about sub-system forced by the many-sheeted space-time and the integration of quantum jump sequences to single quantum as far as conscious experience is considered, suggests that one must introduce entire hierarchy of U-matrices corresponding to p-adic length scale hierarchy and hierarchy of durations for sequences of quantum jumps. The identification as S-matrices is natural since the duration of quantum jump sequence allows identification as counterpart for the duration of time evolution associated with S-matrix in standard physics. Actually subjective time duration is in question and corresponds only in a statistical sense to a definite duration of geometric time. In particular, one expects that these higher level U-matrices do not provide only an approximate description of something more fundamental but express all that can be said and tested by conscious observer.

4. The hierarchy of p-adic number fields and their extensions of increasing dimension should correspond to the hierarchy of U-matrices. This means that the matrix elements of S-matrix should be in extensions of rationals defining finite-dimensional extensions of p-adic numbers possibly involving transcendentals. This would mean that S-matrix theory becomes number theory at the deepest and most challenging level one can imagine.

The lack of explicit formulas for S-matrix elements have been the basic weakness of quantum TGD approach as compared to the concrete perturbative formulas provided by super-string approach. Fortunately, the new number theoretic vision leads to concrete Feynman rules for S-matrix in the approximation that elementary particles can be regarded as CP_2 type extremals. Of course, this is only small piece of quantum TGD but certainly the most important one as far as the empirical testing of the theory is considered.

1.2 The fundamental identification of U- and S-matrices

Single quantum jump corresponds to the sequence

$$\Psi_i \rightarrow U\Psi_i \rightarrow ....\Psi_f \quad .$$

U does not certainly correspond to a genuine time-development in the sense of a unitary time translation operator. A good guess is that U has interpretation as Glebsch-Gordan coefficients between free an interacting representations of Super Virasoro algebra associated with surfaces $\cup_i X^4(Y_i^3)$ and $X^4(\cup_i Y_i^3)$. For a given unentangled subsystem (subsystem in self-organizing self-state) the eigen states of the density matrix of the subsystem becoming unentangled in quantum jump determines what are the final states of the quantum jump. Negentropy Maximization Principle states that the subsystem of unentangled subsystems whose measurement gives rise to maximal entanglement negentropy gain, is quantum measured. U is much more fundamental object than S-matrix. If subsystem is entangled then in a reasonable approximation nothing happens to it during quantum jump sequence and the sub-quantum history remains unchanged.

What could then be the interpretation of S-matrix in this framework?

1. The first glance to the problem is based on macro-temporal quantum coherence. If the dissipative effects caused by the state function reductions and state preparations are absent completely or are not visible in the time resolution defined by the duration of macro-temporal quantum coherence, one might expect that S-matrix is product of U-matrices occurred during the period of the macro-temporal quantum coherence. The system in self state would indeed effectively behave like its own Universe. One could say that time-evolution is discretized with CP_2 time defining the duration of the chronon.

2. Observer is represented by a cognitive space-time sheet drifting towards the geometric future quantum jump by quantum jump along material space-time sheet. Observer is basically interested in the unitary time development induced by the time-translation operator P_0 associated with the modified Dirac operator. This time development can have any duration T and defines time evolution operator $exp(iP_0T)$ if subsystem develops as essentially free system. The measurement of the scattering probabilities defined by S-matrix corresponds to a construction of an empirical arrangement guaranteing that quantum measurement observed by (the sufficiently intelligent!) cognitive space-time sheet at time $t = T$ reduces the quantum state to some of the state of the initial state basis at time $t = 0$. In the ideal situation the measured system would develop unitarily during this interval and stay thus entangled so that it cannot self-organize by quantum jumps.

This picture would suggest that the S-matrix could be defined as a generalization of the exponential $exp(iP_0T)$ of the second quantized Poincare energy operator P_0 associated with the modified Dirac action for the interacting space-time surface $X^4(\cup_i Y_i^3)$ and acts on the tensor product of the state spaces associated with $X^4(Y_i^3)$. This picture might make sense at the limit when wave mechanics is a good approximation but does not work in elementary particle length scales where CP_2 type extremals whose M^4_+ projection is a random light like curve, are expected to dominated.

Interacting space-time surfaces defined by a connected sum of CP_2 type extremals can be regarded as a Feynman graph with lines thickened to 4-manifolds. This suggests the assignment of the exponent with the internal lines of the generalized Feynman graph acting as translation operators whereas vertices where lines join give rise to vertex operators which can be regarded as Glebch-Gordan coefficients for super-Kac-Moody representations.

Since the Hamiltonian in question is quadratic in oscillator operators, the theory is free in the standard sense of the word, and it is only the absolute minimization of Kähler action which induces interactions and makes the theory nontrivial. Feynman diagram structure has purely topological origin. For instance, topological sums of CP_2 type extremals can be regarded as an example of Feynman diagrams with lines thickened to 4-manifolds. The absence of interaction terms guarantees that there are no sources of divergences. For the CP_2 type extremals one can develop rather detailed form of the Feynman rules.

TGD inspired theory of consciousness suggests that one should not take too dogmatic view about S-matrix as a summary for the predictions of quantum theory. Even the idealizations about experimental situation involved with the S-matrix formalism might be quite too strong since experimental observations are always about subsystems.

1.3 Super conformal symmetries and U-matrix

Super-canonical and super Kac-Moody symmetries should dictate also the dynamics of the theory to a very high degree. As already explained [B4, C1], super Kac-Moody conformal symmetries act as conformal transformations in the complex plane containing super-canonical conformal weights as punctures. This means that the expectation values of the elements of the enveloping algebra of super-canonical algebra can be interpreted as n-point functions of a conformal field theory. This must have deep implications for the calculability of the theory.

For instance, the model for a scalar field propagator defined as a super-canonical partition function [C5] leads to highly non-trivial new physics predictions. The expectation is that all massive states are accompanied by an infinite number of resonances which do not correspond to poles but delta function like singularities and have a universal mass spectrum determined by the zeros of Zeta. p-Adic length scale defining the cutoff length scale appears explicitly in the propagator and also cutoff in the number of non-trivial zeros of Riemann Zeta characterizing the sub-algebra of super-canonical algebra is necessary. This cutoff can be interpreted physically as being related to phase resolution. The model for scalar propagator seems to generalize to the case of stringy propagator in a rather straightforward manner. In this chapter this generalization is not however discussed explicitly.

This U-matrix defined in terms of the operators representing translation along the lines of Feynman diagrams constructed from CP_2 type extremals could indeed correspond to the TGD counterpart of string model S-matrix. The construction of S-matrix should reduce more or less to that encountered in super string models. It is even possible that the two-dimensional commutative sub-manifolds of space-time surface might effectively represent space-time surfaces so that at this limit TGD would reduce to super-string model type theory.

One can indeed formulate the Feynman rules for S-matrix when elementary particles are modelled as CP_2 type extremals (this took 23 years of hard work!). Various properties of the CP_2 type extremals allow to reproduce the Feynman diagrammatics of quantum field theories topologically. There are however some important the from standard physics basically due to the non-determinism of CP_2 type extremals and implying that self-organization is present already in elementary particle length scales. Infrared cutoff is coded automatically by the intrinsic characteristics of the CP_2 type extremals representing virtual particles: particle stays in box, not because of the boundary conditions but because the command of staying in box is coded into the structure of the particle. One can understand color confinement elegantly as being due to this kind of 'learned' infrared cutoff. One can say that even elementary particles resemble living creatures in the sense that they can adapt to the surrounding world and p-adic length scale hierarchy represents a hierarchy of this kind of adaptations.

Spin glass analogy at the quantum level means that the predictions for the scattering rates must be averaged over an ensemble of quantum field theories with propagators and coupling constants depending on an arbitrary function of CP_2 coordinates.

1.4 7–3 duality, conformal symmetries, and effective 2-dimensionality

Thanks to the non-determinism of Kähler action, also light like 3-surfaces X_l^3 of space-time surface appear as causal determinants (CDs). Examples are boundaries and elementary particle horizons at which Minkowskian signature of the induced metric transforms to Euclidian one. This brings in a conformal symmetry related to the metric 2-dimensionality of the 3-D CD. This symmetry is identifiable as TGD counterpart of the Kac Moody symmetry of string models.

The possibility of spinorial shock waves at X_l^3 leads to the hypothesis that they correspond to particle aspect of field particle duality whereas the physics in the interior of space-time corresponds to field aspect. More generally, field particle duality in TGD framework states that 3-D light like CDs and 7-D CDs are dual to each other. In particular, super-canonical and Super Kac Moody symmetries are also dually related.

What is new that these $N = 4$ complex super-conformal symmetries do not give rise to global super-symmetries, not even $N = 1$ supersymmetry. Second new aspect is that the solutions of the modified Dirac equation have interpretation as generators of super gauge symmetries whereas generalized eigen modes of the modified Dirac operator correspond to physical states and replace off mass shell states in the formalism of standard quantum field theory.

The underlying reason for 7–3 duality be understood from a simple geometric picture in which 3-D light like CDs X_l^3 intersect 7-D CDs X^7 along 2-D surfaces X^2 and thus form 2-sub-manifolds of the space-like 3-surface $X^3 \subset X^7$. One can regard either canonical deformations of X^7 or Kac-Moody deformations of X^2 as defining the tangent space of configuration space so that 7–3 duality would relate two different coordinate choices for CH.

The assumption that the data at either X^3 or X_l^3 are enough to determine configuration space geometry implies that the relevant data is contained to their intersection X^2. This is the case if the deformations of X_l^3 not affecting X^2 and preserving light likeness corresponding to zero modes or gauge degrees of freedom and induce deformations of X^3 also acting as zero modes. The outcome is effective 2-dimensionality. One cannot over-emphasize the importance of this conclusion. In particular, this almost implies the equivalence of all generalized Feynman diagrams identified as space-time surfaces with initial and final states fixed and represented by 2-D surfaces X_i^2. Combining this observation with the new understanding of conformal symmetries leads to explicit Feynman rules for the space-time S-matrix which in turn induces configuration space S-matrix to a high degree.

1.5 Number theory and U-matrices

It is becoming clear that number theory might allow very profound views about the structure of U-matrix (or hierarchy of U-matrices).

1.5.1 U-matrices exist simultaneously in all number fields

The requirement that U-matrix exists simultaneously in all number fields when finite-dimensional extensions of p-adic numbers are allowed, is extremely powerful. In accordance with the idea about cognitive hierarchy, U-matrices can be assumed to form a hierarchy labelled by p-adic primes p and by the dimensions of the extensions of p-adics. Already the construction of Cabibbo-Kobayashi-Maskawa matrix for the mixing of quarks assuming rationality [F5] demonstrates how strong the rationality constraint combined with simple physical inputs can be.

The basic idea is that U-matrices in different number fields are obtained by an algebraic continuation from extensions of rationals defining finite-dimensional extensions of p-adic numbers. This implies automatically pinary cutoff since the notions of p-adic and real continuity are practically diametric opposites of each other. Also a hierarchy of extensions of p-adic numbers reflecting increasing p-adic phase resolution emerges. This implies an infinite hierarchy of U-matrices corresponding to the hierarchy of p-adic number fields and their finite-dimensional extensions. The hierarchy of phase resolutions corresponds to a hierarchy of U-matrices labelled by the number of non-trivial zeros of Zeta appearing in the sub-algebra of the super-canonical algebra defining physical states. These resolution hierarchies are not fictive but reflect both a genuine hierarchy of phases of the physical system and hierarchy of cognitive structures. There are good reasons to believe that hierarchies of type II_1 factors of von Neumann algebra correspond directly to the hierarchies associated with the phase resolution since the infinite-dimensional Clifford algebra of configuration space gamma matrices is von Neumann algebra. This picture leads also to beautiful connections with braid groups and quantum groups.

1.5.2 Equivalence of loop diagrams with tree diagrams and cancellation of infinities in Quantum TGD

The work with topological quantum computation inspired a cascade of ideas leading to the vision that generalized Feynman diagrams are analogous to knot and link diagrams in the sense that they allow also "moves" allowing to identify classes of diagrams and that the diagrams containing loops are equivalent with tree diagrams, so that there would be no summation over diagrams. This would be a generalization of duality symmetry of string models.

TGD itself provides general arguments supporting same idea. The identification of absolute minimum of Kähler action as a four-dimensional Feynman diagram characterizing particle reaction means that there is only single Feynman diagram instead of functional integral over 4-surfaces: this diagram is expected to be minimal one. At quantum level S-matrix element can be seen as a representation of a path defining continuation of configuration space (CH) spinor field between different sectors of CH corresponding to different 3-topologies. All continuations and corresponding Feynman diagrams are equivalent. The idea about Universe as a computer and algebraic hologram allows a concrete realization based on the notion of infinite primes, and space-time points become infinitely structured monads [E10]. The generalized Feynman diagrams differing only by loops are equivalent since they characterize equivalent computations.

The basic objection against the new view about Feynman diagrams is that it is not consistent with the notion of coupling constant evolution involving loops in an essential manner. The objection can be circumvented. Quantum criticality requires that Kähler coupling constant α_K is analogous to critical temperature (so that the loops for configuration space integration vanish). The hypothesis motivated by the enormous vacuum degeneracy of Kähler action is that α_K has an infinite number of possible values labelled by p-adic length scales and also also by the dimensions of effective tensor factors defined hierarchy of II_1 factors (so called Beraha numbers) as found in [E10].

The dependence on the p-adic length scale L_p corresponds to the usual renormalization group evolution whereas the latter dependence would correspond to a finite angular resolution and to a hierarchy of finite-dimensional extensions of p-adic number fields R_p. The finiteness of the resolution is forced by the algebraic continuation of rational number based physics to real and p-adic number fields since p-adic and real notions of distance between rational points differ dramatically. The higher the algebraic dimension of the extension and the higher the value of p-adic prime the better the angular (or phase) resolution and nearer the p-adic topology to that for real numbers.

The proposed practical realization for the equivalence of loop diagrams with tree diagrams is in terms of categories generalizing Hopf algebras and related structures. The basic algebra and co-algebra axioms would state this equivalence. This requires however a modification of the notion of Feynman diagram forced by the algebraic approach meaning that vacuum lines are allowed. In TGD framework vacuum lines corresponding to the identity element of the algebra correspond to vacuum extremals. In terms of ordinary Feynman diagrams the equivalence of loop diagrams with tree diagrams means the vanishing of loops.

Concerning the construction of S-matrix this approach means that one can assign to a given S-matrix element an infinity of mutually equivalent generalized Feynman diagrams having also interpretation as different continuations between the sectors of configuration space characterizing initial and final states of the reaction. The minimal tree diagram if obviously in a preferred role as far as the actual computation of S-matrix element is considered.

The equivalence of loop diagrams with tree diagrams meaning the vanishing of Feynman graph loop corrections is a strong constraint and there are indications that their vanishing relates very intimately to the zeros of Riemann polyzetas and to the assumption that the zeros of Riemann

Zeta and polyzetas characterizes the super-canonical conformal weights of particles and of their many-particle bound states [C5].

1.5.3 Infinite primes and the construction of U-matrix

The ideas about the possible relevance of infinite primes for the construction of S-matrix are considerably more speculative than the basic stuff discussed above. Infinite primes seem extremely natural from the view point of consciousness theory since they imply a generalization of the notion of number field when multiplication by infinite rationals having unit real norm is allowed. In p-adic sense these multiplicative factors are not units. This means that the points of various number fields would be infinitely structured but that this structure is completely invisible in real physics sense but is absolutely essential for understanding the p-adic physics of mathematical cognition. One could say that single space-time point becomes the Platonia and is able to represent any algebraic structure, even the physical state of entire universe in its structure.

The original hypothesis was that infinite primes are crucial for the understanding of the space-time physics. The hypothesis was motivated by the fact that the construction of infinite primes has high resemblance to the quantization of an arithmetic quantum field theory. Platonia idea corresponds to the realization of algebraic hologram in terms of infinitely structured space-time points, whereas the original idea was to characterize the entire universe in terms of infinite primes and rationals. The two applications are diametrical opposites of each other. If one takes the idea of algebraic hologram and Brahman=Atman identity seriously then following considerations might be easier to follow.

The task of assigning to the surfaces Y_i^3 the free space-time surfaces $X^4(Y_i^3)$ and interacting space-time surface $X^4(\cup_i Y_i^3)$ is the basic stumbling block for the construction of U-matrix. Nothing less that the solution of the field equations for absolute minimization of Kähler action and for induced Dirac equation on space-time surface would be required to achieve this goal in practice. The vision about TGD as a generalized number theory led to a dramatic understanding about how these challenges might be solved and gives even hopes that this might be achieved at practical level.

Space-time surfaces would be coded by infinite primes mapped to products of irreducible polynomials with complex rational coefficients. The Fock states coded by the infinite primes correspond to the states of a hyper-octonionic arithmetic quantum field theory second quantized again and again [E3] Quantum field theory which is based on the notion of point-like particles cannot describe quantum TGD based on generalization of particle concept. A possible interpretation of the Fock states is as ground states of representations of super-Kac Moody algebra representations. This view is consistent with the interpretation of the physical states as configuration space spinor fields assigning to a given 3-surface infinite number of possible states.

U-matrix elements can be identified as matrix elements between the incoming states of super-conformal representations created from the ground states associated with the tensor product of the ground states associated with Y_i^3. The super-algebra generators creating the excited incoming states are super-algebra generators associated with Y_i^3 whereas the outgoing states are created by the super-algebra generators associated with $\cup_i Y_i^3$. What is so beautiful that $X^4(Y_i^3)$ correspond to space-time surfaces associated with infinite primes P_i representing ground states of super-conformal representations whereas $X^4(\cup_i Y_i^3)$ corresponds to the space-time surface associated with the infinite integer $N = \prod_i P_i^{k_i}$. This means that the worst part of the problem is solved.

If this picture were correct, the remaining challenge would be to relate the super-algebra basis to each other: this is necessary for both S- and U-matrix.

There is an objection against this picture. Feynman diagrammatics for CP_2 type extremals relies on the representation of both incoming and outgoing states as legs of the classical Feynman diagram and on the interpretation of vacuum to zero energy state amplitudes as representations for S-matrix elements. This requires a generalization of the proposed interpretation of infinite primes explaining why two kinds of infinite primes are needed. The two kinds of infinite primes constructed from the vacua $V_{\pm} = X \pm 1$ interpreted as associated with positive and negative energy states, naturally represent incoming and outgoing states in case that a zero energy state representing S-matrix element is in question. The infinite integer associated with the zero energy state representing S-matrix element is given by the product N_+N_- of the integers associated with these two kinds of infinite primes. Different Feynman graphs in perturbative expansion would represent the representations of N_+N_- corresponding to different values of fiber coordinates of CH.

1.5.4 U-matrix and arithmetic QFT defined by hyper-octonionic primes

One can assign to each infinite prime a Fock state of a super-symmetric arithmetic quantum field theory. According to the vision discussed in [E3], hyper-octonionic primes could label single particle states of this theory at the lowest level of the hierarchy but also bound states and interacting N-particle states defined as infinite integers are in spectrum.

The relationship of infinite integers with the standard QFT descriptions is not obvious although they should correspond to many particle states formed as tensor products of super conformal representations. It would be rather surprising if this QFT would not have a deep relationship with the QFT limit of quantum TGD. Since QFT is in question one expects that it only describes the ground states of super Kac-Moody conformal representations. This is also required by the fact that configuration space spinors and geometric degrees of freedom represent independent degrees of freedom. Mass squared spectrum is integer valued for this arithmetic QFT which means that the ground states of Super Virasoro representations can have all possible mass squared values. In arithmetic QFT there are only two single-particle states with given four-momentum. For super Kac-Moody Virasoro representation there exists a large number of single particle states with with given four-momentum and their number is determined by mass squared which is essentially the value of conformal weight.

The Fock states defined by infinite primes are not orthogonal with respect to the natural inner product and the overlap matrix G for the states is Hermitian: $G = G^{\dagger}$. The experience with the work with Riemann hypothesis suggests that one might identify G as the matrix $TT^{\dagger}$ associated with the U-matrix $U = 1 + iT$ at QFT limit. This U-matrix could perhaps be related to that part of U-matrix which describes the dynamics in zero modes.

1.5.5 p-Adic co-homology as a BRST type symmetry of the p-adic S-matrices?

The problems related to unitarity in the formal perturbative construction of S-matrix inspired the idea that p-adicity might lead to a totally new view about S-matrix. In p-adic context unitarity conditions of S-matrix $S = 1 + iT$ reads as $i(T - T^{\dagger}) + TT^{\dagger} = 0$. The conditions $T = T^{\dagger}$, $T^2 = 0$ provide in p-adic context a nontrivial solution of these conditions reducing the construction of the S-matrix to co-homology theory. If real scattering probabilities are obtained by canonical identification followed by normalization, p-adic co-homology leads to nontrivial physical predictions. One can construct extremely general family of solutions to p-adic co-homology, and for a while it indeed seemed that p-adic co-homology might provide a royal road to the construction of S-matrix in accordance with Wheeler's "Boundary has no boundary" intuition.

Unfortunately, the new vision about finite-p p-adic physics as physics of cognition forces to give up this view unless p-adic co-homology makes sense for infinite-p p-adics. This does not seem to be the case, basically because $\sqrt{-1}$ does not seem to exist as infinite-p p-adic number. p-Adic co-homology might however make sense for finite-p p-adic S-matrices providing cognitive representations for quantum dynamics.

The most natural interpretation of p-adic co-homology is as a symmetry of S-matrix: the replacements $S \rightarrow S + iT$ for p-adic S-matrix serve as analogs of BRST symmetries leaving the predicted scattering probabilities invariant in a given resolution and these symmetries might be of help in the actual construction of S-matrix. If p-adic probabilities summing up to zero can be regarded as rational numbers, some of them must be negative as real numbers and do not therefore allow frequency interpretation. One could consider the possibility that the tensor factors of p-adic S-matrix satisfying p-adic co-homology represent what might be called imagined dynamics since the total p-adic cross-sections vanish.

1.6 Various approaches to the construction of S-matrix

The gigantic symmetries of quantum TGD are bound to lead to a highly unique U-matrix but the practical construction of U-matrix remains still a formidable challenge. Despite this one can write Feynman rules for the S-matrix in the approximation that the consideration is restricted to elementary particles modelled as CP_2 type extremals. This approximation might well be all that is needed for practical purposes and leads to precise predictions.

1.6.1 7–3 duality as a key to the construction of S-matrix

The notion of 7–3 duality emerged from the (one might say violent) interaction between TGD and M-theory [A2]. The attempts to construct quantum TGD have gradually led to the conclusion that the geometry of the configuration space ("world of classical worlds") involves both 7-D and 3-D light like surfaces as causal determinants. 7-D light like surfaces X^7 are unions of future and past light cone boundaries and play a role somewhat resembling that of branes. 3-D light like surfaces X^3_l can correspond to boundaries of space-time sheets, regions separating two maximally deterministic space-time regions, and elementary particle horizons at which the signature of the induced metric changes.

7–3 duality states that it is possible to formulate the theory using either the data at 3-D space-like 3-surfaces resulting as intersections of the space-time surface with 7-D CDs or the data at 3-D light like CDs [B4]. This results if the data needed is actually contained by 2-D intersections $X^2 = X^3_l \cap X^7$. This effective 2-dimensionality has far-reaching implications. It simplifies dramatically the basic formulas related to the configuration space geometry and spinor structure, it leads to the explicit identification of the generalized Feynman diagrams at space-time level as light like 3-D CDs. The basic philosophy is that quantum-classical correspondence stating that space-time sheets provide a description for the physics associated with the configuration space spin degrees of freedom (fermionic degrees of freedom).

The generalized Feynman diagrammatics is simple. The fermions do not carry four-momenta but are on mass shell particles characterized by the eigenvalues of the modified Dirac operator D. There is no propagator associated with 3-D CDs: only a unitary transformation U_λ representing braiding in spin and electroweak spin degrees of freedom can be present. Vertices are the inner products at X^2 for the positive energy states and negative energy states entering to the vertex, finite, and in principle computable. The equivalence of generalized Feynman diagrams with tree diagrams is expected on basis of the effective 2-dimensionality, and indeed follows from on mass shell property directly. Unitarity follows trivially. No loop summations are thus involved.

1.6.2 Quantum criticality and Hopf algebra approach to S-matrix

Quantum criticality leads to a generalization of duality symmetry of string models stating that the generalized Feynman diagrams with loops are equivalent with diagrams having no loops. This means that each S-matrix element correspond to a unique tree diagram. The conditions for this equivalence can be formulated as algebraic conditions characterizing a Hopf algebra like structure, and, using the language of ordinary Feynman diagrams, correspond to the vanishing of the loop corrections in the configuration space integral crucial for the p-adicization. This symmetry is expected to be of crucial importance for practical evaluation of S-matrix elements as should be also the reduction of the matrix elements of generators of the enveloping algebra of super-canonical algebra to n-point functions of a conformal field theory in the complex plane of super-canonical conformal weights.

1.6.3 von Neumann algebras and S-matrix

The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors of type II_1 could provide the mathematics needed to develop a more explicit view about the construction of S-matrix [C6].

1. Inclusions of hyper-finite II_1 factors as a basic framework to formulate quantum TGD

1. The effective 2-dimensionality of the construction of quantum states and configuration space geometry in quantum TGD framework makes hyper-finite factors of type II_1 very natural as operator algebras of the state space. Indeed, the elements of conformal algebras are labelled by discrete numbers and also the modes of induced spinor fields are labelled by discrete label, which guarantees that the tangent space of the configuration space is a separable Hilbert space and Clifford algebra is thus a hyper-finite type II_1 factor. The same holds true also at the level of configuration space degrees of freedom so that bosonic degrees of freedom correspond to a factor of type I_∞ unless super-symmetry reduces it to a factor of type II_1.

2. Four-momenta relate to the positions of tips of future and past directed light cones appearing naturally in the construction of S-matrix. In fact, configuration space of 3-surfaces can be regarded as union of big-bang/big crunch type configuration spaces obtained as a union of light-cones with parameterized by the positions of their tips. The algebras of observables associated with bounded regions of M^4 are hyper-finite and of type III_1. The algebras of observables in the space spanned by the tips of these light-cones are not needed in the construction of S-matrix so that there are good hopes of avoiding infinities coming from infinite traces.

3. Many-sheeted space-time concept forces to refine the notion of sub-system. Jones inclusions $\mathcal{N} \subset \mathcal{M}$ for factors of type II_1 define in a generic manner imbedding interacting sub-systems to a universal II_1 factor which now corresponds naturally to infinite Clifford algebra of the tangent space of configuration space of 3-surfaces and contains interaction as $\mathcal{M} : \mathcal{N}$-dimensional analog of tensor factor. Topological condensation of space-time sheet to a larger space-time sheet, formation of bound states by the generation of join along boundaries bonds, interaction vertices in which space-time surface branches like a line of Feynman diagram: all these situations could be described by Jones inclusion characterized by the Jones index $\mathcal{M} : \mathcal{N}$ assigning to the inclusion also a minimal conformal field theory and conformal theory with k=1 Kac Moody for $\mathcal{M} : \mathcal{N} = 4$. $\mathcal{M} : \mathcal{N}$=4 option need not be realized physically as quantum field theory but as string like theory whereas the limit $D = 4 - \epsilon \rightarrow 4$ could correspond to $\mathcal{M} : \mathcal{N} \rightarrow 4$ limit. An entire hierarchy of conformal field theories is thus predicted besides quantum field theory.

4. von Neumann's somewhat artificial idea about identical a priori probabilities for states could replaced with the finiteness requirement of quantum theory. Indeed, it is traces which produce the infinities of quantum field theories. That $\mathcal{M} : \mathcal{N} = 4$ option is not realized physically as quantum field theory (it would rather correspond to string model type theory characterized by a Kac-Moody algebra instead of quantum group), could correspond to the fact that

dimensional regularization works only in $D = 4 - \epsilon$. Dimensional regularization with space-time dimension $D = 4 - \epsilon \to 4$ could be interpreted as the limit $\mathcal{M} : \mathcal{N} \to 4$. $\mathcal{M}$ as an $\mathcal{M} : \mathcal{N}$-dimensional $\mathcal{N}$-module would provide a concrete model for a quantum space with non-integral dimension as well as its Clifford algebra. An entire sequence of regularized theories corresponding to the allowed values of $\mathcal{M} : \mathcal{N}$ would be predicted.

2. Generalized Feynman diagrams are realized at the level of $\mathcal{M}$ as quantum space-time surfaces

The key idea is that generalized Feynman diagrams realized in terms of space-time sheets have counterparts at the level of $\mathcal{M}$ identifiable as the Clifford algebra associated with the entire space-time surface X^4. 4-D Feynman diagram as part of space-time surface is mapped to its $\beta = \mathcal{M} : \mathcal{N} \leq 4$-dimensional quantum counterpart.

1. von Neumann algebras allow a universal unitary automorphism $A \to \Delta^{it} A \Delta^{-it}$ fixed apart from inner automorphisms, and the time evolution of partonic 2-surfaces defining 3-D light-like causal determinant corresponds to the automorphism $\mathcal{N}_i \to \Delta^{it} \mathcal{N}_i \Delta^{-it}$ performing a time dependent unitary rotation for $\mathcal{N}_i$ along the line. At configuration space level however the sum over allowed values of t appear and should gives rise to the TGD counterpart of propagator as the analog of the stringy propagator $\int_0^t exp(iL_0 t)dt$. Number theoretical constraints from p-adicization suggest a quantization of t as $t = \sum_i n_i y_i > 0$, where $z_i = 1/2 + y_i$ are non-trivial zeros of Riemann Zeta.

2. At space-time level the "ends" of orbits of partonic 2-surfaces coincide at vertices so that also their images $\mathcal{N}_i \subset \mathcal{M}$ also coincide. The condition $\mathcal{N}_i = \mathcal{N}_j = ... = \mathcal{N}$, where the sub-factors $\mathcal{N}$ at different vertices differ only by automorphism, poses stringent conditions on the values t_i and Bohr quantization at the level of $\mathcal{M}$ results. Vertices can be obtained as a vacuum expectations of of the operators creating the states associated with the incoming lines (crossing symmetry is automatic).

3. The equivalence of loop diagrams with tree diagrams would be due to the possibility to move the ends of the internal lines along the lines of the diagram so that only diagrams containing 3-vertices and self energy loops remain. Self energy loops are trivial if the product associated with fusion vertex and co-product associated with annihilation compensate each other. The possibility to assign quantum group or Kac Moody group to the diagram gives good hopes of realizing product and co-product. Octonionic triality would be an essential prerequisite for transforming N-vertices to 3-vertices. The equivalence allows to develop an argument proving the unitarity of S-matrix.

4. A formulation using category theoretical language suggests itself. The category of space sheets has as the most important arrow topological condensation via the formation of worm-hole contacts. This category is mapped to the category of II_1 sub-factors of configurations space Clifford algebra having inclusion as the basic arrow. Space-time sheets are mapped to the category of Feynman diagrams in $\mathcal{M}$ with lines defined by unitary rotations of $\mathcal{N}_i$ induced by Δ^{it}.

1.6.4 U-matrix as Glebsch-Gordan coefficients

U-matrix relates 'free' and 'interacting' representations of the super-canonical and quaternio conformal algebras acting as symmetries of quantum TGD. The construction is based on the association of 3-surfaces Y_i^3 and corresponding absolute minima $X^4(Y_i^3)$ to incoming states as well as the interacting four-surface $X^4(\cup_i Y_i^3)$ describing the interactions classically. The generators for various super-algebras associated with $X^4(\cup_i Y_i^3)$ are modified by interactions so that the generator basis is not just a union of the generator basis associated with $X^4(Y_i^3)$. U-matrix relates the tensor product for the representations associated with the incoming 'free' space-time surfaces $X^3(Y_i^3)$ and the interaction representation associated with $X^4(\cup Y_i^3)$: generalized Glebch-Gordan coefficients are clearly in question and unitarity is obvious.

1.6.5 Number theoretic approach to U-matrix

The task of assigning to the surfaces Y_i^3 the free space-time surfaces $X^4(Y_i^3)$ and interacting space-time surface $X^4(\cup_i Y_i^3)$ is the basic stumbling block for the construction of U-matrix. The super-algebra generators creating the excitations of the incoming ground states are super-algebra generators associated with $\cup X^4(Y_i^3)$ whereas the outgoing states are created by the super-algebra generators associated with $X^4(\cup_i Y_i^3)$. The surfaces $X^4(Y_i^3)$ correspond to the space-time surfaces associated with infinite primes P_i representing ground states of super-conformal representations whereas $X^4(\cup_i Y_i^3)$ corresponds to the space-time surface associated with the infinite integer $N = \prod_i P_i^{k_i}$. This means that the worst part of the problem is solved.

1.6.6 Perturbation theoretic approach to U-matrix

This formal approach starts from the identification of U-matrix elements as Glebsch-Gordan coefficients relating free and interacting states and tries to construct U-matrix perturbatively by reducing it to stringy perturbation theory. The starting point is that U-matrix must follow from Super Virasoro invariance alone and that the condition $L_0(tot)\Psi = 0$ (plus the corresponding conditions for other super-Virasoro generators) must determine U-matrix. Here $L_0(tot)$ corresponds to the Virasoro generators associated with the interacting space-time surface $X^4(\cup_i Y_i^3)$ whereas $L_0(free, i)$ correspond to the free generators associated with $X^3(Y_i^3)$. It is however not at all obvious whether the generators $L_0(tot)$ are perturbatively related to the the generators $L_0(free, i)$ and whether U-matrix allows perturbative expansion.

1.6.7 Construction of the S-matrix at high energy limit

It is possible to write Feynman rules for the S-matrix in the approximation that only CP_2 type extremals appear as virtual and real particles. All CP_2 type extremals are locally isometric with CP_2 itself and only the random lightlike curve is dynamical. The classical dynamics is actually isomorphic with stringy dynamics since classical Virasoro conditions are satisfied. Fermions belong to the representations of Super-Kac-Moody algebra of $M^4 \times SO(3,1) \times SU(3) \times U(2)_{ew}$. The classical nondeterminism of the dynamics implies that Feynman graph expansion is topologized. This saves from the troubles caused by fermionic divergences since the exponent of the momentum generator effecting translation along the line of the Feynman graph corresponds to that associated with the modified Dirac action and thus to a free quantum theory for fermions.

Vertex operators $V(a, b, c)$ are generalizations of the vertex operators of string theory: instead of strings 3-surface inside CP_2 type extremal fuse together. Propagator factors are products of the exponent of the Kähler action for CP_2 type extremal proportional to the volume of the CP_2 type extremal; the 'stringy' $1/(L_0 + i\epsilon)$ factor, which comes from the vertices; and a unitary translation operator (counterpart of S-matrix as time translation operator) along the geodesic representing average cm motion.

The theory has some features which are characteristic for quantum TGD.

1. One can assume that each quantum jump involves localization in zitterbewegung degrees of freedom. The resulting S-matrix is independent of the choice of the representative for the zitterbewegung orbit as long as the cm motion connects the lines of the vertices. The predictions depend however on an arbitrary function of U of CP_2 coordinates giving rise to a decomposition of CP_2 to 'time slices'. The dependence of the propagator is only through the volume of CP_2 type extremal determined by U whereas coupling constants have more

complicated, but presumably very mild dependence on U. The dependence on the function U means that one must average the scattering rates over the allowed spectrum of functions U. This dependence of the fundamental coupling constants on U is in accordance with spin glass analogy and means that fundamental coupling constants are not strictly speaking constants.

2. The volume of the internal line, which is a fraction of CP_2 volume determines the value of the exponent of Kähler action and provides thus a suppression factor serving as an infrared cutoff. A constraint to the allowed functions U results from the topological condensation of CP_2 in particle like space-time sheet (for instance, massless extremal), which implies that CP_2 type extremals cannot extend outside the region with size of order p-adic length scale L_p. The only plausible interpretation seems to be that the information about the infrared cutoff length scale is coded into the structure of particle: particle in the box is quite not the same as free particle. This suggests new view about color confinement: quarks and gluons correspond to CP_2 type extremals which cannot exist too long time as free particles and therefore cannot leave hadron.

2 Approximate construction of S-matrix

There are good hopes that the S-matrix for the CP_2 type extremals representing elementary particles provides an excellent approximation for the purposes of the particle physicist. The question whether the S-matrix in question can be interpreted as local tensor factor of the U-matrix must be left open. One can develop rather explicit form for the Feynman rules giving S-matrix elements and the construction of the vertex operators seems to be much simpler than in string model context.

2.1 Basic properties of CP_2 type extremals

CP_2 type extremal has the following explicit representation

$$m^k = f^k(u(s^k)) \ , \quad m_{kl}\frac{df^k}{du}\frac{df^l}{du} = 0 \ . \tag{1}$$

The function $u(s^k)$ is an arbitrary function of CP_2 coordinates and serves effectively as a time parameter in CP_2 defining a slicing of CP_2 to time=constant sections. The functions f^k are arbitrary apart from the restriction coming from the light likeness. When one expands the functions f^k to Fourier series with respect to the parameter u, light likeness conditions reduce to classical Virasoro conditions $L_n = 0$.

It is possible to write the expression for m^k in a physically more transparent form by separating the center of mass motion and by introducing p-adic length scale L_p as a normalization factor.

$$\frac{m^k}{L_p} = p^k u + \sum_n \frac{1}{\sqrt{n}} a_n^k exp(i2\pi nu) + c.c. \ . \tag{2}$$

The first term corresponds to the center of mass term responsible for rectilinear motion along geodesic line and second term corresponds to the zitterbewegung motion. p^k serves as an effective classical momentum which can be normalized as $p_k p^k = \epsilon$, $\epsilon = \pm 1$ or $\epsilon = 0$. What has significance is whether p^k is time like, light like, or spacelike. Conformal invariance corresponds to the freedom to replace u with a new 'time parameter' $f(u)$.

The physically most natural representation of u is as a function $f(U)$ of the fractional volume U for a 4-dimensional sub-manifold of CP_2 spanned by the 3-surfaces $X^3(U = 0)$ and $X^3(U)$:

$$u = f(U) \ , \quad U = \frac{V(s^k)}{V(CP_2)} = \frac{S_K(u)}{S_K(CP_2)} \ . \tag{3}$$

The range of the values for U is bounded from above: $U \leq V_{max}/V(CP_2)$ and the value $U = 1$ is possible only if CP_2 type extremal begins and ends as a point. U represents also Kähler action using the value of the Kähler action for CP_2 as a unit.

The requirement that CP_2 type extremal extends over an infinite time and spatial scale implies the requirement

$$f(U_{max}) = \infty \quad . \tag{4}$$

For $f(U_{max}) < \infty$ CP_2 type extremal can exist only in a finite temporal and spatial interval for finite values of 'momentum ' components p^k. This suggest a precise geometric distinction between real and virtual particles: virtual particles correspond to the functions $f(U_{max}) < \infty$ in contrast to the incoming and outgoing particles for which one has $f(U_{max}) = \infty$. This hypothesis, although it looks like an ad hoc assumption, is at least worth of studying.

The mere requirement that virtual CP_2 type extremal extends over a temporal or spatial distance of order $L > L_p$ implies that for $L < L_p$ the value of U is smaller than one. Kähler action, which is given by

$$S_K(X^4) = U \times S_K(CP_2) \quad , \tag{5}$$

remains small for distances much smaller than L. For $f(U_{max}) = \infty$ this is even more true. This has an important implication: below a certain length scale the exponential of the Kähler action associated with the internal line of a Feynman diagram does not give rise to a suppression factor whereas above some characteristic length L and time scale there is an exponential supression of the propagator by the factor $exp(-S_K(CP_2))$ practically hindering the propagation over distances larger than this length scale.

The presence of the exponential obviously introduces an effective infared cutoff: this cutoff is prediction of the fundamental theory rather than adhoc input as in quantum field theories. Of course, infrared cutoff results also from the condition $f(U_{max}) < \infty$. Physically the infrared cutoff results from the topological condensation of the CP_2 type extremals to larger space-time sheets. These could correspond to massless extremals (MEs). p-Adic length scale L_p is an excellent candidate for the cutoff length scale in the directions transversal to ME.

The supression factor coming from the exponent of the Kähler action implies a distance dependent renormalization of the propagators. In the long length scale limit the suppression factor approaches to a constant value

$$exp\left[-\frac{V_{max}}{V(CP_2)}S_K(CP_2)\right] \quad ,$$

and can be absorbed to the coupling constant so that the dependence on the maximal length of the internal lines can be interpreted as an effective coupling constant evolution. For instance, the smallness of the gravitational constant could be understood as follows. Since gravitons propagate over macroscopic distances, the virtual CP_2 type extremals develops a full Kähler action and there is huge supression factor reducing the value of the gravitational coupling to its observed value: at short length scales the values of the gravitational coupling approaches to $G_{short} = L_p^2$ which means strong gravitation for momentum transfers $Q^2 > 1/L_p^2$. The values of V_{max} and thus those of the suppression factor can vary: only at the limit when CP_2 extremal has point like contact with the lines it joins together, one has $V_{max} = V(CP_2)$. If the boundary component characterizing elementary particle family belongs to CP_2 type extremal (it could be associated with a larger space-time sheet), CP_2 type extremal contains a hole: also this reduces the maximal volume of the CP_2 extremal.

2.2 Quantized zitterbewegung and Super Virasoro algebra

Calculating various Fourier components of right left hand side of the light likeness condition $m_{kl}p^kp^l = 0$ for $p^k = dm^k/du$ explicitly using the general expansion for m^k separating center of mass motion from zitterbeweegung, one obtains classical Virasoro conditions

$$\begin{aligned} p_0^2 &= L_0 \ , \\ L_n|phys\rangle &= 0 \ , \ . \end{aligned} \qquad (6)$$

where L_n are defined by by their classical expressions as bilinears of the Fourier coefficients. Therefore interior degrees of freedom give Virasoro algebra and zitterbewegung is more or less equivalent with the classical string dynamics.

It is not however not obvious whether a quantization of this dynamics is needed. If quantization is needed (perhaps to formulate the unitarity conditions in zero modes properly), it corresponds to the construction of the bosonic wave functionals in zero modes defined by the zitterbewegung degrees of freedom. Quantization could be carried out in the same manner as in string models.

The simplest assumption motivated by the Euclidian metric of CP_2 type extremal is that the commutator of p^k and m^k is proportional to a delta function as in ordinary quantization. One can Fourier expand m^k and p_k in the form

$$\begin{aligned} m^k &= m_0^k + p_0^k s + \frac{1}{K}\sum \frac{1}{n} a_n^{k,\dagger} exp(inKs) + \sum \frac{1}{n} a_n^k exp(-inKs) \ , \\ p^k &= p_0^k + i\sum a_n^{k,\dagger} exp(inKs) - i\sum a_n^k exp(-inKs) \ . \end{aligned} \qquad (7)$$

Here cm motion has been extracted and the formula is identical with the formula expressing the motion for a fixed point of string. The parameter K is Kac Moody central charge. Note that the exponents $exp(iKns)$ exist provided that Ks is p-adically of order $O(p)$ or, if algebraic extension by introducing $\sqrt{p}$ is allowed, of order $O(\sqrt{p})$.

The commutator of p_i and m^j is of the standard form if the oscillator operators obey Kac-Moody algebra

$$\begin{aligned} \left[p_{i,0}, m_0^j\right] &= m_i^{\ j} \ , \\ Comm(a_{i,m}^\dagger, a_n^j) &= Km \ .\delta(m,n) m_i^{\ j} \end{aligned} \qquad (8)$$

Here K appeas Kac-Moody central charge, which must be integer in the real context at least.

Expressing the light likeness condition as quantum condition, one obtains an infinite series of conditions, which give the quantum counterparts of the Virasoro conditions

$$\begin{aligned} p_0^2 &= kL_0 \ , \\ L_n|phys\rangle &= 0 \ , \ n < 0 \ . \end{aligned} \qquad (9)$$

k is some proportionality constant. One can solve these conditions by going to the transverse gauge in which physical states are created by oscillator operators orthogonal to an arbitrarily chosen light like vector. What quantization means physically is that zitterbewegung amplitudes are constrained by a Gaussian vacuum functional. A good guess motivated by the p-adic considerations is that the width of the ground state Gaussian is given by a p-adic length scale L_p: this is achieved if m^k is replaced with m^k/L_p in the general expression for $m^k(u)$. The experience with string models would suggests that vacuum functionals might be crucial for the understanding of graviton emission.

2.3 Feynman diagrams with lines thickened to CP_2 type extremals

CP_2 type extremals are just what the on-mass-shell and off-mass shell particles of string models are expected to be.

1. The variation of the modified Dirac operator with respect to the imbedding space coordinates implies Euler-Lagrange equations for the Kähler action and this in turn implies that massless Dirac equation is satisfied. Quaternion- analyticity allows to write the solutions of the modified Dirac equation explicitly and the requirement that the supercharges associated with $M^4 \times SO(3,1) \times SU(3) \times U(2)_{ew}$ generate super-Kac-Moody algebra, fixes the anti-commutation relations of the fermionic super charges which come in two varieties corresponding to the supercharges associated with the conserved fermion numbers and isometry charges. The super-Kac-Moody algebra in question gives rise to the physical states satisfying Super Virasoro conditions. Mass squared is quantized for the representations of Super Virasoro. There is degeneracy caused by the cm degrees of freedom forcing to introduce plane waves and color partial waves. Note that the degeneracy in CP_2 degrees of freedom is present because CP_2 type extremal is not the entire CP_2. Genus-generation correspondence requires the presence of 3-dimensional boundary either inside CP_2 type extremal or on the space-time sheet at which CP_2 type extremal is condensed at.

2. Kähler action results as a c-number term from the normal ordering of the modified Dirac action and appears in the exponent of the modified Dirac action defininining the vacuum functional of the theory. The exponent of the Kähler action for a piece of CP_2 type extremal defined by the line of the Feynman diagram appears as a factor in each internal line of the Feynman diagram.

3. For CP_2 type extremals the spectrum of the conserved momenta is continuous. The reason is that the random motion with light velocity can be regarded as a superposition of classical random zitterbegung motion and an average motion along time or spacelike geodesic line. This means that mass squared operator associated with the CP_2 type extremal is continuous and CP_2 type extremals can represent virtual particles appearing in the internal lines of Feynman diagrams.

4. The condition that the orbit is light like random curve reduces to classical Virasoro conditions and the mass squared of the particle corresponds classical to the 'momentum ' squared associated with the zitterbewegung motion.

2.4 Feynman rules

The heuristic view about Feynman rules (there are certainly delicacies involved not taken into account in this simplistic discussion) is following.

1. There is a sum over all possible Feynman graphs with CP_2 type extremals appearing as lines. This means integration over the positions of the vertices characterized by points of $M^4_+ \times CP_2$ corresponding to cm degrees of freedom. One must assign to each external particle a plane wave in M^4_+ degrees of freedom and color partial wave in CP_2 center of mass degrees of freedom.

2. To each 3-vertex of the Feynman graph one assigns a Glebsch-Gordan coefficient $V(a,b,c)$ for the tensor product of the incoming super-Kac-Moody representations besides the factor taking care of the conservation of quantum numbers, in particular four-momentum and color and electro-weak quantum numbers.

The lines of the Feynman diagram contain two factors: the exponent of Kähler action and translation operator along line.

1. The time development operator of wave mechanics is replaced with the unitary translation operator along the line connecting the two vertices P_1 and P_2. Translation operator is expressible as the exponent of the conserved four-momentum associated with the modified Dirac operator. The momentum operator is in the direction of the propagator line automatically. By using an eigen state basis of four-momenta, translation operator along the line connecting P_1 and P_2 can be expressed as

$$U(P_1,P_2) = exp(iP_k\Delta m^k) \ \ , \quad \Delta m^k = m_2^k - m_1^k \ \ . \tag{10}$$

   Rather remarkably, the contribution of the time development operator in the dynamics trivializes totally and there is no need to construct explicit representation of the momentum generators.

2. In order to get the propagator pole correctly it is necessary to assing with the propagator line the factor

$$I = \frac{1}{L_0 + i\epsilon} \ \ ,$$

   where L_0 is the representation of the Virasoro generator representing scaling in the Super-Kac-Moody algebra defined by $M^4 \times SO(3,1) \times SU(3) \times U(2)_{ew}$. In string models the propagator factor follows from the Hamiltonian time development operator defined by L_0. In present case propagator-factor should result from the vertex operators. The vertices at the ends of the Ramond type propagator line should be proportional to $1/G_0$ resp. $1/G_0^\dagger$. When the internal line corresponds to N-S-representation, the vertices at the ends of the propagator line should be proportional to the inverses of the super-generator $G^{\pm 1/2}$ resp. $G^{\mp 1/2 \dagger}$.

3. Internal lines contain also an exponential suppression factor $f(V)$ given by the exponent of the Kähler function for the piece of CP_2 type external defined by the line. This factor is given by

$$F(V) \quad = \quad exp(-S_K(X^4)) = exp\left[-\frac{V}{V(CP_2)}S_K(CP_2)\right] \ \ . \tag{11}$$

   X^4 is the four-dimensional submanifold of CP_2 having as its boundaries the 3-surfaces $X^3(U = 0)$ and $X^3(U = V/V_{CP_2})$. The latter form follows from the fact that Kähler action density is constant for CP_2 type extremals so that Kähler action is proportional to the volume of X^4. All functions U for which the internal line defines the same CP_2 volume give rise to the same Kähler action. In accordance with the conformal invaraince, there is no explicit dependence on the zitterbewegung orbit.

   The presence of the plane wave factors implies that the integration over the vertex positions multiplies the stringy propagator $1/(L_0 + i\epsilon)$ with an infrared suppression factor given by the Fourier transform of $F(V)$ which on basis of Lorentz invariance is only a function of invariant line length of M_+^4 (V and invariant line-length are alternative parameters for the internal line).

Scattering amplitude is obviously very sensitive to this factor and since the suppression factor determines the momentum dependence of the propagators, one can say that the laws of physics depend on the distribution for the functions $u(s^k)$ sensitively. This distribution is in turn constrainted by the requirement that CP_2 type extremals have suffered topological condensation on larger space-time surfaces.

2.5 Fundamental coupling constants as Glebsch-Gordan coefficients

The Glebsch-Gordan coefficients associated with the super-Kac-Moody algebra $M^4 \times SO(3,1) \times SU(3) \times U(2)_{ew}$ should be determined by a construction analogous to the vertex operator construction encountered in string models. In present case also a dramatically simpler approximative treatment suggests itself.

2.5.1 Vertex operator construction

The construction of the vertex operators could proceed roughly as follows.

1. If one requires that CP_2 type extremals form smooth surfaces one must assume that the vertex regions are deformed so that the vertex represents topological sum of two CP_2 type extremals. This means that vertex region has higher than 1-dimensional M^4_+ projection and is presumably non-vacuum classically. A simple analogy is that of gluing a cylindrical tube to another cylindrical tube smoothly. In principle there are three functions $U = V/V(CP_2)$ involved: denote them by U_i, $i = 1, 2, 3$. U_2 and U_3 are associated with the outgoing CP_2 type extremals and have value $U_i = 0$ at the vertex.

2. Since only 3-vertices are involved one can visualize the situation as flows associated with two incoming lines combining to single flow along the outgoing line. The CP_2 'time' coordinate $U(s^k)$ serves as the time parameter for the flow. One can continue the flowlines of the incoming flows such that they intersect the outgoing 3-surface $X^3(U = 0, out)$ surface. Thus it seems possible to divide the outgoing 3-surface $X^3(U = 0, out)$ to two parts $X^3_1(out)$ and $X^3_2(out)$ such that flow lines of the flows U_i, $i = 1, 2$ from two external legs $X^3_i(U, in)$, $i = 1, 2$ enter these regions.

3. This inspires the hypothesis that the fermionic quantum states associated with the two incoming lines are constructible using the oscillator operators constructed from the fermion fields of $X^3(U = 0, out)$ restricted to the region $X^3_i(in)$, $i = 1, 2$. This would allow to express the fermionic state at $X^3(U = 0, out)$ using the fermionic oscillator operators associated with the outgoing line and one would obtain a superposition of various states restricted by the conservation of basic quantum numbers.

4. Coupling constants $V(a, b, c)$ are not genuine constants of Nature since they are parameterized by arbitrary functions $U(s^k)$ associated with the incoming and outgoing lines. The dependence on these functions is expected to be very weak. This dependence is present irrespective of whether a complete localization occurs in zero modes or whether wave functionals are possible in zero modes.

The general construction is clearly akin to the construction of vertex operators in string models. For string models the fusion of incoming strings defines splitting of outgoing string to two parts and essentially similar relationship between incoming and outgoing oscillator operators results. In present case the situation is complicated by the fact that the fusing objects are 3-dimensional submanifolds of CP_2 rather than strings propagating in some higher-dimensional Minkowski space. On the other hand, the dynamics of the basic objects is almost trivial since CP_2 geometry is not affect at all by the warped imbedding. In any case, the vertex operator is in principle functional of the incoming and outgoing 3-surfaces X_i^3.

2.5.2 Simplified model for the vertices

One can construct a simplifed model giving a good idea about what for the vertex operators look like.

1. Idealize the projection of the vertex region to a point in M_+^4 so that the CP_2 type extremals are not deformed in any manner in the vertex region. To get a minimally non-singular surface one must assume that the functions u_i for the CP_2 type extremals define same 3-surface X^3 at the vertex. This means that the conditions $U_1 = constant$ for the incoming line and the conditions $U_2 = 0$ and $U_3 = 0$ for the outgoing lines define same 3-surface. This means that the three 'time-coordinates' U_i have same 3-surface as a common time=constant slice. What this condition means geometrically is that CP_2 type extremal branches: Y-shaped 1-dimensional surface is the homological equivalent of the resulting surface. In fact, the branching means that the situation is effectively 1-dimensional just as it is quantum field theories. Although this surface is singular it might provide a realistic idealization for the construction of vertices.

2. The picture suggests the possibility that, apart from creation or annhilation of fermion pairs, the Fock state representing the incoming particle simply splits into a product of the Fock states associated with the outgoing lines. This assumption is analogous to Zweig rule and would trivialize the vertex construction. If this approximation is sensical, vertices would be simply Fock space inner products between the initial state and the state created by the product of the operators creating final final states. QFT limit suggests that the operators creating the states are analogous to the products of quantum fields $\psi(x)$ at same point x, say $x = 0$. This requires that operators can be constructed as products of the operators which are sums of positive energy creation operators for fermion and negative energy annihilation operator for antifermion. This would perhaps make it possible to have nontrivial vertices since annihilation and creation of fermion pairs becomes possible in the vertex provided that the annilating fermions belong to different lines: this is essentially what Zweig rule states. For 'Zweig option' fermionic statistics implies that same fermionic oscillator operator cannot occur in each line. It is not clear whether the vertices for the emission of graviton can be non-vanishing in this approximation. For photon graviton vertex the total number of oscillator operators involved is just the minimal one to allow graviton emission. If this picture is correct, the effective values of various coupling constants at QFT limit should be determined by the average value of the exponential of the Kähler action associated with the propagator lines representing the particles.

3. Besides the conservation of various quantum numbers 'Zweig rule' suggests the conservation of the vacuum weight h_{vac}. This conservation law could be an excellent approximation quite generally. The conservation of h_{vac} eliminates very large number of the vertices involving exotic particles and gives strong constraints on the vacuum weights of the observed particles. For instance, in the emission of neutral gauge bosons vacuum weight is conserved. This means that Z^0, photon, gluon, and graviton must correspond to particles having a vanishing vacuum weight. Furthermore, the differences of the vacuum weights for the fermions inside electro-weak doublets must differ by the vacuum weight of W boson.

4. One must somehow take into account the fact that the fermions inside CP_2 type extremals move in different directions. The momentum directions of the incoming state and outgoing states are related by a rotation. This rotation corresponds to a unitary operator U_i, $i = 2, 3$

represented as an exponent of the angular momentum operator associated with the modified Dirac action. Therefore a natural idea is to perform the transformation $a \rightarrow U_i a U_i^{-1}$ for the fermionic oscillator operators of the incoming state.

5. The requirement that the vertices involve smooth topological sum of CP_2 type extremals implies that vertex regions cannot be vacua in a finite region surrounding the vertex point. Therefore it is not possible to have vertices which are too close to each other so that the sizes of the loops have lower bound, which saves from ultraviolet divergences. It is quite probable that the loop diagrams using the vertex operators obtained by allowing singular vertices give rise to ultraviolet divergences unless one introduces the ultraviolet cutoff by hand.

2.6 How to treat the zitterbewegung degeneracy?

Since CP_2 type extremals are non-deterministic, the calculation of S-matrix elements involves integration over all possible Feynman graphs with lines thickened to manifolds such that the positions of the vertices vary freely. One can however wonder what to do with the the infinite-dimensional degeneracy associated with the zitterbewegung.

2.6.1 Integral over zitterbewegung degrees of freedom diverges

It is easy to see that the functional integral over the zero modes without wave functional is non-sensical being like a path integral with a vanishing action (apart the classical Virasoro constraints) and thus horribly divergent. The constraints $L_n = 0$ can be conveniently represented as delta function in Fourier representation:

$$X = exp(i \sum \lambda_n L_n) \tag{12}$$

The functional integral over the real coefficients λ_n indeed gives factor $\delta(L_n)$ to the path integral. These terms give rise to effectively Gaussian functional integral over the zitterbewegung degrees of freedom. A little formal calculation demonstrates that the result is integral over λ_n for the inverse for the square root of the Gaussian determinant $det(\sum \lambda_n l_n)$, where l_n denotes the matrix determined by L_n in the Fourier basis for m^k as a function of u. The determinant is a homogenous polynomial of the variables λ_n so that one has Hilbert space integral of a homogenous polynomial. Taking Hilbert space distance $\sqrt{\sum \lambda_n^2}$ as the radial variable, one can decompose the functional integral into a product of radial integral and an integral over unit sphere. It is easy to see that the integral over the radial variable diverges.

2.6.2 In which degrees of freedom localization can occur?

The basic question is whether a localization in zitterbewegung modes occurs in each quantum jump or not.

1. Quaternion-conformal degrees of freedom correspond to zero modes in the sense that they do not contribute to the configuration space metric. If the localization in quantum jump occurs also in these zero modes in all length scales, one avoids difficult problems related to the definition of the functional integral over zero modes. Zitterbewegung indeed corresponds to a conformal degeneracy since light likeness conditions correspond to classical Virasoro conditions. On the other hand, to treat unitary condition properly, and in particular, to estimate the probability that particular zero modes appear as a final state, one must introduce orthonormal state functional basis in zero modes.

2. Zitterbewegung Feynman diagrams characterize the superposition of a generalized 3-surfaces (association sequences) defined as minimal sequences of spacelike surfaces with time like separations fixing the selection of a given zitterbewegung orbit. They characterize the final state of the quantum jump rather than being a fictive theoretical construct. CP_2 type extremals are actually continuous association sequences of 3-surfaces and effective 4-dimensional objects. One could assume that initial and final states have functional-integrable state functionals in zero modes. This assumption would correspond to a weakened form for the localization in zero modes: localization would occur only in a resolution defined by the p-adic length scale L_p.

3. The positions of the vertices do not contribute to the configuration space metric and are thus zero modes. One must however assign wave functions to them and these wave functions partially characterize the quantum numbers of the incoming and outgoing particles. Thus one must partially give up the dogma of a complete localization and perhaps replace it with a localization below the p-adic length scale L_p. It must be however emphasized that cm degrees of freedom are 'different' in the sense that they correspond to the imbedding space isometries rather than conformal degrees of freedom. These degrees of freedom also correspond to $n = 0$ generators of Super-Kac-Moody algebra.

4. Perturbation expansion should have genuine objective content. In particular, the sum over different Feynman graphs cannot correspond to a sum over discrete zero modes since in this case nothing would forbid the restriction of the sum to a contribution coming from a single graph. Rather, an integration over un-controllable configuration space fiber degrees of freedom should be in question. For purely topological reasons vertex regions cannot correspond to exact vacuum extremals of the Kähler action. Thus the vertices contribute to the configuration space metric and it is perhaps possible to regard the summation over Feynman graphs as an integral over fiber degrees of freedom.

2.6.3 Two options for how spin-glass property is realized

The randomness of CP_2 type extremals brings in two arbitrary functions $f(U)$ and $U = V/V(CP_2)$. From foregoing it is clear that there are two options for the treating this degeneracy.

1. Spin-glass property at quantum level:
   For each geodesic of the Feynman diagram extending from point P_1 to P_2 in Minkowski space along average curve which is geodesic line, one must integrate over all possible zitterbewegung orbits with a weight factor representing wave function in zero modes.

2. Spin glass property at classical level:
   Localization in zitterbewegung degrees of freedom is assumed and average over the scattering rates over some probability distribution for the zitterbewegung orbits is performed. One might hope that this distribution is determined to a reasonable degree by the exterbal conditions such as the space-time sheet at which CP_2 type extremal has suffered topological condensation. The presence of this distribution reflects the basic fact that the physical laws for spin glass are state dependent in the sense that there is probability distribution for the coupling constants. Certainly the classical option is simpler mathematically and might provide excellent approximation even in case that one localization in zitterbewegung zero modes does not occur.

Irrespective of whether one allows wave functionals in zitterbewegung zero modes or whether one assumes complete localization in these degrees of freedom, the prediction is that there are fluctuations in the values of the effective coupling constants and the functional form of the propagators since the exponent of the Kähler action depends on the function $u = f(U)$ appearing as effective time coordinate in the equation for CP_2 type extremal as as well as the functional form of $U = V/V(CP_2)$. This means that there are two arbitrary functions playing the role of hidden variables and probably not fixable by experimental conditions. This arbitrariness implies fluctuation in the values of coupling constants of effective low energy theories.

2.6.4 Does localization in zitterbewegung zero modes really occur?

One can represent an objection against the localization in zitterbewegung zero modes. The assumption motivated by the second quantization of the zitterbewebung motion is that the wavefunctionals in zitterbewegung zero modes correspond to the states created from the ground state Gaussian satisfying the quantum counterparts of the Virasoro conditions for zitterbewegung zero modes. The fact that zitterbewegung degrees of freedom should represent the bosonic conformal degrees of freedom, raises the question whether the Gaussian ground state wave functionals in zitterbewegung modes should be identified as a part of the ground state for a super-conformal representation. This picture would give a very close relationship with superstring models.

Even in case that the localization in the zitterbewegung zero modes does not occur, the independence of the vacuum functional factor on the zitterbewegung modes means the separation of the super-conformal degrees of freedom and of the vacuum functional, and one can indeed assume effective gauge fixing to a single zitterbewegung orbit. The oscillator Gaussian defines a cutoff in the zitterbewegung degrees of freedom and p-adic length scale emerges naturally, if it is the scaled coordinates m^k/L_p which are effectively quantized in the construction of the zero mode wave functionals.

2.6.5 Propagators are not sensitive to the choice of the zitterbewegung orbit but depend on the choice of the function U

Both the Glebsch-Gordan coefficients appearing in vertices and the exponent of the Kähler action determining the supression factor associated with the internal depends only on the function $U = V/V(CP_2)$ defining effective decomposition of CP_2 to time=constant slices. There is no dependence on the details of the zitterbewebung orbit itself. This suggests that one could indeed regard the choice of zitterbewegung orbit more or less as a choice of Virasoro gauge apart from the constraint that the quantized four-momentum is in the direction of the cm motion for the zitterbewegung orbit.

This simplifies enormously the situation since only the average over the functions U remains. The vertex factors are expected to depend rather weakly on the functions U associated with the incoming lines. The supresssion factor coming from the Kähler action is however exponentially sensitive to the choice of the function U so that propagators are sensitive functionals of U. Obviously one must perform some kind of averaging over the allowed functions U. Since Fourier transforms of the functions $u = f(U)$ appear in propagator lines, the averaging cannot be done in x-space.

As already found, the weight factor $F(V) = exp(-S_K)$ must very near to unity below some critical length scale above which Kähler action becomes 'full'. This length scale is naturally the p-adic length scale L_p. The values of $F(V)$ as function of the length of the propagator line are characterized by the function $U(s^k) = V/V(CP_2)$ associated with the propagator line and defining decomposition of CP_2 to 'time'=constant slices. The couplings in vertices are determined by Glebsch-Gordan coefficients. The dependence of $F(V)$ on length scale L determines the momentum dependence of the propagator on momentum transfer and implies effective infrared cutoff in length scale of order L_p. If the function $u = f(U)$ appearing in the definition of CP_2 type extremal is finite, infrared cutoff is absolute.

2.6.6 Do even elementary particles adapt?

The experience with quantum field theory suggests that the length scale L_p could be regarded as dynamically generated infrared cutoff length scale. The most plausible origin of this length scale is due to the topological condensation of CP_2 type extremals on, say MEs, having transversal thickness of order L_p. This naturally restricts the radii of the zitterbewegung orbits below L_p. This suggest that for physical particles CP_2 type extremals are glued to MEs of thickness of order L_p. This picture is in accordance with TGD based phenomenological view about what the observed particle should be.

The infrared cutoff coded by the extremely small value of the exponent of Kähler function above length scale L_p is coded into the geometric properties of the particle rather than forced by boundary conditions (closing the particle into a box). Thus even elementary particles are somewhat like biological systems able to adapt to their environment. This suggests a new view about color confinement and about the stability and coherence of the biological systems.

The previous considerations suggest that macroscopic predictions should be averaged over the functions U with the constraints coming from the requirement that zitterbewegung orbits stay within the space-time surfaces at which particles have suffered topological condensation. The distribution of allowed functions is to some degree result of evolution.

This picture conforms with the idea that self-organization and evolution occur already at elementary particle level. The p-adic length scales allowed by the p-adic length scale hypothesis can be seen as survivors in the selection induced by the dissipation caused by quantum jumps between quantum histories. Thus the values of the effective coupling constants and thus also physical laws would be to certain degree results of a generalized Darwinian selection.

2.7 Can one avoid infrared suppression and how the values of the coupling constants are determined?

CP_2 type extremals of infinite duration ($u = f(U) \rightarrow \infty$ at the limit $U \rightarrow U_{max}$) can appear as incoming and outgoing states since wave function normalization (division of the propagator factors of external lines away) compensates the supression factors coming from the external legs of the Feynman diagrams. In case of internal lines the situation is different. An interesting question is whether the exponential IR supression could be mildened by some mechanism and whether the mildened IR supression could in fact determine the values of the effective coupling constant strengths as proportional to the supression factors associated with the propagator lines emerging from the vertex. If virtual CP_2 type extremals have finite length ($f(U)$ is finite for all values of U), there is always also absolute length scale cutoff involved with the interactions induced by them. This cutoff could explain color confinement and imply deviations from QED at large distances.

2.7.1 Could CP_2 type extremals have a small volume?

CP_2 type extremals could have a volume which is only a small fraction of the full volume of the entire CP_2 type extremal: the exponents of Kähler action for the virtual particles in the vertex would thus define the values of the effective coupling constant strenghs.

1. This would be the case if generation-genus correspondence is due to the holes inside CP_2 type extremals and if CP_2 type extremals have considerable volume at the moment of absorption and emission. On the other hand, the volumes of the virtual CP_2 type extremals should be essentially the same irrespective of the genus of the hole since the couplings of the fermionic generations to photons are in an excellent precision the same. If the hole gives rise to a large reduction of the volume of CP_2 type extremal, it is difficult to understand why the reduction factor would not depend on the topology of the hole. The safest conclusion is that the hole should give rise to a negligibly small reduction of volume.

2. The simplified model for the emission of CP_2 extremal assumes that the 3-surfaces associated with the incoming and outgoing particles are identical at the vertex. Since one of these particles is the incoming particle, it is natural to assume that these 3-surfaces are far from point like so that a considerable reduction of the volume would automatically occur. One can also consider the possibility that CP_2 volume increases rapidly to its asymptotic value so that the incoming surfaces at the vertices are always in the asymptotic region and have volume near the maximal volume. This implies that the values of the effective coupling constants are determined by the the averages volumes of the CP_2 type extremals.

3. Gravitons could differ from other particles basically because the size of the gravitonic 3-surface at the moment of emission is very small. This could be understood if the vertex for the emission of graviton vanishes in the approximation in which vertex represents singular manifold homologically equivalent with a 3-vertex of QFT. This is quite possible and in accordance with the standard physical intuition that quantum field theory description of graviton is not possible but requires genuinely higher-dimensional vertices.

2.7.2 Magnetic compensation of the Kähler action

The magnetic fields associated with the space-time sheet at which the CP_2 type extremal representing the charged particle is condensed, could compensate the Kähler action of the CP_2 type extremal. This allows to circumvent the constraint coming from the large value of the Kähler action if one allows virtual CP_2 type extremals to have $f(U_{max}) = \infty$. This mechanism might be involved with the phase transitions of p-adic CP_2 type extremals in p-adic regions of space-time surface to matterlike real regions existing as virtual intermediate states but is not a plausible mechanism at the level of elementary particle physics.

What is needed is weak magnetic of Z^0 magnetic field whose net action is positive and larger than Kähler action. For a CP_2 type extremal of duration T the minimum value of the magnetic field strength must scale like $B \propto (L_p/T)^{1/2}$ so that at the limit of an infinite duration the magnetic field strength goes to zero. Charged particles generate dipole type magnetic with a positive Kähler action and Coulombic electric fields fields with a negative Kähler action. The net Kähler action should compensate the Kähler action of the CP_2 type extremal.

2.7.3 Could CP_2 type extremals be transformed to $D < 4$ extremals?

Infrared suppression could be avoided if CP_2 type extremals could be transformed to $D < 4$ extremals for which which the number of the compactified dimensions is $D = 0, 1, 2$ and possibly to $D = 3$. Of course, also the reverse of this mechanism is needed and here lies the first strong objection against this mechanism. Also this mechanism would relate effective coupling constants to the average value of the volume of the virtual CP_2 type extremal at the moment when the transformation to $D < 4$ extremal occurs. The simplest possibility is that CP_2 extremal transforms to $D < 4$ extremal with the same quantum numbers. It must be emphasized that this process very probably does *not* correspond to the dressing of bare particles not hadronization process of QCD as one might first think.

The value of the coupling constant for the emission of a particle would be exponentially sensitive to the rate of the transformation to $D < 4$ extremal and depends on the quantum numbers of the particle and on the function U characterizing CP_2 type extremal. Thus the coupling constant evolution for the particles appearing in the asymptotic states would be reduced to the estimation of this transformation rate as a function of p-adic prime. Nonperturbative effect would certainly be in question since quantum phase transition involving the reverse of compactification is involved.

In case of gravitons this transformation should occur so slowly that CP_2 type extremal could develop full Kähler action making the propagator extremely small and thus reducing the effective gravitational coupling constant from its value L_p^2 to G. In case of photon this transformation process should be so fast so that the reduction of the effective coupling of photon would not be so drastic. Also this picture suggests strong gravitation below length scale L_p which correspond to momentum transfers larger than $1/L_p$.

Of course, what 'slow' and 'fast' mean depends on the function $u = f(U)$ characterizing CP_2 type extremal and it could be that the transformation processes occur with same absolute rate for both photon and graviton and graviton and photon differ from each other in the sense that for photon the function U achieves its maximal value much more slowly than in case of graviton. This would mean that photons would have suffered Darwinian selection by self-organization so that they couple relatively strongly. This looks rather strange from the viewpoint of standard physics but TGD inspired theory of consciousness forces to take very seriously the idea about evolution and adaptation occurring already at elementary particle level.

There are however an immediate objection against this view.

1. In case of charged leptons the only possibility is the transformation to $D = 3$ extremals. This works if $D = 3$ extremals have small Kähler action and they exist and are stable, which is not plausible. Thus this mechanism could be at work only for massless particles like photons and gravitons.

2. A grave counter argument against this idea is that also the reverse transformations should occur in the interactions with charged particles and this does not seem plausible. Of course, these transformations could still take place.

3. The transformation leading from CP_2 sized objects to objects which have at least one macroscopic spatial dimension seems also implausible. Rather, the opposite process (say the decay of cosmic strings to CP_2 type extremals) seems more plausible on basis of reversibility arguments.

2.7.4 Some implications of the infared cutoff

If the volumes of the CP_2 type extremals associated with virtual massive particles are a considerable fraction of CP_2 volume, the presence of the infared cutoff coded to the properties of the CP_2 type extremal has definite consequences at the low energy limit of the theory and is thus a testable prediction. Deviations from QED behavior above electron length scale and color confinement are the most obvious predictions.

1. Are virtual charged lepton exchanges suppressed above Compton length scale?

The prediction of the infrared cutoff characterized by the p-adic length scale associated with the massive particle should have testable implications at the level of QED in long length scales.

Infared suppresions could be avoided if there is some mechanism compensating the negative Kähler action of CP_2 type extremal representing virtual electron and if virtual electron can propagate over arbitrarily long distances: this is however in conflict with p-adic length scale hypothesis and even the existence of this mechanism means deviations from QED.

If infrared cutoff is present, the diagrams containing virtual charged particles are strongly suppressed at low momentum transfers and theory becomes effectively classical in the sense that the tree approximation allowing only diagrams without virtual charged leptons becomes much more than a mere approximation at low momentum transfers. Creation of real pairs is possible but virtual pairs of charged leptons appear only in a space-time volume with size maller than the p-adic length scale L_p characterizing charged lepton. Also loops containing fermions are excluded.

If this picture is correct, QED would not be a correct theory of electromagnetic interactions of electrons above electron Compton length of electron since the suppression of the diagrams involving propagation of electrons over distances larger than electron Compton length is not taken into account. Atomic and molecular physics would represent basic examples of the phase in which fermions are effectively classical.

2. Mechanism of color confinement

The infrared cutoff coded into the structure of quarks and gluuons suggests an attractive mechanism of color confinemement. The command to stay confined would be coded to the functions U characterizing quarks and gluons so that these particles could not exist as free particles for time longer than the p-adic time T_p characterizing hadrons. Emission or absorption of gluon should occur sufficiently often to replace the old CP_2 extremal with a new one so that free quark and gluon would be extremely social creatures and could not exist with this strong interaction.

3 Does S-matrix at space-time level induce S-matrix at configuration space level?

The concrete construction of S-matrix continues to be a basic challenge of quantum TGD. Quantum-classical correspondence and the understanding of the equivalence of the generalized Feynman diagrams with tree diagrams in terms of a geometric symmetry realized at the space-time level encourages to think that configuration space S-matrix could be induced from space-time S-matrix. Stating it from a different angle, micro-locality would be realized at the configuration space level in the sense that the core element of the configuration space S-matrix would be space-time S-matrix acting in configuration space spin degrees of freedom identifiable in terms of states created by second quantized induced spinor fields.

7–3 duality implying effective 2-dimensionality provides quite a concrete view about how to construct S-matrix satisfying the basic constraints. By quantum classical correspondence space-time surface represents not only quantum states but also sequences of quantum jumps, and even the anatomy of single quantum jump. Indeed, the gauge invariance due to the deformations of light like 3-D CDs could be interpreted as a space-time counterpart for an infinite number of different quantum jumps consisting of unitary process, state function reduction and preparation and connecting given initial and final states. Quantum classical correspondence states that not only quantum states but also quantum jump sequences and even the complex anatomy of quantum jump must have representation at space-time level. This has far reaching implications. Single quantum jump and thus a particular space-time S-matrix should correspond to a finite space-time region, perhaps single space-time sheet with the maximal deterministic regions of the space-time sheet correlating with with the anatomy of quantum jump.

Since space-time surfaces represent quantum jump sequences and quantum states are configuration space spinor fields in the set of these space-time surfaces, quantum states become self referential referring to entire quantum jump sequence which led to it. This view is of considerable help in the attempts to formulate configuration space S-matrix in terms of much more concrete space-time S-matrix.

3.1 General ideas

It is appropriate to describe the general ideas relating to the second quantization of the induced spinor fields before the introduction of generalized Feynman rules.

3.1.1 The new way to second quantize

What is new is that physical states correspond to the generalized eigen states of the modified Dirac operator D [B4] satisfying $D^2 = 0$ whereas the solutions of $D\Psi = 0$ generate super conformal symmetries acting as gauge symmetries. Induced spinor fields are second quantized at the intersections of $X^2_\pm = X^3_l \cap X^7_\pm$ and obey anti-commutation relations fixed by the super-canonical algebra. Anti-commutation relations are posed on spinor fields associated with X^4_+ and X^4_- assumed to have identical eigenvalue spectra.

The second quantization differs from the standard one in TGD framework since besides ordinary positive energy fermions also negative energy fermions are possible. At the space-time level it does not make sense to speak about energy and momentum: rather, the modes of the modified Dirac operator are characterized by its eigen values λ expected to have both signs. There is no deep reason why the spectrum should be symmetric under the reflection $\lambda \rightarrow -\lambda$.

This could explain matter antimatter asymmetry. Matter would correspond to positive values of λ and antimatter to negative values of λ, and the dynamics would favor matter at positive energy space-time sheets and antimatter at negative energy space-time sheets. For instance, the number of negative eigenvalues λ could be simply smaller than the number of positive eigen values at positive energy space-time sheets. Positive energy matter and negative energy antimatter would be created in "big bang" from vacuum so that most of the matter and antimatter would be at different space-time sheets. Negative energy fermions would be analogous to phase conjugate photons allowing interpretation as negative energy photons propagating to the direction of the geometric past.

The modes having $\lambda > 0$ could correspond to creation operators and those with $\lambda < 0$ to annihilation operators. The alternative option would that the oscillator operators associated with both signs of λ are creation operators for Ψ. The situation would be analogous to a second quantization using a vacuum with a negative energy fermion sea and positive energy anti-fermion sea guaranteing a vanishing net vacuum energy. If the real energies are of same sign for $\lambda > 0$ and $\lambda < 0$ modes, both fermions and anti-fermions possess the same sign of energy at a given space-time sheet. This option does not however allow a representation of photon state as a local current like bilinear.

3.1.2 Generalized Feynman diagrams as generalizations of braid diagrams

The ability of simple planar diagrams to represent extremely abstract mathematical structures such as 3-topologies, knots and braids, allowing concrete calculations is almost magic. This inspired the idea that also the incredibly abstract configuration space geometry and dynamics could be represented by a generalization of braid diagrams [E10, C5, E9]. Braid diagrams indeed define a topological S-matrix by simple local rules having deep relevance to anyon physics and topological quantum computation [E9]. This adds more weight to the view that a generalization of braid diagrams would be much more appropriate notion than Feynman diagram in TGD framework. The discovery of the 7–3 duality and effective 2-dimensionality realized this dream.

1. Braiding must be introduced by a flow

One can imagine of assigning two kinds of braidings with the generalized Feynman diagrams.

1. The first braiding would be associated with the X^3_l interpreted as an orbit of X^2 and induced by some naturally occurring flow in complex coordinates for X^2. Marked points would correspond to the points assigned with the local operators creating the state. This braiding would allow interpretation as a flow.

2. Second braiding would be assigned to the sphere S^2 associated with $\delta M^4_\pm = S^2 \times R_+$ (actually projective sphere). One can indeed imagine that $\delta M^4_\pm$ is translated continuously along the line connecting the two vertices connected by X^3_l, and that the braiding is induced by the projection of the trivial flow $w_t(z) = z$ at X^3_l to $S^2 \times R_+$ and inducing a trivial braiding in X^2 coordinates. This braiding would not have any obvious generalization to a flow and seems thus an implausible option.

The connection with braidings and central extensions of Kac Moody and Virasoro algebras [17, 18] suggests that either of the braidings is non-trivial.

Consider now the situation in more detail in case 1), which seems to be the only plausible candidate.

1. The metric 2-dimensionality of X_l^3 means that one can decompose it locally into a product $X^2 \times R_+$ such that that the embedding of the complex surface X^2 varies with the light like coordinate t of R_+. The anti-commutation relations are two-dimensional in the sense that the anti-commutator involves a delta function in X^2 coordinates.

2. It is not quite obvious that the anti-commutator $\{\overline{\Psi}(z_1, t_1), \Psi(z_2, t_2)\}$ is the simplest possible one and thus proportional to $\delta(z_1, z_2)$. There could be a flow taking the points z of $X^2(t_1)$ to points $w_t(z)$ of $X^2(t_2)$ such that the anti-commutator (w and x do not denote complex coordinate here)

$$\{\overline{\Psi}(w_t(z)), \Psi(z)\}$$

   is non-vanishing (essentially delta function). This flow could involve also a unitary time evolution $U(t)$ mixing components of Ψ for a given eigenvalue λ. This flow or the flow induced by it at $\delta M_+^4 \times CP_2$ could define a non-trivial unitary representation $U(t)$ of the braid group with any n points of X^2 defining the initial configuration of the braid. If one can allow also a unitary transformation $U(t)$ of the modes Ψ_λ along X_l^3, the representation of the braid group becomes non-Abelian. Both ordinary rotation and electro-weak rotation in spinor degrees of freedom could contribute to the braiding operation.

2. Particle massivation and braiding

A strong grasp on the physics behind braiding comes from the realization that the flow induced by the braiding could explain the decay of correlations in turn giving rise to the mass of the parton. The individual contributions to the general mass squared eigenvalues of partons are not fixed uniquely in the general mass formula discussed in [B4, F2]. The proposed identification of parton masses is only the simplest possibility, and one could add to the individual parton conformal weights contributions compensated by the super-canonical conformal weights. This might however be a blessing rather than a curse since it suggests a manner to understand particle massivation at the fundamental level.

1. A natural requirement is that parton masses are consistent with the poles of the S-matrix elements. This assumption is quite general and certainly makes sense if the S-matrix elements allow a decomposition to vertices and propagators for a tree diagram.

2. The time evolution for the quantum states of individual partons, which are always on mass shell (that is generalized eigen states of the modified Dirac operator) is a unitary process, and corresponds to a braiding for an N-puncture system defined by the product of N local operators creating the parton state. The basic requirement is that the flow contains information about the presence of other partons and thus about the normal derivatives of the imbedding space coordinates at X_l^3. Hence the S-matrix indeed contains information about the interior of the space-time surface and the effective two-dimensionality is indeed only effective. The condition that the S-matrix elements remain unchanged in conformal transformations obviously poses explicit conditions on the normal derivatives and can be regarded as conditions stating the vanishing of the corresponding beta function.

   The best candidate for the flow is as the hydrodynamic flow defined by the discontinuity of the energy momentum tensor associated with the Kähler action at X_l^3 representing what might be regarded as a hydrodynamical shock wave. This flow is in general not integrable in the sense that one could assign a global coordinate varying along the flow lines. By identifying the points of X_i^2 and X_f^2 having the same value of the complex coordinate z, the flow $X_i^2 \rightarrow X_f^2$ defines a map $w : (x,y) \rightarrow (u(x,y), v(x,y))$ mixing the points of X_i^2. The inner products of the states created by the local operator $\Psi_n(x,y)$ creating one-parton state at X_i^2 with the state created by the transformed operator $\Psi_n(u(x,y), v(x,y)$ define correlation functions, which vanish above some length scale determining the mass of the parton. Massivation occurs if this map fails to be a conformal transformation.

3. By quantum classical correspondence this braiding can be also regarded as a braiding for the points of X_i^2, which correspond to the super-canonical conformal weights just like the points of the celestial sphere correspond to momenta. If the number of operators creating the parton state is larger than two, the minimal number three of threads in the braid is present and the conformal weights become "off mass shell" conformal weights. The massivation however can in principle occur always. In [E9, C5] the proposal was made that the bound state conformal weights could correspond to the zeros of poly-zetas: obviously this is a very strong prediction. Riemann Zeta would correspond to Higgs zero phase with the minimum of 2 operators with conjugate super-canonical conformal weights creating the state.

4. The successful description of particle massivation in terms of p-adic thermodynamics for the Virasoro generator L_0 (with p^2 not included) of the partons encourages to think that the change of the parton conformal weight could be understood as a generation of a thermal conformal weight by the flow induced by an ergodic braiding flow. This interpretation would allow to circumvent the problem created by the fact that the thermalization for mass squared is not consistent with the Lorentz invariance.

3. What is the flow defining the braiding?

The basic condition on the braiding is that it contains information about the interior of X^4 and thus about interactions with other partons. Second constraint is that the braiding flow is trivial for massless particles such as photon for which the space-time correlate should correspond to X_l^3 carrying a light-like energy momentum current.

The components $X^{n\alpha}$ of some tensor field with α restricted to X_l^3 define the most natural candidate for the braiding flow. The existence of the preferred light-like normal coordinate x^n constant at X_l^3 (in the case of δM_+^4 the light like coordinates would be $x^{\pm} = t \pm r$) is essential to achieve general coordinate invariance.

The discontinuity of the normal component $T^{n\alpha}$ of the energy momentum tensor associated with the Kähler action is a good candidate. At light like CDs the discontinuity of $T^{n\alpha}$ could be non-vanishing if allows light like CD to carry a shock wave also in imbedding space degrees of freedom as suggested by the super-symmetry. The discontinuity of $T^{n\alpha}$ would have an interpretation as a shock wave like hydrodynamic flow at the boundary. For massless particles the energy momentum current would have only a longitudinal component, the braiding would be trivial and particle would remain massless. The appearance of the energy momentum tensor in the definition of the S-matrix conforms with the hydrodynamic character of field equations and with the fact that the theory must be also a quantum theory of gravitation.

This guess is supported by the modified Dirac equation. By multiplying the modified Dirac equation at X_l^3 for shock waves localized at X_l^3 with the o_t defined by the light like gamma matrix along X_l^3 and doing the anti-commutations with the modified Dirac operator D, one finds

$$T^{\alpha n} D_\alpha \Psi = 0 . \tag{13}$$

The equation states that Ψ is covariantly constant along the flow lines of the flow defined by $T^{\alpha n}$. The equation can be written as

$$\begin{aligned} D_t\Psi + v^i D_i\Psi &= 0 \ , \\ v_i &= \frac{T^{ni}}{T^{nt}} \ . \end{aligned} \tag{14}$$

This differs from a standard flow equation for a quantity Ψ moving along field lines only by the fact that ordinary derivatives ∂_α are replaced by covariant derivatives D_α. This means that Ψ suffers a braiding transformation in spin and electro-weak degrees of freedom. Obviously, this equation states super-conformal invariance in the sense that it is not possible to poses the values of Ψ arbitrarily in entire X^3_l but only at X^2. One could regard TGD as anyonic shydrodynamics at the space-time level. The usual dispersion characterizing Schödinger equation emerges only at the level of imbedding space when one assigns wave equations to propagators defined by S-matrix elements.

Since the induced spinor connection is continuous at X^3_l, the discontinuity for this equation reads as

$$\begin{aligned} w^i D_i\Psi &= 0 \ , \\ w_i &= \Delta\left[\frac{T^{ni}}{T^{nt}}\right] \ , \end{aligned} \tag{15}$$

thus defines an adiabatic series of braiding flows.

The naive guess is that unitarity requires that the ordinary inner product for scalar functions is preserved in the flow so that the flow defined by v^i is volume preserving. This would require that the vector field $v^i = T^{ni}/kT^{nt}$, which is analogous to a velocity field, has a vanishing diverge for some choice of the function k and thus defines a symplectic flow of X^2 generated by some Hamiltonian. This would pose an additional condition to the solution of field equations at X^3_l.

3.1.3 Minimal braiding without crossings and the number of fermion families

Elementary particle vacuum functionals vanish for genera $g > 2$ if X^2 is hyper-elliptic: this explains why there are only three fermion families. The assumption that X^2 corresponds to a complex sub-manifold of the imbedding space regarded as an octonionic manifold means that the tangent space at each point of X^2 can be regarded as a complex plane and therefore allows a conjugation operation. If this operation can be continued to a global conformal Z_2 symmetry, hyper-ellipticity follows and X^2 would always have $g < 3$. The task is to understand what $g < 3$ condition could mean.

Suppose that X^2 represents forward scattering. If the preferred points of X^2 behave like anyons rather than ordinary fermions and bosons, they can suffer also non-trivial topological scattering represented by a topological S-matrix representing unitarily the braiding suffered by the lines. In the recent case this braiding is defined by the ribbons composed from the points with conjugate super-canonical conformal weights.

The natural requirement is that the topology of X^2 is such that it is able to represent the braiding operation without crossing of the lines. This is always achieved by just adding a handle along which the line goes over the other one. The question is whether the braiding operation is possible using only $g < 3$ topologies. It seems that two handles are enough.

Consider a trivial braid with N lines, and imbed it on a surface of a cylinder. Any braiding inducing a cyclic permutation of the lines can be performed by just twisting all the lines in the same direction around the cylinder so that a permutation $12..n \to k...n...n-k+1$ results. The braiding $12...n \rightarrow 21...n$ is possible without crossings only if one introduces a handle connecting the positions of 1 and 2. 2 goes to 1 along the surface of the cylinder and 1 to 2 along the handle. The braidings leading to permutations $12...k(k+1)..n \rightarrow 12...(k+1)k..n$ can be performed by performing first the cyclic permutation $12...k(k+1)...n \rightarrow k(k+1)....(k-1)$, doing then the braiding $k \leftrightarrow k+1$ using the handle, and then carrying out the inverse of the original cyclic permutation. Compactifying the cylinder to a torus by connecting its ends gives rise to a $g=2$ surface.

Thus $g=2$ topology is the minimal requirement for carrying arbitrary braiding. The requirement that the braiding is always possible implies that $g=0$ and $g=1$ surfaces can represent only the braidings of $N<3$-particle states whereas $g=2$ surface can represent classically arbitrarily high number of particles. One could say that 3 particle generations are enough for representational purposes.

This argument looks attractive and leaves only braiding induced by the flow lines of the vector potential A_K of Kähler form into consideration. This braiding does not allow a global coordinate varying along the flow lines in the general case but this does not lead to any obvious problems.

3.1.4 Vertices do not correspond to smooth 4-surfaces

The question is whether the new conceptual framework allows to describe elegantly the most essential element of interacting quantum field theory: the decay of a boson to a fermion pair, which necessitates a non-linear interaction terms in quantum field theory framework and leads to the divergence difficulties. In contrast to what one might think first, the decays and fusions of 3-D CDs described by smooth 4-surfaces do not help. These topological decays only distribute the probability amplitude between two different CDs much like in a double slit experiment in which photon travels through two paths.

The solution of the puzzle was based on the old vision of Feynman about final state particles as particles moving to the direction of past in terms of space-time topology and implied by crossing symmetry. Now negative energy states however have a concrete physical interpretation as phase conjugate states. Pair annihilation of a boson to a fermion pair can be regarded as a process in which boson from past and fermion pair from future meet at the 2-dimensional surface X^2 at a bosonic 3-sheet $X^3 \subset X^7$: the two kinds of space-time sheets emerge from the two sides of δM^4_+. The meeting of light like 3-D CDs entering from different sides of a 7-D CD corresponds to various particle vertices, and must be distinguished from time reflection in which the sheets are at the same side of the 7-D CD.

This description allowed by crossing symmetry forces naturally the branching of 2-surfaces at vertices. Crossing symmetry however leads to an equivalent description for the decay as a branching of bosonic 2-surface to two fermionic 2-surfaces, all possessing positive energies. Homologically the ordinary Feynman diagram describes the process correctly and the original belief that a smooth 4-manifold describing a decay 3-surface could describe particle decay is wrong. The elimination of non-linear interaction term requires to give up smooth manifold topology and to allow branching. Also boson-fermion scattering should involve tangential discontinuity of the space-time surface. The discontinuities of space-time surfaces, obviously closely related to the 3-D CDs, reflect the discontinuity of quantum jump at space-time level.

3.1.5 The lines of generalized Feynman graph as on mass shell particles and triviality of loops

The fundamental gauge invariance for the extremals of Kähler action states that all generalized Feynman diagrams defined by light like 3-D CDs are equivalent with tree diagrams. One can ask whether this gauge invariance generalizes so that both positive and negative energy space-time sheet are involved. If so it could be applied also the the initial state at X^7_+ at which pair of positive and negative energy states with vanishing net quantum numbers is created. Any loop going to future and returning back should give back the initial state. This would give nothing but the unitarity of the S-matrix since the S-matrix associated with positive and negative energy space-time sheets would be hermitian conjugates of each other. This assumption together with on mass property of propagating fermions implies the equivalence of loop diagrams with tree diagrams.

1. One mass shell condition implies the equivalence of loop diagrams with tree diagrams

The key observation is that the eigen states of D at light like 3-D CDs representing the lines of the generalized Feynman diagram are propagating on mass shell modes with the momentum replaced with the eigen value λ of the modified Dirac operator D. One can assign four-momenta only to the incoming and outgoing particles: they do not appear as labels of the intermediate states of the generalized Feynman diagram. This is due to the fact that four-momentum cannot be assigned to a fermion at space-time sheet but to a configuration space spinor field. Indeed, M^4 translational degrees of freedom are associated with the sectors of the configuration space sectors defined by the unions of $X^7_{\pm}$.

The on mass shell property suggests an amazingly simple manner to understand why loops do not affect the Feynman diagram. Already in the ordinary quantum field theory the restriction of virtual particles in the loop on mass shell gives nothing but $SS^{\dagger} = 1$ for the space-time S-matrix acting in the fermionic degrees of freedom.

2. A delicate point related to the sign of α_K

This raises a rather delicate point. The change of the sign of α_K is an attractive idea and might be also important in guaranteing that the actions of CP_2 type extremals associated with positive and negative energy particles cancel each other. If the values of the Kähler coupling strength are same for the two time orientations, space-time sheets and their time mirror images would be complete copies and the time reflected evolutions of fermionic Fock states would be identical with the original. The change of the sign of α_K however means that Kähler electric (magnetic) fields are favored for X^4_+ (X^4_-) so that an asymmetry results and the states are not identical. This would however mean the failure of the generalized form of the gauge invariance and unitarity in the proposed form.

There are thus two options.

1. If one wants to keep the gauge invariance in the most general sense, the sign of α_K must be the same for X^4_+ and X^4_-. In this case genuine loops are possible and the possibility to eliminate them would imply also unitarity condition $SS^{\dagger} = 1$ for the space-time S–matrix associated with a particular loop.

   This would also allow to resolve at quantum level the original paradox that led to the idea about different signs of α_K. At the moment of big bang corresponds to time reflection of negative energy cosmic strings at $\delta M^4_+ \times CP_2$ to positive energy cosmic strings. The problem was that the dissipative dynamics is expected to imply the decay of cosmic strings also at the negative energy space-time sheets but in a reverse time direction of geometric time. How it is possible that the cosmic string from the geometric future have been able to survive? The change of the sign of α_K would favor magnetic fields instead of electric fields and would make cosmic strings stable final states of self-organization. The quantum solution of the paradox is to see the situation is as creation of matter from vacuum rather than time reflection. Whether the dissipative evolution for the extremals of Kähler action proceeds in the same time direction at both space-time sheets must be left open. If it does then negative energy cosmic strings are stable.

2. If the signs of α_K are different, all loops that can be eliminated should preserve the character of the space-time sheet as X^4_+ or X^4_-. The annihilation of a boson to a virtual fermion anti-

fermion pair could not occur at all. The only loops would correspond to smooth manifold topologies and to a temporal flow of single fermion probability amplitude along two different routes rather than a creation of a virtual pair which is a process occurring at Fock space level. This option looks implausible.

3.2 Feynman rules

The Feynman rules are in a concise form following.

1. Positive/negative energy partons, identifiable as partons and their phase conjugates, reside at future/past directed space-time sheets. Crossing symmetry allows to relate a process in which negative energy partons appear in the initial state to a process in which they are replaced by positive energy partons in the final state. The vacua associated with partons and their phase conjugates relate like Dirac's bras and kets to each other.

2. There are no propagators associated with the parton lines. They are replaced by unitarity matrices $U_\lambda(t)$ acting in spin and electro-weak spin degrees of freedom representing single particle unitary time evolution U_λ with respect to the light like coordinate t along X^3_l connecting the vertices. U_λ defines braiding matrices in the projective sphere defined by $\delta M^4_\pm$.

3. Vertices correspond to surfaces $X^2 \subset X^7_\pm$ at which parton surfaces branch are branched. Vertices are vacuum expectation values for the operators creating the resulting zero energy state.

An important difference with respect to the standard Feynman diagrams is that vacuum lines are possible. Vacuum extremals are obviously the space-time counterpart for them and for them all spinor degrees of freedom are pure gauge. For instance, the smooth decay of a quark 2-surface to two can be understood as a formation of superposition of states for which either line is in a vacuum state. In the ribbon algebra formalism based on the generalization of braid diagrams to tree diagrams vacuum lines have identity morphism as their counterpart [C5]. Note that vacuum extremals are not vacua with respect to gravitational energy and gravitational quantum numbers.

The cancellation of divergences can be understood from the replacement of local composite states with many particle states of a free super-conformal fermionic field theory at 2-surfaces X^2. The vertex functions become essentially weighted averages of N-point correlation functions appearing in the construction of states, with N determined by the total number of fermions and anti-fermions in the vertex.

3.2.1 Partons and particles

The basic structures are 7-D light like CDs of form $X^7_\pm = \delta M^4_\pm \times CP_2$ and 3-D light like CDs X^3_l representing lines of generalized Feynman diagrams. Each incoming/outgoing particle corresponds to a 3-surface $X^3_\pm \subset X^7_\pm$ containing X^2_i as partons. Bound states result by connecting 3-surfaces by join along boundaries bonds so that they correspond to same 7-D CD. Each incoming/outgoing free particle corresponds to its own $X^7_{+/-}$. The 3-D CDs connecting 7-D CDs brings in mind strings connecting branes in M-theory. The notions of composite particle and parton emerge at the level of the basic definitions and the theory also describes the reactions in which partons arrange to new particles. Hadronic physics is obviously the fundamental applications for the theory. Perturbative QCD would correspond to a restriction of the theory to single X^3.

Some notation is in order. Let capital letters $I, J, ...$ label the 7-D CDs and small letters $i, j, ...$ label the 3-D CDs. One can in principle continue the eigen modes of the induced spinor field from the partons $X^2_{+,I,i}$ along the generalized Feynman diagram defined by $X^3_{l,I,i} \subset X^7_{+,I}$ to the parton surfaces $X^2_{-,J,j} \subset X^7_{-,J}$. The eigen modes continued from $X^7_{+,I}$ through the generalized

Feynman diagram without encountering vertices $X^7_\pm$ is traversed are in general superpositions of eigen modes at X^7_-.

3.2.2 Vertices: $BF\overline{F}$ vertex as an example

The eigen values λ replace four-momenta in the propagator lines. Parton lines can contain arbitrarily many fermions each accompanied by unitary operator $U_\lambda(t)$ relating Ψ_λ at the end of line to Ψ_λ at the beginning of the line. Local composites, such as photon interpreted as a local composite of fermions and anti-fermions, are allowed since everything commutes at a given space-time sheet and there are no normal ordering problems. The local composite property is preserved along the line since the evolution of Ψ in the longitudinal direction is induced by a flow allowing no dispersion. Vertices can be regarded as having the eigen values λ_i associated with various incoming and outgoing particles as labels instead of the momenta.

The equivalence of generalized Feynman diagrams with tree diagrams requires that the space-time S-matrix is same for all space-time surfaces for which $X^2_{\pm,I,i}$ are identical and for which also 3-D tangent space of $X^3_{l,I,i}$ at $X^2_{\pm,I,i}$ are same. The generalized Feynman diagrams are like paths for analytic continuation and this suggests that the notion analogous to homotopy equivalence might be relevant. Hence a possible additional condition is that equivalent Feynman diagrams are related by a conformal symmetry. The existence of a large number of conformal homotopy classes could relate to the p-adic coupling constant hierarchy. On the other hand, p-adic mass calculations support the conclusion that p-adic prime characterizes X^2 rather than conformal homotopy class of different surfaces X^3_l having X^2 as its its end. Also the fact that X^3_l with different topologies should define equivalent generalized Feynman diagrams dis-favors the notion of conformal homotopy.

It is instructive to consider $BF\overline{F}$ vertex as an example.

1. The gauge boson must correspond to a bi-local fermion-antifermion operator since for a local operator the norm would be infinite. The bi-local operator involves a kind of structure function in $X^2 \times X^2$ allowing visualization as a line connecting two points x and y having fermion and anti-fermion at its ends. The bi-local current would be sum of two terms

$$B = \int_{X^2\times X^2} dV_x dV_y B(x,y)\left[\overline{\Psi}(x)\Gamma(y)\Psi(y) + \overline{\Psi}(x)\Gamma(x)\Psi(y)\right] \ . \tag{16}$$

   Here Γ involves various vertex operators acting on spinor fields such as a contraction of a polarization vector with gamma matrices in case of spin one bosons, and M^4 derivative operators/color isometry generators in case of graviton/gluon. The vacuum expectation value determining the vertex would boil down to a correlation function defined as integral over $X^2 \times X^2$ and bilinear of functions formed from positive energy fermion and anti-fermion. $B(x,y)$ is determined by the super conformal invariance essentially as a correlation function [F2].

2. The vertex involves a vacuum expectation value for the product of operators creating boson and fermion and anti-fermion. The definitions of fermionic operators involve an integral over X^2. Fermionic anti-commutators boil down to delta functions so that a two-point weighted average over $X^2 \times X^2$ of the correlation function with kinematical factors at the ends ends is the outcome.

Conformal theory alone gives no hint about how the coupling constant appears, and configuration space-integral is necessary to understand the emergence of the gauge coupling.

1. A strong hint comes from the facts that all coupling constants, except possibly gravitational constant, must be proportional to Kähler coupling g_K. The most natural manner to achieve this is to require that the bosonic configuration space spinor fields vanish at the maximum of the Kähler function where the perturbation series is developed. That bosons should correspond to small perturbations around the maximum of the Kähler function is in accordance with the assumption that quantum fields correspond to the perturbations around the extrema of the action functional. This means that one can write $B(x,y)$ in the form

$$\begin{aligned} B(x,y) &= \partial_I K B^I(x,y) \ , \\ \partial_I K(X^3) &= 0 \ \text{at the maximum of } K \ . \end{aligned} \tag{17}$$

Gere $\partial_I K$ denotes partial derivatives of Kähler function with respect to the configuration space coordinates X^I vanishing at the maximum of K.

2. The functional integral in the lowest order approximation is obtained by expanding $B(x,y)$ in lowest order to functional Taylor series in using the coordinates X^I

$$B(x,y) = \partial_R K \times B^I(x,y) \times X^R \ , \tag{18}$$

It is understood that also $B^I(x,y)$ allows functional power series expansion as a functional of X^3. In the lowest order approximation the norm N of the boson state is given by the functional integral

$$\begin{aligned} N &= \langle \int_{X^2\times Y^2} \overline{B}(x,y)B(x,y)dV_xdV_y\rangle = A_{IJ}\times B^{IJ} \ , \\ A_{IJ} &= \partial_I\partial_R K \times \partial_J\partial_S K \times \langle X^R X^S\rangle \ , \\ B^{IJ} &= \int_{X^2\times Y^2} \overline{B}^I(x,y)B^J(x,y)dV_xdV_y \ . \end{aligned} \tag{19}$$

Here $\langle X^R X^S\rangle$ is a two point function defined by the functional integral over small perturbations around the maximum of Kähler function Specifying the coordinates to complex coordinates and using the covariant Kahler metric $G_{K\overline{L}} = \partial_K\partial_{\overline{L}}K$ as the kinetic term. Since the contravariant Kähler metric defines the propagator, the lowest order approximation gives

$$N = G_{K\overline{L}} \times B^{K\overline{L}} \ . \tag{20}$$

What is nice that the symmetry considerations allow to determine the covariant metric highly uniquely and the propagator disappears from the final formula. The normalization factor $1/\sqrt{N}$ of the boson state is obviously proportional to g_K since the Kähler function K is proportional to $1/\alpha_K$.

3. Fermion boson vertex is indeed proportional to g_K. $B(x,y)$ must be expanded in a functional Taylor series up to a second order term

$$B(x,y) = \partial_R K B^I(x,y)X^R + \partial_R K \times \partial_S B^I(x,y) \times X^R X^S + \cdots \ . \tag{21}$$

The general expression of the $BF\overline{F}$ vertex is

$$\begin{aligned} V_{BF\overline{F}} &= \frac{1}{\sqrt{N}}\langle\int_{X^2\times Y^2}\overline{B}(x,y)\Gamma dV_x dV_y\rangle = \frac{1}{\sqrt{N}}A \ , \\ A &= \int_{X^2\times Y^2}\partial_I\overline{B}^I(x,y)\Gamma dV_x dV_y \ . \end{aligned} \quad (22)$$

The propagator compensates the second order derivatives of Kähler function in the functional integral average, and the vertex is indeed proportional to g_K.

3.2.3 How to deduce the exponent of Kähler function?

The exponent of Kähler function appears in the Feynman diagrams. If the exponent for the difference of Kähler actions at neighboring maximal deterministic space-time regions, call them R_n, equals to the ratio of Dirac determinants for the modified Dirac operator in the region separating these regions then it is possible to deduce the value of Kähler function assuming that there is a maximal deterministic region, say R_{n_0}, in which Kähler action vanishes. Of course, one can also take the attitude that the Dirac determinants are the fundamental quantities and just hope that their ratios are expressible using the exponents of Kähler action.

Consider arbitrarily region R_{n_1} and construct a path $R_{n_1}R_{n_2}...R_{n_K}R_{n_0}$ leading from R_n to R_0. Form the product $E_{n_1} = D_{n_1n_2}D^{-1}_{n_2n_3}D_{n_3n_4}....D^{(-1)^{K+1}}_{n_Kn_0}$ of Dirac determinants. This product gives the exponent of Kähler action in R_{n_1}. The same procedure applies to any region inside connected space-time region. By multiplying the resulting exponents E_{n_k}, one obtains the value of the exponent of the Kähler function.

Dirac determinant D_{mn} is for the determinant bundle what curvature form is for ordinary bundle and therefore one might expect that it contains all the physics. Hence it is quite possible that there is some kind of gauge principle which might simplify the situation: for instance, the normalization of the configuration space spinor fields might imply that Dirac determinants are all that is needed and that there is no need for the proposed construction.

A second problem is computation of the configuration space propagator as the inverse of the Kähler metric. This should be possible from the expression of the Kähler metric deduced from super-canonical invariance.

3.2.4 How to understand gauge coupling evolution?

The proportionality of gauge couplings to g_K fixes the bulk part of the dependence of the coupling constants on the length scale and phase resolution characterized by the p-adic length scale and Beraha numbers B_n determining the value of $\hbar(n)$. Quite generally, the general coupling constant evolution would trace a path in the 2-dimensional discrete space with points (p,n) determined by p-adic length scales and Beraha numbers $B_n = 4cos^2(\pi/n)$, $n \geq 3$ directly related to the algebraic extensions of p-adic numbers.

The length scale evolution of Kähler coupling strength is typical $U(1)$ evolution and cannot explain the different length scale evolutions of electro-magnetic, weak, and color coupling strengths. The value of Kähler coupling strength for $L(k=2)$ which corresponds to CP_2 length scale is $\alpha_K \simeq .04$ as predicted from its value at $L(k=127)$ very near to fine structure constant $\alpha \simeq 1/137$. Color coupling strength according to QCD decreases as high energies and the increase of fine structure constant is slower ($1/\alpha \simeq 128$ and $1/\alpha_K \simeq 101$ at intermediate boson length scale $L(89)$).

The different dependence of the n-point functions on p-adic length scale should explain the different dependence on length scale. The typical size of the surfaces X^2 determined by the p-adic length scale is expected to affect the coupling constant evolution and the effects would depend on the detailed structure of the particles involved. By quantum criticality the beta functions of the conformal field theory associated with X^2 vanish but the dependence of the fixed point values of the coupling constants of the super-conformal field theory on p-adic length scale would affect the length scale dependence. What should be done is to try to understand how the spectrum of the critical values of coupling constants emerges in conformal field theory framework.

The hypothesis that the perturbative character is preserved would force the increase of $\hbar$ to compensate for the increase of color coupling strength. The maximal reduction would be by factor $1/2$ in the range $n \geq 4$. At the last step a transition to $n = 3$ confining phase with universal properties would occur [D6]. An interesting possibility suggested by TGD based view about coupling constant evolution is that there is entire hierarchy of QCDs which are not asymptotically free which would mean that coupling constant would start to increase above and below certain critical p-adic length scales. The hypothesis that perturbative character is preserved by the increase of $\hbar$ would lead to a decrease of $\hbar$ and the last means of achieving this would be transition to $n = 3$ confining phase with universal properties.

3.3 S-matrix

The first guess is that configuration space S-matrix is induced from space-time S-matrix so that only convolution to construct the full S-matrix. This picture is over-simplification but also in a more general case similar factorization of the dynamics in "orbital" and "spin" degrees of freedom occurs.

3.3.1 Space-time S-matrix

S-matrix at the space-time level is obtained by constructing the amplitudes using propagators and vertices defined in the proposed manner at surfaces X^2 at which positive and negative energy space-time sheets coming from different sides of 7-D CDs meet.

The equivalence of generalized Feynman diagrams with tree diagrams suggests the existence of a hierarchy of unitary S-matrices. Indeed, loops starting from X^7_+ and ending up to a fixed X^7_- and returning back to X^7_+ would give rise to a unitary sub-S-matrix when the fermionic states at both ends are allowed to vary. In fact, all continuations of configuration space spinor fields between different sectors of the configuration space give rise to S-matrices so that the family of S-matrices is labelled by the initial and final sectors. Also the hierarchy of space-time sheets labelled by p-adic length scales would reflect itself as a hierarchy of S-matrices. The existence of these hierarchies would fit nicely with the thinking of practicing experimental physicist. Space-time S-matrix corresponds to S-matrix at single point of the configuration space restricted to configuration space spin degrees of freedom.

3.3.2 Configuration space S-matrix

The simplest guess is that configuration space S-matrix $\mathcal{S}$ could be constructed by taking matrix elements of the space-time S-matrix in fermionic degrees of freedom regarded as a functional in the space of 3-surfaces between purely bosonic state functionals $\Omega(X^3)$ defined in the configuration space.

Let $\Omega_I(X^3)$ define a complete set of configuration space spinor fields. Define the elements of S-matrix $\mathcal{S}$ as

$$\mathcal{S}_{Ii,Jj} = \int \overline{\Omega}_I(X^3) S_{ij}(X^3) \Omega_J(X^3) DV \ . \tag{23}$$

The unitarity condition for $\mathcal{S}$ would reduce to the completeness of the basis $\Omega_I(X^3)$ and unitary of the space-time S-matrix. Here X^3 refers to the 3-surface characterized by the the intersections of X^4 with X^7_+ or X^7_-. dV denotes the integration measure for configuration space.

One can argue that the absence of the dispersion is only an approximation. One can indeed generalize the ansatz to by replacing the S-matrix with $\mathcal{S}(X^3, Y^3)_{ij} = S_0(X^3, Y^3)S_{ij}(Y^3)$. Here $S_0(X^3, Y^3)$ would satisfy unitarity condition with X^3 and Y^3 playing the role of matrix index. Note however that here i and j refer to spinors in X^3 and Y^3. Quantum classical correspondence allows to argue that the non-determinism of the Kähler action always implies the existence of a space-time surface for which X^3 and Y^3 correspond to 3-surfaces resulting as intersections of X^4 with X^7_+ and X^7_- so that the generalization would not actually bring in anything new. Stated in a different manner, the path of the configuration space connecting X^3 to Y^3 would have a representation as a space-time surface. This would also make obvious why all generalized Feynman graphs with same initial and final stats are equivalent.

Configuration space integration gives rise to Feynman graphics using α_K as a coupling constant but by the non-locality of the Kähler function as a functional of 3-surface does not involve infinities. An interesting question is whether loops vanish at this level too. The decomposition of the configuration space to a union of symmetric spaces indeed suggests that a generalization of Duistermaat-Heckman theorem [16] from the finite-dimensional case holds true and implies that the integration effectively reduces to a Gaussian integral around unique maximum of Kähler function for given values of zero modes and that the matrix elements of configuration space spinor fields in irreducible representations of the super-canonical algebra are determined by symmetry considerations.

3.4 Some intriguing resemblances with M-theory

There are some intriguing resemblances with M-theory, which need not be purely accidental.

1. Since configuration space geometry can be coded in terms of the surfaces X^2 and 3+4 dimensional CDs, TGD can be said to almost reduce to a membrane theory with 3+4 dimensional CDs taking a role analogous to that of like light like 6-branes connected by light like 2-branes. M-theory would correspond to a situation in which 2-branes connect the outer boundaries of the 6-branes and also this kind of situation might be realized in TGD framework and could provide a representation of dynamics in terms of quantum states (light-like surfaces would make possible time-like–space-like duality). Of course, the physical interpretations of TGD and M-theory are totally different, and these resemblances force to think that although the physical interpretation of M-theory is badly misguided the theory contains at least some correct mathematical pieces.

2. Witten [19] has shown that the maximally helicity violating n-gluon amplitudes of $N = 4$ super symmetric Yang Mills theory, which are holomorphic functions of a twistor variable, allow an expression as integrals over 2-dimensional surfaces of 6-real-dimensional twistor space CP_3, which can be given a structure of a Calabi-Yau manifold. TGD allows $N = 4$ complex local super symmetry in leptonic and quark sectors. $\delta M^4_\pm \times CP_2$ is metrically 6-dimensional but not Calabi-Yau space, which might be interpreted in terms of the breaking of $N = 1$ global super-symmetry, which has turned out to be a more like a curse than blessing in M-theory.

   One ends up with the twistor space CP_3 from δM^4_+ interpreted as a representation of massless four-momenta p by the following procedure. Assign to p, $p^2 = 0$ a pair of row and column spinors λ and $\tilde{\lambda}$, whose tensor product $\tilde{\lambda}\otimes\lambda$ defines 2×2 SL(2,C) matrix $\sigma\cdot p$ with determinant equal to $p^2 = 0$. λ and $\tilde{\lambda}$ are defined only modulo projective transformation $(\lambda, \tilde{\lambda}) \rightarrow$

$(u\lambda, \tilde{\lambda}/u)$. A possible choice is $\tilde{\lambda} = \pm\overline{\lambda}$, where the sign factor defines the sign of energy. The assignment $p \rightarrow \lambda$ is not a function and the assignment corresponds to Hopf bundle $S^3 \rightarrow S^2$.

A polarization vector ϵ orthogonal to p can be represented as a tensor product of a two-component spinor μ and λ. Thus one has a pair (μ, λ) of 2-spinors defined modulo a projective scaling, and this gives rise a twistor defining an element of CP_3. 10-dimensionality of $CP_3 \times CP_2$ brings in mind super string models. It must be however made clear that twistor description is rather limited even in its super-symmetric form and does not seem to be plausible in TGD framework.

4 Construction of U-matrix in 'stringy' approach

Perturbative approach to the construction of U-matrix relies on the assumption that U-matrix follows from Super Virasoro invariance alone and that the condition $L_0(tot)\Psi = 0$, where $L_0(tot)$ is Virasoro generator for interacting space-time surface, must determine U-matrix. In the following it will be found that one indeed ends up with a general expression of stringy U-matrix using the following ingredients.

1. Poincare and Diff^4 invariances.
2. Decomposition of the Virasoro generator $L_0(tot)$ of $X^4(\cup_i X^3_i)$ to a sum of 'free' Super Virasoro generators $L_0(n)$ for various asymptotic 3-surfaces $X^3_i(a \rightarrow \infty)$ plus interaction terms. 'Free' Super Virasoro generators are defined by regarding these 3-surfaces as independent universes characterized by their own absolute minima $X^4(X^3_n)$ of Kähler action.
3. Representation of the solutions of the Virasoro condition $L_0(tot)\Psi = 0$ in a form analogous to the scattering solution of Schrödinger equation.

Contrary to earlier expectations, it seems that one cannot assign explicit Schrödinger equation with the U-matrix although the general structure of the solutions of the Virasoro condition is same as that of scattering solutions of Schrödinger equation in time dependent perturbation theory and U-matrix is completely analogous to that obtained as time evolution operator $U(-t, t)$, $t \rightarrow \infty$ in the perturbation theory for Schrödinger equation.

4.1 Poincare and Diff^4 invariance

Virasoro generators contain mass squared operator. Poincare invariance of the U-matrix requires that one must use Diff^4 invariant momentum generators $p_k(a \rightarrow \infty)$ in the definition the Super Virasoro generators and of U-matrix. At the limit $a \rightarrow \infty$ the generators of $Diff^4$ invariant Poincare algebra $p_k(a)$ should obey standard commutation relations. One can even assume that states have well defined Poincare quantum numbers and Poincare invariance becomes exact if one can assume that the states are eigen states of four-momentum. Therefore very close connection with ordinary quantum field theory results.

4.2 Decomposition of L_0 to free and interacting parts

At the limit $a \rightarrow \infty$ 3-surfaces $X^3(n)$ associated with particles can be assumed to behave in good approximation like their own independent universes. This means that one can assign to each particle like 3-surface X^3_n its own $Diff^4$ invariant generators $p_k(n, a \rightarrow \infty)$, whose action is defined by regarding $X^3_n(a \rightarrow \infty)$ as its own independent universe so that Diff^4 invariant translations act on the absolute minimum space-time surface $X^4(X^3_n)$ associated with X^3_n rather than $X^4(X^3)$ associated with the entire universe X^3.

This means effective decomposition of the configuration space to a Cartesian product of single particle configuration spaces and the gamma matrices associated with various sectors, in particular those associated with center of mass degrees of freedom, are assummed to anti-commute. It is assumed that each sector corresponds to either Ramond or NS type representation of Super Virasoro. The Virasoro generator $L_0(tot)$ for for entire Universe contains sum of Virasoro generators $L_0(n)$ for X^3_n plus necessary interaction terms. The Super Virasoro representation of entire universe in turn factors into a tensor product of these single particle Super Virasoro representations. Quite generally, Super Virasoro generators for the entire universe can be expressed as sums of the Super Virasoro generators associated with various 3-surface X^3_n plus interaction terms.

4.3 Analogy with time dependent perturbation theory for Schrödinger equation

Time dependent perturbation theory for ordinary Schrödinger equation is constructed by using energy eigen states as state basis and the basic equation is formal scattering solution of the Schrödinger equation

$$\Psi = \Psi_0 + \frac{V}{E - H_0 + i\epsilon}\Psi \ . \tag{24}$$

Here ϵ is infinitesimally small quantity. Ψ_0 (Ψ) is eigen state of H_0 (H) with eigen energy E. With these assumptions Schrödinger equation is indeed satisfied and one can construct Ψ perturbatively by developing right hand side to a geometric series in powers of the interaction potential V. This expansion defines the perturbative expansion of U-matrix, when perturbative solution is normalized appropriately.

Since ordinary Schrödinger equation is consistent with the scattering matrix formalism avoiding elegantly the difficulties with the definition of the limit $U(-t,t)$, $t \to \infty$, it is natural to take this form of Schrödinger equation as starting point when trying to construct explicit form of the 'time' evolution operator U. One can even forget the assumption about time evolution and require only that the basic algebraic information guaranteing unitarity is preserved. This information boils down to the Hermiticity of free and interacting Hamiltonians and to the assumption that the spectra non-bound states for free and interacting Hamiltonians are identical.

4.4 Scattering solutions of Super Virasoro conditions

One ends up with stringy perturbation theory by decomposing $L_0(tot)$ to a sum of free parts and interaction term. In this basis Super Virasoro condition can be expressed as

$$L_0(tot)|m\rangle = \left[L_0(free) + L_0(int)\right]|m\rangle = 0 \ . \tag{25}$$

Various terms in this condition are defined in the following manner:

$$\begin{aligned} L_0(free) &= \sum_n L_0(n) = \sum_n \left[p^2(n) - L_0(vib,n)\right] \equiv P^2 - L_0(vib) \ , \\ P^2 &\equiv 0 \textstyle\sum_n p^2(n) \ ; \qquad L_0(vib) \equiv \sum_n L_0(vib,n) \ ; \\ L_0(n) &= p^2(n) - L_0(vib,n) \ . \end{aligned} \tag{26}$$

Note that the mass squared operator $p^2(n)$ act nontrivially only in the tensor factor of state space associated with X^3_n.

One can write the general scattering solution to this equation as

$$|m\rangle = |m_0\rangle - \frac{L_0(int)}{L_0(free) + i\epsilon}|m\rangle \ . \quad (27)$$

ϵ is infinitesimal parameter defining precisely the momentum space-time integrations in presence of propagator poles. $L_0(int)$ is defined uniquely by the decomposition of the L_0 associated with the entire universe to a sum of $L_0(n)$:s associated with individual 3-surfaces X^3_n regarded as independent sub-universes plus interaction term.

$|m_0\rangle$ is assumed to satisfy the Virasoro conditions of the 'free theory' stating that all particles are on mass shell particles:

$$L_0(n)|m_0\rangle = \left[p^2(n) - L_0(vib, n)\right]|m_0\rangle = 0 \ . \quad (28)$$

These conditions are satisfied if Ψ_0 belongs is expressible as tensor product of solutions of Super Virasoro conditions for various sectors X^3_n. Ψ_0 runs over the entire solution spectrum of 'free' Super Virasoro conditions.

The momentum operators $p_k(n)$ are generators of $p_k(n, a \to \infty)$ Diff^4 invariant translations acting on the 3-surface $X^3_n(a \to \infty)$ associated with particle n regarding it as its own independent universe. The perturbative solution of the equation is obtained by iteration and leads to stringy perturbation theory with $L_0(n)$ appearing in the role of propagators and $L_0(int)$ defining interaction vertices. These conditions define Poincare invariant momentum conserving U-matrix if $L_0(int)$ defines momentum conserving vertices. This should be the case at the limit $a \to \infty$.

An explicit expression for the scattering solution is as geometric series

$$\begin{aligned}
|m\rangle &= \frac{1}{1+X}|m_0\rangle \ , \\
\langle m| &= \langle m_0|\frac{1}{1+X^\dagger} \ , \\
X &= \frac{}{L_0 + i\epsilon}L_0(int) \ , \\
X^\dagger &= L_0(int)\frac{1}{L_0 - i\epsilon} \ .
\end{aligned} \quad (29)$$

4.5 "Proof" of unitarity using a modification of formal scattering theory

The solution of the Virasoro condition for L_0 has same general structure as the scattering solution of Schrödinger equation. The action of "time development" operator U means the replacement of the superposition of the solutions of "free" Super Virasoro conditions with a superposition of the corresponding normalized scattering solutions of the full super Virasoro conditions. It does not seem however useful to assign explicit Schrödinger equation with Super Virasoro conditions. It is not clear whether this is even possible. One can however modify formal scattering theory to "prove" the unitarity of U-matrix.

1. One has the basic equation

$$|m\rangle = |m_0\rangle - \frac{1}{(L_0(free) + i\epsilon)}L_0(int)|m\rangle \ . \quad (30)$$

2. One can multiply this equation by $L_0(free)+i\epsilon$ and move the terms of type $...|m\rangle$ to the left hand side to get $(L_0(tot)+i\epsilon)|m\rangle$. Right hand side gives $(L_0(free)+L_0(int)+i\epsilon)|m_0\rangle - L_0(int)|m_0\rangle$ by adding and subtracting $L_0(int)|m_0\rangle$. Solving $|m\rangle$ one obtains

$$|m\rangle \quad = \quad |m_0\rangle - \frac{1}{(L_0(tot)+i\epsilon)}L_0(int)|m_0\rangle \ . \tag{31}$$

3. One can also solve $|m_0\rangle$ from the first equation

$$|m_0\rangle \quad = \quad |m\rangle + \frac{1}{(L_0(free)+i\epsilon)}L_0(int)|m\rangle \ . \tag{32}$$

Consider now the matrix element $\langle m|n\rangle$: one must show that this is $\langle m_0|n_0\rangle$ in order to prove unitarity.

1. Expressing $\langle m|$ in terms of $\langle m_0|$ using equation 31 gives

$$\langle m|n\rangle = \langle m_0|n\rangle - \langle m_0|L_0(int)\frac{1}{L_0(tot)-i\epsilon}|n\rangle \ .$$

2. One can use the fact that $L_0(tot)$ annihilates $|n\rangle$ to remove $1/(L_0(tot)+i\epsilon)$ term in front of $L_0(int)$ and write the resulting $1/i\epsilon$ as $1/(L_0(free)+i\epsilon)$ using the fact that $L_0(free)$ annihilates $\langle m_0|$. This gives

$$\langle m|n\rangle = \langle m_0|n\rangle + \langle m_0|\frac{1}{L_0(free)+i\epsilon}L_0(int)|n\rangle = \langle m_0|n_0\rangle \ ,$$

where equation 32 is used. Thus one has $\langle m|n\rangle = \langle m_0|n_0\rangle$ and unitarity holds true formally.

The basic counter argument against this "proof" is that scattering states contain of mass shell contributions so that the space of "free states" is subspace of scattering states. If scattering states form a complete orthonormal set, unitarity conditions become $\sum_n S_{mn}S^*_{mr} = P_{nr}$, where P denotes projection operator to the space of "free" states. Thus probability conservation is not achieved.

4.6 Formulation of inner product using residy calculus

It is not clear how the dirty looking formulas for the scattering states containing $\epsilon \rightarrow 0$ can give rise to a finite U-matrix: the relevant part of the inner product is proportional to $1/i\epsilon$. One gets rid of this difficulty by using a proper representation for the projection operator. The represention is obtained by replacing the states $|n(\epsilon)\rangle$ with states $|n(z)\rangle$, where ϵ is replaced with complex number z.

$$\begin{aligned} |n(z)\rangle &= |n_0\rangle + \frac{1}{L_0(free)+iz} = \frac{1}{1+X(z)}|n_0\rangle \ , \\ X(z) &= \frac{1}{L_0+iz}L_0(int) \ , \\ X^{\dagger}(z) &= L_0(int)\frac{1}{L_0-i\bar{z}} \ . \end{aligned} \tag{33}$$

Projection operator can be written in two forms

$$\begin{aligned} P &= \frac{1}{2\pi}\oint_C dz \frac{1}{L_0(free)+iz} \equiv \oint_C dz p(z) \ , \\ P &= \frac{1}{2\pi}\oint_C d\bar{z} \frac{1}{L_0(free)-i\bar{z}} \equiv \oint_C d\bar{z} p(\bar{z}) \ , \\ p(z) &= \frac{1}{2\pi}\frac{1}{L_0(free)+iz} \ , \\ p(\bar{z}) &= \frac{1}{2\pi}\frac{1}{L_0(free)-i\bar{z}} \ . \end{aligned} \tag{34}$$

C is very small curve surrounding origin containing no other poles than states annihilated by L_0. By acting on arbitrary state decomposed to eigen states of L_0 one finds that the integration picks up only the states annihilated by $L_0(free)$.

The inner product for scattering states reads as

$$\langle m|P|n\rangle = \oint_C dz \oint_C d\bar{z} \langle m(\bar{z})|p(\bar{z})p(z)|n(z)\rangle \ . \tag{35}$$

In this manner the dirty limiting procedure for defining states and their inner products is replaced with elegant formalism based on residy calculus and formulation becomes mathematically more rigorous.

4.7 Unitarity conditions

U-matrix is defined between the projections $P|n\rangle$ of scattering states to "free" states satisfying free Virasoro conditions. Therefore the Hilbert spaces of "free" and projected scattering states are at least formally identical. This means that off-mass-shell states appear only as intermediate states in the perturbative expansion of the U-matrix just as they do in the standard quantum field theory.

U-matrix is unitary if outgoing states are orthogonal to each other. This follows from the definition of U-matrix as

$$S_{n,m} = \langle m_0|n\rangle \ , \tag{36}$$

where m_0 is incoming state and n is scattering state normalized to unity. Unitary condition reads as

$$\sum_r S_{m,r}(S_{n,r})^* = \delta(n,m) \ . \tag{37}$$

where summation is over the "free" states $|r_0\rangle$ to which quantum jump occurs.
Unitarity condition reads explicitly as

$$\sum_r S_{m,r}(S_{n,r})^* = \sum_r \langle n|r_0\rangle\langle r_0|m\rangle = \langle n|m\rangle \ . \tag{38}$$

Here the completeness of the "free" state basis has been used. Hence unitarity holds true if one has

$$\langle m|n\rangle \propto \delta(m,n) \ . \tag{39}$$

provided that the normalization constant for the outgoing states are finite. In quantum field theories this is not usually the case and this could be the reason for why p-adics are necessarily needed.

In case of Schrödinger equation one can prove orthogonality of the scattering states by noticing that "free" and scattering state basis are related by a unitary time development operator, which preserves the orthonormality of the incoming states. Now the situation is different. The combinatorial structure is same as in wave mechanics but genuine time development operator need not exist and one must resort to the hermiticity of $L_0(free)$ and $L_0(int)$ plus the general algebraic structure of the scattering states plus possible additional assumption in order to prove the unitarity.

Using geometric series expansions and the expression of the inner product based on residy calculus one can write unitarity conditions as

$$\begin{aligned}
&\oint_C d\bar{z} \oint_C dz \langle m_0|\tfrac{1}{1+X^\dagger(\bar{z})}p(\bar{z})p(z)\tfrac{1}{1+X(z)}|n_0\rangle \ = G(m,n) \ , \\
&G(m,n) =< m|P|m\rangle\delta(m,n) \ , \\
&X(z) = \tfrac{1}{L_0(free)+iz}L_0(int) \ , \\
&X^\dagger(\bar{z}) = \tfrac{1}{L_0(free)-i\bar{z}}L_0(int) \ .
\end{aligned} \tag{40}$$

G is the matrix formed by the wave function renormalization constants. Note that in these conditions outgoing states are defined as *on mass shell projections of the scattering states* just as in quantum field theory. In the formal scattering theory this projection is not included. The necessary presence of projection operators suggests strongly that the formal proof of unitarity fails and that additional condition guaranteing unitarity is needed.

4.8 A condition guaranteing unitarity

The naive expectation supported by the formal proof is that the unitarity of the U-matrix follows automatically if free and interacting Virasoro generators L_0 can be regarded as Hermitian operators. The fact that time development operator need not exist might somehow make unitarity impossible without additional conditions. In fact, unitarity is by no means obvious even in standard scattering theory. Potential difficulties are also caused by the fact that normalization constants can diverge: this is indeed what they typically do in quantum field field theories. There is also the problem caused by the fact that the state bases formed by "free" and scattering states are not identical: this is obvious from the fact that scattering states contain off mass-shell contributions coming from particles, which do not satisfy the free Virasoro conditions $L_0(n) = 0$. Therefore it is of considerable interest to see whether some additional constraint could guarantee unitarity and perhaps provide a precise mathematical realization for quantum criticality and perhaps even explain why the p-adicization of U-matrix is necessary.

Experimentation with various possibilities guided by critical comments of Hitoshi Kitada (he pointed out the possibility of complex formalism and demonstrated that my first guess did not work) indeed led to a promising candidate for the additional condition. The condition for the unitarity is that $L_0(int)$ annihilates the projections of the genuine scattering contributions to the space of "free" states:

$$\begin{aligned} L_0(int)P|n_1\rangle &= 0 \ , \\ |n\rangle &\equiv |n_0\rangle + |n_1\rangle \ . \end{aligned} \tag{41}$$

It turns out that these conditions guarantee unitarity and implies that the wave function and coupling constant renormalizations are trivial as indeed expected on basis of quantum criticality. In real context the condition forces U-matrix to be trivial but in p-adic case situation is different. One can also construct very general family of unitary p-adic U-matrices forming "category", which is closed with respect to direct sum and direct product.

4.9 Formal proof of unitarity

Consider now the formal proof of the unitarity. Orthogonality condition guaranteing unitarity can be expressed also as the condition

$$\begin{aligned} \frac{1}{1+X^\dagger}P\frac{1}{1+X} &= G \ , \\ G(m,n) &= \delta(m,n)\langle m|m\rangle \ . \end{aligned} \tag{42}$$

This condition can be written in the form

$$\langle m_0|n_0\rangle + \langle m_0|P|n_1\rangle + \langle m_1|P|n_0\rangle + \langle m_1|P|n_1\rangle = G(m,n) \ . \tag{43}$$

The proof of unitarity splits in two basic steps.

1. Consider first the last term appearing at the left hand side:

$$\langle m_1|P|n_1\rangle = \oint_C d\bar{z}\langle m_0|\sum_{k>0}\left[L_0(int)\frac{1}{L_0(free)-i\bar{z}}\right]^k \frac{1}{L_0(free)-i\bar{z}}P|n_1\rangle \ . \tag{44}$$

   The first thing to observe is that $\langle m_1|$ has operator $L_0(int)\frac{1}{L_0(free)-i\bar{z}}P$ outmost to the right. Since projection operator effectively forces L_0 to zero, one can commute $L_0(int)$ past the operators $1/(L_0(free)-i\bar{z})$ so that it acts directly to $P|n_1\rangle$. But by the proposed condition $L_0(int)P|n_1\rangle = 0$ vanishes!

2. Consider next second and third terms at the left hand side of the unitarity condition. The sum of these terms can be written as

$$\begin{aligned} &\langle m_0|P|n_1\rangle + \langle m_1|P|n_0\rangle \\ &= \tfrac{1}{2\pi}\oint_C dz\langle m_0|\tfrac{1}{L_0(free)+iz}\textstyle\sum_{k>0}X^k|n_0\rangle \\ &+\tfrac{1}{2\pi}\oint_C d\bar{z}\langle m_0|\textstyle\sum_{k>0}(X^\dagger)^k L_0(int)|\tfrac{1}{L_0(free)-i\bar{z}}|n_0\rangle \ . \end{aligned} \tag{45}$$

   One might naively conclude that the sum of these terms is zero since the overall sign factors are different (this looks especially obvious in the dirty $1/i\epsilon$-approach). This is however the case only if on mass shell states do not appear as intermediate states in terms X^k. Unless this is the case one encounters difficulties.

3. One can project out on mass shell contribution to see what kind of contributions one obtains: what happens that the conditions $L_0(int)P|m_1\rangle =$ guarantees that these contributions vanish! Consider the second term in the sum to see how this happens. The on mass shell contributions from terms $X^k|n_0\rangle$ can be grouped by the following criterion. Each on mass shell contribution can be characterized by an integer r telling how many genuinely off mass shell powers of X appear before it. The on mass shell contributions which come after r:th X can be written in the form X^rPX^{k-r} The sum over all these terms coming from $\sum_{n>0} X^n$ is obviously given by

$$X^rP\sum_{k>r} X^{k-r}|m_0\rangle = X^rP|m_1\rangle = 0$$

and vanishes sinces X^r is of form $...L_0(int)$ and hence annihilates $P|m_1\rangle$. Thus the condition implying unitarity also implies that on mass shell states do not contribute to the perturbative expansion.

The condition implies not only the unitarity of the U-matrix but also that wave function renormalization constants are equal to one so that these cannot serve as sources of divergences. The condition implies not only the unitarity of the U-matrix but also that wave function renormalization constants are equal to one so that these cannot serve as sources of divergences. What is important is that unitarity holds true for the inner products of the on mass shell projections of the scattering states so that incoming and outgoing states span same state space. In the formal scattering theory the proof of unitarity fails because the presence of off mass shell particles implies that incoming and outgoing state spaces are not identical.

4.10 About the physical interpretation of the conditions guaranteing unitarity

The conditions guaranteing unitarity allow a nice physical interpretation in p-adic context but in real context they lead to a trivial U-matrix. This could be seen as an indication for the failure of the perturbative approach in real context.

As already found, the condition

$$L_0(int)P|m_1\rangle = 0$$

guaranteing unitarity implies that wave function renormalization is trivial. The condition also says that the effect of the vertex operator on the "dressed" state $|m\rangle$ is same as on the "bare" state $|m_0\rangle$:

$$L_0(int)|m\rangle = L_0(int)|m_0\rangle \ .$$

A pictorial interpretation of this is that the contribution of the virtual particle cloud to any vertex is trivial. This is very much like vanishing of the radiative corrections to coupling constants implying that various coupling constants are not renormalized.

The invariance of the p-adic Kähler coupling strength under renormalization group is one of the basic hypothesis of quantum TGD and there are reasons to believe that quantum criticality is more or less equivalent with this property. The condition $L_0(int)P|m_1\rangle = 0$ however suggests that this condition is much more general: all vertices are renormalization group invariants. In real context this certainly does not make sense since the coupling constants in the real quantum field theories for the fundamental interactions are known to run. In p-adic context situation is however different. One can interpret RG invariance as the symmetry of the p-adic U-matrix holding true in each sector D_p of the configuration space. The dependence of the Kähler coupling strength on p-adic length scale L_p means that continuous coupling constant evolution is effectively replaced with a discrete one. The dependence of α_K on p dictates the dependence of the other coupling constants on p-adic length scale.

Expressing U-matrix as

$$S = 1 + T \ ,$$

the conditions guaranteing unitarity can be written in the form

$$T + T^{\dagger} + T^{\dagger}T = 0 \ .$$

As already found, the conditions guaranteing unitarity imply that much stronger conditions

$$T + T^{\dagger} = 0$$

and

$$T^{\dagger}T = 0$$

hold true. These conditions obviously state that iT is hermitian and nilpotent matrix. The rows and columns of iT are orthogonal vectors with vanishing length squared such that diagonal components of T are real. These conditions do not certainly make sense in real context since real or complex valued hermitian nilpotent matrices are impossible mathematically. p-Adic probability concept however allows in principle to circumvent the difficulty.

Somewhat loosely speaking, the conditions satisfied by T imply that the absorptive parts of the forward scattering amplitudes given by $T + T^{\dagger}$ vanish identically. Therefore scattering amplitudes would be analytic functions lacking the cuts characterizing the scattering amplitudes in real context. By unitarity the absorptive parts are proportional to $TT^{\dagger}$ which therefore also vanishes: this means vanishing total reaction rates. Thus the conditions $L_0(int)P|m_1\rangle = 0$ imply trivial U-matrix in real context.

The content of the conditions $L_0(int)P|m_1\rangle = 0$ is that the total p-adic probability for the scattering from a state $|m_0\rangle$ to the states $|n_0\rangle \neq |m_0\rangle$ vanishes. This means that the p-adic probability for the diagonal scattering $|m_0\rangle \rightarrow |m_0\rangle$ is exactly one. As far as total scattering rates are considered, p-adic many-particle states behave therefore like many-particle states of a free field theory.

This mechanism would imply an elegant description of elementary particles. In real context the concept of elementary particle has some unsatisfactory features: the reason is basically that the concept of free particle is in conflict with the non-triviality of the interactions. For instance, in case of unstable particles one is in practice forced to introduce decay widths Γ making particle energies complex: $E \rightarrow E + i\Gamma$. This kind of mathematical trickery takes into account the finite lifetime of the particle in a rather ugly manner. p-Adic decay widths however vanish and particles behave like stable particles as far as total p-adic decay rates are considered. Real decay widths are of course non-vanishing and are in TGD framework parameters related to the time evolution by quantum jumps rather than unitary time evolution by U and real decay widths have absolutely nothing to do with the energy of the particle.

It has been also found that total p-adic probabilities for the transitions between sectors D_{p_1} and D_{p_2} $p_1 \neq p_2$ of the configuration space must vanish by internal consistency requirements. The proposed scenario generalizes this hypothesis from the level of the configuration space sectors to the level of quantum states. One consequence of the generalized hypothesis is that the total p-adic probability for a transition changing the values of the zero mode coordinates vanishes although U-matrix elements for the transitions changing the values of the zero modes and even the value of p, are non-vanishing.

Real scattering probabilities can be deduced from the p-adic probabilities by canonical identification map followed by normalization to one and total reaction rates are determined by real probabilities. Unitarity does not make sense in the real context since in general it is not possible to assign U-matrix to real reaction rates. There are however reasons to expect that at the limit of large p-adic prime real unitarity is good approximation although total p-adic reaction rates must still vanish. p-Adic unitarity provides also an elegant solution to the infrared divergences leading to infinite total reaction rates and forward scattering amplitudes.

It must be emphasized that p-adic co-homology is possible only for p-adic U-matrices describing physics of cognitive regions. It is also quite possible that p-adic co-homology could be regarded as a symmetry of p-adic U-matrix realized as replacement $S \rightarrow S + iT$, $[S, T] = 0$ leaving total p-adic scattering rates invariant. p-Adic U-matrix could also have $S_1 = 1 + iT$ representing p-adic co-homology as a tensor factor: the vanishing of p-adic total cross sections could perhaps be seen as a mathematical expression of the fact that cognition is pure imagination and has no physical effects.

5 Number theoretic approach to the construction of U-matrix

The bridge between classical and quantum provided by infinite primes allows to make one step in the task of deriving the U-matrix of quantum TGD.

5.1 U-matrix as Glebch-Gordan coefficients

U-matrix elements can be identified as Glebsch-Gordan coefficients between interacting and free representations of the super-canonical algebra. These representations are in turn defined as a union of absolute minima for composite 3-surfaces and by the absolute minimum for the union of composite 3-surfaces. These surfaces are coded by infinite primes mapped to products of irreducible polynomials with complex rational coefficients.

The Fock states coded by the infinite primes correspond to the states of a hyper-octonionic arithmetic quantum field theory second quantized again and again. Quantum field theory which is based on the notion of point like particles cannot describe quantum TGD based on generalization of particle concept. Thus the natural interpretation of the Fock states is as ground states of super-canonical representations. This view is consistent with the interpretation of the physical states as configuration space spinor fields assigning to a given 3-surface infinite number of possible states.

U-matrix elements can be identified as matrix elements between the incoming states of super-canonical representations created from the ground states associated with the tensor product of the ground states associated with Y_i^3. The superalgebra generators creating the excited incoming states are super-algebra generators associated with Y_i^3 whereas the outgoing states are created by the super-algebra generators associated with $\cup Y_i^3$. The challenge is to relate the super-algebra basis to each other.

The basic idea of rational physics approach suggests that U-matrix is complex rational or at most belongs to an extension of rationals defining finite extensions of p-adic numbers for every prime p. Rationality would imply that the phase factors involved are Pythagorean. This requirement must be extremely strong when combined with general physical principles. Even the weaker, and presumably more realistic, condition that phase factors belong to finite-dimensional extensions of p-adic numbers is very strong. An analogous constraint has been analyzed already earlier in the model of CKM mixing and together with some general physical inputs it was found to fix CKM matrix highly uniquely [F4]. Complex rationality would allow to define p-adic counterparts of the U-matrix.

5.2 Zeros of Riemann Zeta and U-matrix

The observation that the zeros of Riemann Zeta are excellent candidates for the conformal weights labelling the generators of super-canonical algebra [B2, B3] predicts that Rieman Zeta and U-matrices are closely related. A further natural speculation is that the zeros of polyzetas $\zeta(z_1, ..., z_K)$ label the super-canonical conformal weights of K-particle bound states. The vanishing of loop corrections could be understood as being due to the fact that they are proportional to polyzetas having super-canonical conformal weights as arguments. This speculation was inspired by the fact that polyzetas with integer arguments emerge in loop corrections of quantum field theories.

The construction of scalar propagator discussed in "Equivalence of Loop Diagrams with Tree Diagrams and Cancellation of Infinities in Quantum TGD" was based on the assumption that scalar propagator can be regarded as a partition function in super-canonical algebra. The masses for the predicted universal spectrum of resonances is expressible in terms of zeros of Riemann Zeta. A similar universal spectrum of resonances (which are not poles but delta functions) is predicted also when $1/L_0$ replaces scalar propagator. Ultraviolet cutoff appears automatically and in p-adic context one must identify it as p-adic length scale by number theoretical existence requirement. p-Adic length hierarchy scale emerges thus naturally.

A further prediction is that a hierarchy of propagators results and is labelled by the the hierarchy of sets $\{y_1 < y_2 < ... < y_K\}$ of imaginary parts $y_i > 0$ of the non-trivial zeros of Zeta ordered by there magnitude. This cutoff hierarchy corresponds to a finite p-adic phase resolution (the better the phase resolution the higher the algebraic dimension of the extension of p-adic numbers needed). The hypothesis that q^{iy}, q any prime, belongs to a finite extension of R_p for all primes p is necessary for the p-adicization of the propagator.

5.3 Reduction of the construction of U-matrix to number theory for infinite integers?

The fact that integers correspond to many-particle states in arithmetic quantum field theory, suggests that the space-time surface associated with an infinite integer N provides the representation for the interacting space-time surface and the space-time surfaces associated with the prime factors P of N provide the representation for the space-time surfaces associated with incoming states. This means extremely elegant solution to three problems: that of finding interacting absolute minima of Kähler action; that of finding ground states of super-canonical representations; and that of constructing the operators creating free (incoming) and interacting (outgoing) states. The knowledge of the ground states as functionals of 3-surface labelled by infinite primes are absolutely essential for the construction of U-matrix but this is not enough: the classical description of the interactions by assigning to the infinite integer interacting space-time surface $X^4(\cup_i Y_i^3)$ is equally important element of the construction.

Interacting space-time surfaces should approach to non-interacting surfaces in asymptotic regions. This is possible if the conditions $\partial_q P_i(p,q) = 0$ and $P_i(p,q) = 0$ are satisfied simultaneously in the asymptotic regions associated with various particles present in the many particle state represented by the infinite integer N. These conditions are exactly satisfied only in single point but in practice the failure to satisfy these conditions might be very small everywhere except in the regions near the interaction vertices.

If this picture is correct, number theory for infinite integers would provide the description of the interactions between the composite primes of the infinite integer representing the physical state. Note however that appearance of higher powers of same prime means that single four-surface must be counted several times as incoming state geometrically: this is somewhat counter-intuitive but does not have any effects at Fock space level.

This step does not fully solve the problem of constructing U-matrix. U-matrix elements are functionals of the initial and final 3-surface and actually kind of a local kernel for U-matrix is in question. To get U-matrix proper one must perform configuration space integrations. Integration over the zero modes is not needed if a localization in the zero modes occurs in each quantum jump. What remains to be done is the integral defining the inner product in the fiber degrees of freedom representing the degrees of freedom where configuration space metric is nontrivial. Since everything in fiber degrees of freedom is fixed by super-canonical symmetry, there are good hopes that the calculation reduces to a purely group-theoretical construction.

5.4 Does U-matrix possess adelic decomposition?

Each p-adic (with p understood in very general sense) observer would see some aspect of the physical state and only by the combination of all these aspects a complete characterization of the physical state would result. This representation of the quantum state is very much analogous to the representation of a real number, and at the higher level of abstraction to the string model vacuum amplitude, provided by the adelic formula.

For instance, U-matrix elements could be adelic products over U-matrix elements associated with p-adic number fields corresponding to ...-adic primes in the algebraic extension of infinite hyper-octonionic primes and that generalization of adelic formula holds true for the moduli squared of U-matrix elements as product of -adic moduli squared for ...-adic U-matrix elements. ...-adic representations would represent the state as seen by a particular observer characterized by a particular ...-adic topology. This view would conform with the vision of TGD inspired theory of consciousness about a p-adic hierarchy of consciousness giving increasingly refined representations of the physical state.

The theory should also be able to describe the physics as seen by an observer identified as a space-time sheet characterized by a given p-adic topology. Thus the p-adicization of at least U-matrix might be necessary in order to describe physics in the language understood by a given observer. p-Adicization for $p\ mod\ 4 = 3$ is achieved if U-matrix is a complex rational matrix so that matrix elements are expressible in terms of Pythagorean phases. For $p\ mod\ 4 = 1$ the existence of $\sqrt{-1}$ as a p-adic number might pose some difficulties probably overcome by using the notion of G-adic numbers in which p-adic p is replaced with the prime of G-adic number field satisfying $G\overline{G} = p$. The idea that p-adic space-time regions code into their geometry quantum numbers suggests that they could also provide a representation for U-matrix (certainly not faithful). Perhaps the dynamics of the second quantized induced spinors on p-adic space-time sheets could provide this representation.

If the p-adic counterparts of U-matrix exists, one can also consider the possibility that the p-adic counterpart of the configuration space vacuum functional appearing in the definition of the U-matrix exists p-adically. Vacuum functional is exponent of the Kähler function and exists p-adically for all values of p only in case that it is a rational number. One cannot exclude the possibility that the exponent of the Kähler function has rational values for the absolute minima of the Kähler action for some critical value of Kähler coupling depending on p. The rationality of the exponent of the Kähler action implies that it corresponds to the hyperbolic counterpart of a Pythagorean phase so that also $cosh(K)$ and $sinh(K)$ are rational numbers very closely related to rational numbers defining Pythagorean phases. For CP_2 type extremals this is true if G/R^2 is a rational number but in the general case situation is different. The assumption that critical value of Kähler strength depends on p allows some freedom in this respect.

6 Appendix: p-Adic co-homology

If the p-adic T-matrix is hermitian it must be nilpotent by the unitarity of S-matrix and therefore defines a co-homology theory. In the following very general construction of T-matrices defining p-adic co-homologies is carried out.

6.1 p-Adic T-matrices could define p-adic co-homology

Let the p-adic matrix iT be a hermitian nilpotent matrix. Therefore one can regard iT as an exterior derivative operator defining co-homology. The construction of the co-homology defined by iT reduces to the task of finding those vectors of the state space which are mapped to zero by iT but which do not belong to the zero norm subspace defined by iT. There is a nice parallel with super-symmetric theories: Hermitian super charges are nilpotent operators. Also the BRST charges appearing in the quantization of Yang Mills theories and defining physical states as BRST co-homology are nilpotent and Hermitian. BRST charges appear also in the construction of physical states satisfying Super Virasoro conditions. Super and BRST charges are presumably not representable as matrices but it is perhaps p-adicity what makes the representation as infinite-dimensional matrix possible. Super symmetric situation suggests that state space has decomposition into states labelled by "T-parity" instead of R-parity: states with R-parity zero are states which do not belong to the image of iT and the states with belong to the image of iT have T-parity one.

In case that T-co-homology is trivial, the states in these two spaces are in one-one correspondence. In a more general case, state space decomposes to the direct sum $V_0 + V_1 + iTV_1$, where V_0 corresponds to co-homologically nontrivial subspace mapped to zero by iT and V_1 corresponds to the states which are not mapped to zero by iT. Apart from a multiplicative constant, iT can defined as a "projection operator" to the space of exact states:

$$iT = \sum_k |Te_k\rangle\langle e_k| \ ,$$

where e_k are p-adic zero norm states. The rows of T span a linear subspace for which every vector has vanishing norm and T maps state space to this zero-norm subspace. Thus the construction of the matrices T reduces to that of finding zero-norm subspaces of the entire state space.

Physically the co-homologically nontrivial states belonging to V_0 and mapped to zero by iT (closed but not exact states) are noninteracting states remaining invariant under the "time evolution" operator U. These states are obviously natural candidates for the fixed points of the time evolution by quantum jumps.

6.2 About the construction of T-matrices

iT matrices are hermitian nilponent matrices and it not at all clear whether this kind of matrices exist at all. Certainly they do not exist in real context. It is quite easy to construct p-adic vectors having vanishing length squared. Possible problems are related to the orthogonality requirement for the rows of T.

It is easy to check that nilpotent hermitian p-adic valued 2×2 matrices exist. Assume that $p \ mod\ 4 = 3$ so that $i = \sqrt{-1}$ is not ordinary p-adic number. The most general form of this matrix is

$$\begin{aligned} iT &= \begin{pmatrix} a & b \\ \bar{b} & -a \end{pmatrix} \ , \\ b &= b_1 + ib_2 \ , \\ a &= \sqrt{-|b|^2} = \sqrt{-b_1^2 - b_2^2} \ . \end{aligned} \tag{46}$$

By hermiticity a must be "p-adically real" number. This is indeed possible in p-adic context but both b_1 and b_2 must be obviously non-vanishing.

One can construct infinite number of $2N \times 2N$-dimensional matrices iT as direct sums $iT_1 \oplus iT_2 \oplus$ ... and tensor products $iT_1 \otimes iT_2 \otimes ...$ of two-dimensional iT-matrices and iT-matrices constructed from them. $T=0$ is also acceptable T-matrix in 1×1-dimensional case and one can include this matrix to direct sum to obtain $2N+1\times 2N+1$-dimensional T-matrices. Clearly, the "category" constructed in this manner is closed with respect to $\oplus$ and $\otimes$ operations. This category is also closed under tensor multiplication by Hermitian matrices since the tensor product of arbitrary Hermitian matrix with Hermitian and nilpotent matrix is also Hermitian and nilponent: Hermitian nilpotency is infectuous disease! More concretely, by taking arbitrary Hermitian matrix and multiplying its elements with Hermitian and nilponent matrix one obtains new Hermitian and nilponent matrix. Also the sum of commuting Hermitian and nilpotent matrices has same properties. It should be noticed that all possible T-matrices form also a "category" in the proposed sense.

An interesting question is whether all $(2N+1)\times(2N+1)$ dimensional matrices are direct sums of $2N\times 2N$-dimensional matrix and 1×1 dimensional zero-matrix. The study of 3-dimensional case suggests that this is indeed the case. In 3-dimensional case it seem that no T-matrices exist. One can write the solution ansatz as

$$
\begin{aligned}
iT &= \begin{pmatrix} a_1 & b_1 & b_2 \\ \bar{b}_1 & a_2 & b_3 & , \\ \bar{b}_2 & \bar{b}_3 & a_3 \end{pmatrix} \\
b_i &= b_{i1}+ib_{i2} \ , \quad i=1,2,3 \ , \\
a_1 &= \epsilon_1\sqrt{-|b_1|^2-|b_2|^2} \ , \\
a_2 &= \epsilon_2\sqrt{-|b_1|^2-|b_3|^2} \ , \\
a_3 &= \epsilon_3\sqrt{-|b_2|^2-|b_3|^2} \ .
\end{aligned} \tag{47}
$$

The constraint that a_i are "p-adically real numbers" is nontrivial. There 6 unkowns b_i. The square roots defining a_i are unique only modulo sign factors ϵ_i. Formally there are 6 orthogonality conditions which can be written as

$$
\begin{aligned}
a_1+a_2 &= -\frac{b_2\bar{b}_3}{b_1} \ , \\
a_1+a_3 &= -\frac{b_1b_3}{b_2} \ , \\
a_2+a_3 &= -\frac{b_1b_2}{b_3} \ .
\end{aligned} \tag{48}
$$

One one writes formally b_i in the form $b_i=x_i^{1/2}exp(i\phi_i)$, where $x_i^{1/2}$ is taken to be real: exponential factor is however not p-adic exponent function. One finds that the equations stating the vanishing of the inner products give same condition for the phases $exp(i\phi_i)$ defined as

$$exp(i\phi_i)\equiv\frac{b_i}{x_i^{1/2}}$$

.

The equation reads as

$$exp(i\phi_1) = exp(i\phi_2)exp(-i\phi_3) \ . \tag{49}$$

The equation has now number theoretic contents and it is not at all obvious that solutions exist. Thus the number of equations reduces to 4.

Taking the squares of both sides of the equations 48 one obtains equations for the moduli of the $x_i \equiv |b_i|^2$. The squares of the equations 48 give

$$\begin{aligned} 2a_1a_2x_1 &= x_2x_3 + x_1(2x_1 + x_2 + x_3) \ , \\ 2a_1a_3x_2 &= x_1x_3 + x_2(2x_2 + x_3 + x_1) \ , \\ 2a_2a_3x_3 &= x_1x_2 + x_3(2x_3 + x_1 + x_2) \ . \end{aligned} \quad (50)$$

These equations are clearly cyclically symmetric.

By taking squares again one obtains 3 equations, which are degree 4 homogenous polynomials in variables x_i. The three cyclically symmetric equations read

$$\begin{aligned} P_i(x_1, x_2, x_3) &= 4x_i^2(x_i + x_{i+1})(x_i + x_{i+2}) \\ &- [x_{i+1}x_{i+2} + x_i(2x_i + x_{i+1} + x_{i+2})]^2 = 0 \ . \end{aligned} \quad (51)$$

where one has $i + 3 \equiv i$. P_i is homogenous polynomial in its arguments having the general form

$$P_i(x_1, x_2, x_3) = 4x_i^4 + ... + x_{i+1}^2 x_{i+2}^2 \ .$$

From a given solution (if it exists) one obtains a new solution by multiplying it with a p-adic number allowing p-adically real square root. This means that one can scale x_i simultaneously by a square of "p-adically real" number. One can fix the solution by fixing say x_3 to some arbitrarily chosen value. This means that one has 3 equations and 2 unknowns. This suggests that the three polynomial equations do not allow any solutions. This would mean that "irreducible" 3×3-matrices iT do not exist. An interesting conjecture is that 2-dimensional T-matrices are the only irreducible T-matrices and hence together with 1×1-dimensional zero matrix generate the category of all T-matrices. This would be in line with the fact that fermionic oscillator operators are used to construct Fock states.

To sum up, the conditions $L_0(int)|m_1\rangle = 0$ make sense only p-adically and force the theory to be as close to free theory as it can possibly be. An especially attractive feature is the reduction of the construction of the p-adic U-matrix to a generalized co-homology theory. What is especially nice and perhaps of practical importance is that allowed U-matrices form "category" with respect to direct sum and direct product operations. TGD based construction of U-matrix could realize Wheeler's great dream that physics could be reduced to the almost trivial statement "boundary has no boundary"! Of course, one can regard the success of the real unitarity as an objection against p-adic approach and one must therefore keep mind open for the weakening of the conditions.

6.3 What is the physical interpretation of the p-adic co-homology?

There are interesting questions related to the physical interpretation of the p-adic co-homology which makes sense only for finite-p p-adic number fields as is easy to see. The simplest example of T-matrix satisfying p-adic co-homology is

$$T = \begin{bmatrix} a & b \\ b & -a \end{bmatrix} \ , \quad a^2 + b^2 = 0 \ .$$

This condition can be satisfied if $\sqrt{-1}$ exists p-adically so that one has $p \ mod \ 4 = 1$. A slightly more complicated example making sense for the extension of $p \ mod \ 4 = 3$ with imaginary unit is

$$T = \begin{bmatrix} a & b \\ \bar{b} & -a \end{bmatrix} \ , \quad a^2 + b\bar{b} = 0 \ .$$

In this case the conditions are satisfied if $-b\bar{b}$ does possesses square root.

p-Adic S-matrix makes sense as an S-matrix for the dynamics of a cognitive model and p-adic co-homology could relate to the physics of cognition. The vanishing of the total p-adic scattering probabilities could have interpretation in terms of the concept of monitoring [E5]: if the cognitive system is not monitored, which means that one is only interested on whether scattering occurs or not, but not on which sub-space of Hilbert space final state is, then total p-adic scattering probability and, of course, also its real counterpart obtained by the canonical identification vanishes. Nothing happens in the cognitive system which is not monitored. This of course is natural in some sense natural since cognitive representation of dynamics is useless unless it is not monitored!

The most conservative view is that p-adic co-homology represents nothing more than a BRST type symmetry of the p-adic S-matrix leaving the measurement resolution defined by the decomposition of the state space to a direct sum of subspaces invariant [C1]. The fact that only the total scattering cross sections remain invariant in the transformations $T \rightarrow T + it$, however suggests that the p-adic T-matrices $T + it$, where the matrix t satisfies the conditions

$$t = t^{\dagger} \ , \quad t^2 = 0 \ , \quad [t, T] = 0 \ ,$$

could define a more detailed finite-p p-adic dynamics accompanying the dispersion between sectors D_P labelled by infinite primes. This dynamics would be invisible at the level of the total scattering probabilities. Also the tensor product $(1 + iT) \otimes (1 + it)$, where t satisfies p-adic co-homology is possible. Here t could represent the detailed finite-p p-adic dynamics. As far as the structure of G is considered, both $T + it$ structure and $(1 + iT) \otimes (1 + it)$ structure are invisible so that G would fix only the topological aspects of the dynamics.

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