Massless States and Particle Massivation

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Contents

Abstract

The massless sector of the TGD and particle massivation is studied in this chapter. The identification of the spectrum of light particles reduces to two tasks: the construction of massless states and the identification of the states which remain light in p-adic thermodynamics.

1. Physical states as representations of super-canonical and Super Kac-Moody algebras

Physical states belong to the representation of super-canonical algebra and Super Kac-Moody algebra of $SO(2) \times E^2 \times SU(3) \times U(2)_{ew}$ associated with the 2-D surfaces X^2 defined by the intersections of 3-D light like causal determinants (CDs) with 7-D CDs $X^7 = X_l^3 \times CP_2$, where X_l^3 is boundary of future or past directed light cone. These 2-surfaces have interpretation as partons, and the effective 2-dimensionality means that the machinery of 2-D conformal field theories can be applied in the state construction.

The recipe is simple. Construct first a null state with a non-positive conformal weight using super-canonical generators, and then apply Super-Kac Moody generators to compensate this conformal weight to get a state with vanishing conformal weight and zero mass. Pose also the conditions that the commutator of super-canonical and super Kac-Moody algebras and corresponding commutator of Virasoro algebras annihilates physical states.

The conformal weights of super-canonical algebra generators are complex and in a well-defined sense expressible in terms of zeros of Riemann Zeta although the connection is much more subtle as thought originally [C1] and conformal weight cannot be regarded as quantum number in the standard sense of the word. More precisely, the arguments of [C1] suggest that radial conformal weight Δ for super-canonical algebra in fact depends on the point of geodesic sphere S^2 in CP_2 and is given in terms of the inverse $\zeta^{-1}(z)$ of Riemann ζ having the natural complex coordinate z of S^2 as argument. This implies a mapping of the radial conformal weights to the points of the geodesic sphere CP_2 serving in the role of "conformal heavenly sphere".

Linear combinations of zeros correspond to algebraic points in the intersections of real and p-adic partonic 2-surfaces and are thus in a unique role from the point of view of p-adicization. They can be also identified as conformal weights associated with parton as an n-particle state in the algebraic sense (these points correspond to arguments of n-point functions of conformal field theory in the construction of S-matrix). This discrete set of points defines in a natural manner number theoretic braid and a connection with braiding S-matrices emerges. This if one believes the basic conjecture that the numbers p^s, p prime and s zero of Riemann Zeta are algebraic numbers.

The original hypothesis was that the conformal weights of physical states are real. This would imply conformal confinement to which color confinement might reduce: what would happen that partons can have complex super-canonical conformal weights but particles have real conformal weights. It has however turned out that there is actually no strong reason for the reality of the conformal weights.

The waves x^s with conformal weights $s = 1/2 + i\sum_k n_k y_k$, where $s_k = 1/2 + iy_k$ is a zero of Zeta, define an orthogonal basis with respect to the inner product defined by the integration measure dx/x. These conformal weights can be assigned to both the eigenvalues of the modified Dirac operator and to radial logarithmic waves multiplying the Hamiltonians at $\delta M^4_\pm \times CP_2$ appearing in the construction of the configuration space geometry.

The imaginary part of the complex weight would allow to distinguish between particle and its phase conjugate (phase conjugate photons obey time reversed dynamics) by assigning to a particle inherent time orientation allowing to distinguish between positive energy particle propagating to the geometric future from a negative energy particle propagating to the geometric past. Obviously a strong correlation with second law of thermodynamics would emerge. It has turned out that the conformal weights $s = 1/2 + iy_k$ correspond to systems critical against transition changing the value of Planck constant.

One can also introduce the notion of bound state conformal weight in terms of Riemann polyzetas, and it turns out that number theoretic constraints imply that the net conformal weights in this case have the same spectrum as in single particle case and that irreducible n-particle bound states are possible only for $n = 2$ and 3. This suggests a connection with valence

quark numbers of mesons and baryons and perhaps also with family replication phenomenon (parton with genus $g = 0, 1, 2$ as conformally bound state of sphere and g handles so that only 3 stable particle families would result). The zeros of Zeta represent essentially non-stringy aspects of TGD being due to the fact that the basic objects are effectively 2-dimensional rather than 1-dimensional.

For spinor harmonics of CP_2 the correlation between color and electro-weak quantum numbers is not correct. Super-canonical generators provide a natural mechanism allowing to cure the problem. Boson states are identified as bi-local bilinears of fermions and anti-fermions in X^2 characterized by charge matrices and conformally invariant correlation function. $BF\overline{F}$ coupling constants can be identified in terms of normalization factors of the boson states. The small value of gravitational coupling can be understood as resulting by a fractal mechanism reducing its value from the square of p-adic length L_p, and a concrete physical interpretation for the expression of gravitational constant in terms of CP_2 length derived from number theoretic arguments emerges. The presence of primes $2, 3, ...23, p$ in the expression of the gravitational constant can be interpreted in terms of multi-p p-adic fractality involving these primes.

If the primes $p = 2, 3, ...23$ are present, the question whether besides p-adic length scales $L_p \propto \sqrt{p}$ also their multiples $\sqrt{\prod_i q_i} L_p$, where $\{q_i\}$ forms a subset of $\{2, 3, ..., 23\}$ define fundamental length scales. The implication would be small-p p-adic fractality for these small primes with each p-adic length scale L_p taking the role of CP_2 length, and there indeed is some evidence for this kind of fractality.

2. Particle massivation

Particle massivation can be regarded as a change of the vanishing parton conformal weights describable as a thermal mixing with higher conformal weights. The observed mass squared is not p-adic thermal expectation of mass squared but that of conformal weight so that there are no problems with Lorentz invariance.

The space-time mechanism of massivation can be articulated in several manners.

a) CP_2 type vacuum extremals representing elementary particles have random light-like curve as an M^4 projection so that the average motion correspond to that of massive particle. Lighlike randomness gives rise to classical Virasoro conditions. p-Adic thermodynamics is consistent with this picture.

b) A possible candidate for the physical mechanism causing the thermal massivation is hydrodynamical mixing by the braiding flow. One can imagine several realizations for this flow. For instance a flow defined by the normal components of energy momentum tensor of the induced Kähler field at light like 3-D CDs describing the orbits of partons. Number theoretical approach leads to a purely number theoretical identification of braids and braiding flow and it seems that this flow might be more fundamental.

c) The fundamental parton level description of TGD is based on almost topological QFT for light-like 3-surfaces. Dynamics is constrained only by the requirement that CP_2 projection is for extremals of Chern-Simons action 2-dimensional and for off-shell states light-likeness is the only constraint. Hence a justification for the ergodic hydrodynamic flow as a fundamental cause of massivation emerges. The symmetries respecting light-likeness property correspond gives rise to Kac-Moody type algebra and super-canonical symmetries emerge also naturally as well as $N = 4$ character of super-conformal invariance. Four-momentum appears as non-conserved Noether charge (mass squared is however conserved) and has identification as gravitational four-momentum. Inertial momentum corresponds to the statistical average of gravitational four-momentum and p-adic thermodynamics is thus a natural description.

This mechanism cannot explain the massivation of electro-weak gauge bosons, which could be caused either by TGD variant of Higgs mechanism or by the fact that the charge matrices of W boson and left handed component of Z^0 are not covariantly constant, which together with the hydrodynamical mixing could lead to a loss of correlations. TGD indeed predicts a candidate for Higgs as a wormhole contact whose throats are identified as lightlike 3-surfaces and carry quantum numbers of fermion and antifermion and it is now clear that this is the correct option.

The underlying philosophy is that real number based TGD can be algebraically continued to various p-adic number fields. This gives justification for the use of p-adic thermodynamics although the mapping of p-adic thermal expectations to real counterparts is not completely unique. Instead of energy, the Super Kac-Moody Virasoro generator L_0 (essentially mass squared) is thermalized in p-adic thermodynamics. This guarantees Lorentz invariance. It is important to notice that four-momentum does not appear in the definition of super Virasoro generators. The reason is simply that four-momentum does not appear in the expression of super Virasoro generators as Noether charges associated with the modified Dirac action. The dependence of Virasoro generators on four-momentum would be in conflict with Lorentz invariance.

p-Adic thermodynamics forces to conclude that CP_2 radius is essentially the p-adic length scale $R \sim L$ and thus of order $R \simeq 10^4\sqrt{G}$ and therefore 10^4 times larger than the naive guess. Hence p-adic thermodynamics describes the mixing of states with vanishing conformal weights with their Super Kac-Moody Virasoro excitations having masses of order 10^{-4} Planck mass.

p-Adic temperature is quantized by purely number theoretical constraints (Boltzmann weight $exp(-E/kT)$ is replaced with p^{L_0/T_p}, $1/T_p$ integer) and fermions correspond to $T_p = 1$ whereas $T_p = 1/2$ seems to be the only reasonable choice for bosons. That mass squared, rather than energy, is a fundamental quantity at CP_2 length scale is also suggested by a simple dimensional argument (Planck mass squared is proportional to $\hbar$ so that it should correspond to a generator of some Lie-algebra (Virasoro generator L_0!)).

There is also modular contribution to the mass squared which can be estimated using elementary particle vacuum functionals in the conformal modular degrees of freedom of the partonic 2-surface. This contribution can be identified as a contribution coming from a thermodynamics in super-canonical Virasoro algebra which generates excitations of the ground states with negative conformal weight.

The predictions of the general theory are consistent with the earlier mass calculations, and the earlier ad hoc parameters disappear. In particular, optimal lowest order predictions for the charged lepton masses are obtained and photon, gluon and graviton appear as essentially massless particles. The negative conformal weight created by super-canonical generators can have arbitrarily large magnitude (ground state corresponds to a null state of super-conformal algebra annihilated by L_n, $n < 0$) so that an infinite hierarchy of exotic massless states is in principle possible. These states receive mass by the proposed mechanism and they are expected to be unstable but it remains to be shown that they do not appear in the spectrum of light particles. Since X^2 can have an arbitrarily large size and can even correspond to black hole horizon, the emergence of this complex structure of states is completely natural.

1 Introduction

This chapter tries to represent the most recent view about particle massivation. The identification of the spectrum of light particles reduces to two tasks: the construction of massless states and the identification of the states which remain light in p-adic thermodynamics. The latter task is relatively straightforward. The thorough understanding of the massless spectrum requires however a real understanding of quantum TGD. It would be also highly desirable to understand why p-adic thermodynamics combined with p-adic length scale hypothesis works. A lot of progress has taken place in these respects during last years.

Zero energy ontology providing a detailed geometric view about bosons and fermions, the generalization of S-matrix to what I call M-matrix, the notion of finite measurement resolution characterized in terms of inclusions of von Neumann algebras, the derivation of p-adic coupling constant evolution and p-adic length scale hypothesis from the first principles, and understanding of Higgs mechanism in terms of the generalized eigenvalues of the modified Dirac operator: these are the most important steps of progress during last years with a direct relevance for the understanding of particle spectrum and massivation although the predictions of p-adic thermodynamics are not affected. What is frustrating is that the joy by every great step of progress is shadowed by the realization that it creates a lot of mammoth bones generating internal inconsistencies (there are fifteen books about TGD so that I have to fight fiercely to avoid total chaos!), and I feel that my first task before continuing is to represent apologies for not being able to identify all of them. Therefore it is better to take these chapters as lab note books rather than final summaries.

1.1 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge from quantum TGD?

What p-adic coupling constant evolution really means has remained for a long time more or less open and detailed attempts to model the situation has suffered from this. The progress made in the understanding of the S-matrix of the theory [C2] has however changed the situation dramatically.

1.1.1 M-matrix and coupling constant evolution

The final breakthrough in the understanding of p-adic coupling constant evolution came through the understanding of S-matrix, or actually M-matrix defining entanglement coefficients between positive and negative energy parts of zero energy states in zero energy ontology [C2]. M-matrix has interpretation as a "complex square root" of density matrix and thus provides a unification of thermodynamics and quantum theory. S-matrix is analogous to the phase of Schrödinger amplitude multiplying positive and real square root of density matrix analogous to modulus of Schrödinger amplitude.

The notion of finite measurement resolution realized in terms of inclusions of von Neumann algebras allows to demonstrate that the irreducible components of M-matrix are unique and possesses huge symmetries in the sense that the hermitian elements of included factor $\mathcal{N} \subset \mathcal{M}$ defining the measurement resolution act as symmetries of M-matrix, which suggests a connection with integrable quantum field theories.

It is also possible to understand coupling constant evolution as a discretized evolution associated with time scales T_n, which come as octaves of a fundamental time scale: $T_n = 2^n T_0$. Number theoretic universality requires that renormalized coupling constants are rational or at most algebraic numbers and this is achieved by this discretization since the logarithms of discretized mass scale appearing in the expressions of renormalized coupling constants reduce to the form $log(2^n) = nlog(2)$ and with a proper choice of the coefficient of logarithm log(2) dependence disappears so that rational number results.

1.1.2 p-Adic coupling constant evolution

One can wonder how this picture relates to the earlier hypothesis that p-adic length coupling constant evolution is coded to the hypothesized $log(p)$ normalization of the eigenvalues of the modified Dirac operator D. There are objections against this normalization. $log(p)$ factors are not number theoretically favored and one could consider also other dependencies on p. Since the eigenvalue spectrum of D corresponds to the values of Higgs expectation at points of partonic 2-surface defining number theoretic braids, Higgs expectation would have $log(p)$ multiplicative dependence on p-adic length scale, which does not look attractive.

Is there really any need to assume this kind of normalization? Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5$ μm (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3.

4. The fundamental role of 2-adicity suggests that the fundamental coupling constant evolution and p-adic mass calculations could be formulated also in terms of 2-adic thermodynamics. With a suitable definition of the canonical identification used to map 2-adic mass squared values to real numbers this is possible, and the differences between 2-adic and p-adic thermodynamics are extremely small for large values of for $p \simeq 2^k$. 2-adic temperature must be chosen to be $T_2 = 1/k$ whereas p-adic temperature is $T_p = 1$ for fermions. If the canonical identification is defined as

$$\sum_{n\geq 0} b_n 2^n \rightarrow \sum_{m\geq 1} 2^{-m+1} \sum_{(k-1)m\leq n<km} b_n 2^n \ ,$$

it maps all 2-adic integers $n < 2^k$ to themselves and the predictions are essentially same as for p-adic thermodynamics. For large values of $p \simeq 2^k$ 2-adic real thermodynamics with $T_R = 1/k$ gives essentially the same results as the 2-adic one in the lowest order so that the interpretation in terms of effective 2-adic/p-adic topology is possible.

1.2 How quantum classical correspondence is realized at parton level?

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or a boundary term of YM action associated with a particle carrying gauge charges of the quantum state. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography. The difference between Chern-Simons action characterizing quantum state and the fundamental Chern-Simons type factor associated with the Kähler form would be that the latter emerges as the phase of the Dirac determinant.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

1.3 Physical states as representations of super-canonical and Super Kac-Moody algebras

Physical states belong to the representation of super-canonical algebra and Super Kac-Moody algebra assignable $SO(2) \times SU(3) \times SU(2)_{rot} \times U(2)_{ew}$ associated with the 2-D surfaces X^2 defined by the intersections of 3-D light like causal determinants (CDs) with 7-D CDs $X^7 = X^3_l \times CP_2$, where X^3_l is boundary of future or past directed light cone. These 2-surfaces have interpretation as partons, and the effective 2-dimensionality means that the machinery of 2-D conformal field theories can be applied in the state construction.

It has taken considerable effort to understand the relationship between super-canonical and super Kac-Moody algebras and there are still many uncertainties involved. What looks like the most plausible option relies on the generalization of a coset construction proposed already for years ago but given up because of the lacking understanding of how SKM and SC algebras could be lifted to the level of imbedding space. The progress in the *Physics as generalized number theory* program provided finally a justification for the coset construction.

1. Assume a generalization of the coset construction in the sense that the differences of super Kac-Moody Virasoro generators (SKMV) and super-canonical Virasoro generators (SCV) annihilate the physical states. The interpretation is in terms of TGD counterpart for Einstein's equations realizing Equivalence Principle. Mass squared is identified as the p-adic thermal expectation value of either $SKMV$ or SCV conformal weight (gravitational or inertial mass) in a superposition of states with $SKMV$ (SCV) conformal weight $n \geq 0$ annihilated by $SKMV - SCV$.

2. Construct first ground states with negative conformal weight annihilated by $SKMV$ and SCV generators G_n, L_n, $n < 0$. Apply to these states generators of tensor factors of Super Viraroso algebras to obtain states with vanishing SCV and $SKMV$ conformal weights. After this construct thermal states as superpositions of states obtained by applying $SKMV$

generators and corresponding SCV generators G_n,L_n, $n > 0$. Assume that these states are annihilated by SCV and $SKMV$ generators G_n, L_n,$n > 0$ and by the differences of all SCV and $SKMV$ generators.

3. Super-canonical algebra represents a completely new element and in the case of hadrons the non-perturbative contribution to the mass spectrum is easiest to understand in terms of super-canonical thermal excitations contributing roughly 70 per cent to the p-adic thermal mass of the hadron. It must be however emphasized that by SKMV-SCV duality one can regard these contributions equivalently as SKM or SC contributions.

1.4 Particle massivation

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3. The fundamental parton level description of TGD is based on almost topological QFT for light-like 3-surfaces. Dynamics is constrained only by the requirement that CP_2 projection is for extremals of Chern-Simons action 2-dimensional and for off-shell states light-likeness is the only constraint. Hence a justification for the ergodic hydrodynamic flow as a fundamental cause of massivation emerges. The symmetries respecting light-likeness property correspond gives rise to Kac-Moody type algebra and super-canonical symmetries emerge also naturally as well as $N = 4$ character of super-conformal invariance. Four-momentum appears as non-conserved Noether charge (mass squared is however conserve

4. and has identification as gravitational four-momentum. Inertial momentum corresponds to the statistical average of gravitational four-momentum and p-adic thermodynamics is thus a natural description.

This mechanism cannot explain the massivation of electro-weak gauge bosons, which could be caused either by TGD variant of Higgs mechanism or by the fact that the charge matrices of W boson and left handed component of Z^0 are not covariantly constant, which together with the hydrodynamical mixing could lead to a loss of correlations. TGD indeed predicts a candidate for Higgs as a wormhole contact whose throats are identified as lightlike 3-surfaces and carry quantum numbers of fermion and antifermion and it is now clear that this is the correct option.

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p-Adic temperature is quantized by purely number theoretical constraints (Boltzmann weight $exp(-E/kT)$ is replaced with p^{L_0/T_p}, $1/T_p$ integer) and fermions correspond to $T_p = 1$ whereas $T_p = 1/2$ seems to be the only reasonable choice for bosons. That mass squared, rather than energy, is a fundamental quantity at CP_2 length scale is also suggested by a simple dimensional argument (Planck mass squared is proportional to $\hbar$ so that it should correspond to a generator of some Lie-algebra (Virasoro generator L_0!)).

There is also modular contribution to the mass squared which can be estimated using elementary particle vacuum functionals in the conformal modular degrees of freedom of the partonic 2-surface. For bosons both Virasoro and modular contributions seem to be negligible and could be due to the smallness of the p-adic temperature. Virasoro contribution can be identified as a contribution coming from a thermodynamics in super-canonical Virasoro algebra which generates excitations of the ground states with negative conformal weight. This contribution will be discussed in the next section.

The predictions of the general theory are consistent with the earlier mass calculations, and the earlier ad hoc parameters disappear. In particular, optimal lowest order predictions for the charged lepton masses are obtained and photon, gluon and graviton appear as essentially massless particles. The negative conformal weight created by super-canonical generators can have arbitrarily large magnitude (ground state corresponds to a null state of super-conformal algebra annihilated by L_n, $n < 0$) so that an infinite hierarchy of exotic massless states is in principle possible. These states receive mass by the proposed mechanism and they are expected to be unstable but it remains to be shown that they do not appear in the spectrum of light particles. Since X^2 can have an arbitrarily large size and can even correspond to black hole horizon, the emergence of this complex structure of states is completely natural.

2 Heuristic picture about massivation

The understanding of particle massivation developed in an inverted manner from p-adic mass calculations to their interpretation and has been a process with several side tracks. The recent picture is that massivation involves two mechanisms. p-Adic thermodynamics gives the dominant contribution to the masses of fermions and involves the contributions from p-adic thermodynamics for Virasoro generator L_0 and thermodynamics in modular degrees of freedom explaining the mass differences between different fermion families. Besides this there is contribution to Higgs particle identified as a wormhole contact carrying net weak isospin assignable to the fermion and antifermion at the lightlike partonic 3-surfaces defining the throats of the wormhole contact. This contribution dominates the masses of gauge bosons.

2.1 The relationship between inertial gravitational masses

It took quite a long time to accept the obvious fact that the relationship between inertial and gravitational masses cannot be quite the same as in General Relativity.

2.1.1 Modification of the Equivalence Principle?

The findings of [D3] combined with the basic facts about imbeddings of Robertson-Walker cosmologies [D5] force the conclusion that inertial mass density vanishes in cosmological length scales. This is possible if the sign of inertial energy depends on time orientation of the space-time sheet. This forces a modification of Equivalence Principle. The modified Equivalence Principle states that gravitational energy corresponds to the absolute value of inertial energy. Since inertial energy can have both signs, this means that gravitational mass is not conserved and is non-vanishing even for vacuum extremals. This differences is dual for the two time times: the experienced time identifiable as a sequence of quantum jumps and geometric time.

More generally, all conserved (that is Noether-) charges of the Universe vanish identically and their densities vanish in cosmological length scales. The simplest generalization of the Equivalence Principle would be that gravitational four-momentum equals to the absolute value of inertial four-momentum and is thus not conserved in general. Gravitational mass density does not vanish for vacuum extremals and, as will be found, one can deduce the renormalization of gravitational constant at given space-time sheet from the requirement that gravitational mass is conserved inside given space-time sheet. The conservation law holds only true inside given space-time sheet.

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since induced color fields are proportional to color Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

2.1.2 # contacts, non-conservation of gauge charges and gravitational four-momentum, and Higgs mechanism

Gravitational # contacts are necessary and if gravitational energy can be regarded in the Newtonian limit as a gauge charge, the contacts feed the gravitational energy regarded as a gauge flux to the lower condensate levels. The non-conservation of gravitational gauge flux means that # contacts can carry gravitational four-momentum and since CP_2 type vacuum extremals are the natural candidates for # contacts, the natural hypothesis is that the non-conserved light-like gravitational four-momentum of # contacts is responsible for the non-conservation of gravitational four-momentum flux. The non-conservation of the light-like gravitational four-momentum of CP_2 type extremals would in turn be responsible for the non-conservation of the net gravitational four-momentum.

contacts could be also carriers of inertial mass which must be conserved in absence of four-momentum exchange between environment and wormhole contact. Therefore Equivalence Principle cannot hold true in a strict sense even at elementary particle level. Equivalence Principle would be satisfied in a weak sense if the inertial four-momentum is equal to the average four-momentum associated with the zitterbewegung motion and corresponds to the center of mass motion for the # contact.

The non-conservation of weak gauge currents for CP_2 type extremals implies a non-conservation of weak charges and the finite range of weak forces. If wormhole contacts correspond to pieces of CP_2 type vacuum extremal, electro-weak gauge currents are not conserved classically unlike color and Kähler current. The non-conservation of weak isospin corresponds to the presence of pairs of right/left handed fermion and left/right handed antifermion at wormhole contacts. These wormhole contacts are excellent candidates for the TGD counterpart of Higgs boson providing the most natural mechanism for the massivation of weak bosons. The dominant contribution to fermion mass would be due to p-adic thermodynamics [F3]. If weak form of Equivalence Principle holds true, inertial mass would result simply as the average of non-conserved light-like gravitational four-momentum.

There would be two contributions to the mass of the elementary particle.

1. Part of the inertial mass is generated in the topological condensation of CP_2 type extremal representing elementary particle involving only single light like elementary particle horizon, say fermion, and would correspond naturally to the contribution to the mass modellable using p-adic thermodynamics. The contribution from primary topological condensation is negligible if the radius of the zitterbewegung orbit is larger than the size of the space-time sheet containing the topologically condensed boson so that the motion is along a light-like geodesic in a good approximation. For gauge bosons this contribution should be very small or vanishing. Systems like superconductors where also photons and even gravitons can become massive [D3] might form an exception in this respect.

2. The space-time sheet representing massless state suffered secondary topological condensation at a larger space-time sheet and viewed as a particle can develop an additional contribution to the mass via Higgs mechanism since the wormhole contacts cannot be regarded as moving along light like geodesics of M^4 in the length and time scale involved. # contacts carrying a net weak isospin would have interpretation as TGD counterparts of neutral Higgs bosons and the formation of coherent state involving a superposition of states with varying number of wormhole contacts would correspond to the generation of a vacuum expectation value of Higgs field. The inertial mass of the wormhole contact must be small, presumably its order of magnitude is given by $1/L_p$, where L_p is the characteristic p-adic length scale associated with a given condensate level.

2.1.3 Gravitational mass is necessarily accompanied by non-vanishing gauge charges

The experience from the study of the extremals of the Kähler action [D1] suggests that for non-vacuum extremals at astrophysical scales Kähler charge doesn't depend on the properties of the condensate and is apart from numerical constant equal to the gravitational mass of the system using Planck mass as unit:

$$Q_K = \epsilon_1 \frac{M_{gr}}{m_{proton}} . \tag{1}$$

The condition $\frac{\epsilon_1}{\sqrt{\alpha_K}} < 10^{-19}$ must hold true in astrophysical length scales since the long range gauge force implied by the Kähler charge must be weaker than gravitational interaction at astrophysical length scales. It is not clear whether the 'anomalous' Kähler charge can correspond to a mere Z^0 gauge or em charge or more general combination of weak charges.

Also for the imbedding of Schwartschild and Reissner-Nordström metrics as vacuum extremals non-vanishing gravitational mass implies that some electro-weak gauge charges are non-vanishing [D1]. For vacuum extremals with $sin^2(\theta_W) = 0$ em field indeed vanishes whereas Z^0 gauge field is non-vanishing.

If one assumes that the weak charges are screened completely in electro-weak length scale, the anomalous charge can be only electromagnetic if it corresponds to ordinary elementary particles. This however need not be consistent with field equations. Perhaps the most natural interpretation for the "anomalous" gauge charges is due to the elementary charges associated with dark matter. Since weak charges are expected to be screened in the p-adic length scale characterizing weak boson mass scale, the implication is that scaled down copies of weak bosons with arbitrarily small mass scales and arbitrarily long range of interaction are predicted. Also long ranged classical color gauge fields are unavoidable which forces to conclude that also a hierarchy of scaled down copies of gluons exists.

One can hope that photon and perhaps also Z^0 and color gauge charges in Cartan algebra could be quantized classically at elementary particle length scale ($p \leq M_{127}$, say) and electromagnetic gauge charge in all length scales apart from small renormalization effects. One of the reasons is that classical electromagnetic fields make an essential part in the description of, say, hydrogen atom.

The study of the extremals of Kähler action and of the imbeddings of spherically symmetric metrics [D3, D1] shows that the imbeddings are characterized by frequency type vacuum quantum numbers, which allow to fix these charges to pre-determined values. The minimization of Kähler action for a space-time surface containing a given 3-surface leads to the quantization of the vacuum parameters and hopefully to charge quantization. This motivates the hypothesis that the electromagnetic charges associated with the classical gauge fields of topologically condensed elementary particles are equal to their quantized counterparts. The discussion of dark matter leads to the conclusion that electro-weak and color gauge charges of dark matter can be non-vanishing [J6, F9].

2.1.4 Equivalence Principle as duality between super-canonical and Super Kac-Moody conformal algebras

A precise formulation of Equivalence Principle came from a deeper mathematical understanding of the relationship between super-canonical (SC) and Super Kac-Moody (SKM) symmetries which has been one of the central themes in the development of TGD. The progress in the understanding of the number theoretical aspects of TGD [E2] gives good hopes of lifting $SKMV$ (V denotes Virasoro) to a subalgebra of SCV so that coset construction works meaning that the differences of SCV and $SKMV$ generators annihilate physical states. This condition has interpretation in terms of Equivalence Principle with coset Super Virasoro conditions defining a generalization of Einstein's equations in TGD framework. Rather concretely: the actions of the imbedding space Dirac operator associated with the generator G_0 in for SC and SKM degrees of freedom are identical so that SKM and SC four-momenta and color quantum numbers identifiable as gravitational and inertial variants of these quantum numbers can be identified. Also p-adic thermodynamics finds a justification since the expectation values of SKM conformal weights can be non-vanishing in physical states.

2.2 The identification of Higgs as a weakly charged wormhole contact

Quantum classical correspondence suggests that electro-weak massivation should have simple space-time description allowing also to identify Higgs boson if it exists. This description indeed exists and allows also to understand the precise relationship between gravitational and inertial masses and how Equivalence Principle is weakened in TGD framework.

The basic observation is that gauge and gravitational fluxes flow to larger space-time sheets through # (wormhole) contacts. If gravitational energy can be regarded in the Newtonian limit as a gauge charge, the contacts feed the gravitational energy regarded as a gauge flux to the lower condensate levels. The non-conservation of gravitational gauge flux means that # contacts can carry gravitational four-momentum. Since CP_2 type vacuum extremals are the natural candidates for # contacts, the natural hypothesis is that the non-vanishing light-like gravitational four-momentum of # contacts is responsible for the non-conservation of gravitational four-momentum flux. The non-conservation of the light-like gravitational four-momentum of CP_2 type extremals is in turn responsible for the non-conservation of the net gravitational four-momentum.

contacts can be also carriers of inertial four-momentum which must be conserved in absence of four-momentum exchange between environment and wormhole contact. Therefore Equivalence Principle cannot hold true in strict sense. Equivalence Principle is satisfied in a weak sense if the inertial four-momentum is equal to the average four-momentum associated with the zitterbewegung motion and corresponds to the center of mass motion for the # contact.

The non-conservation of weak gauge currents for CP_2 type extremals implies a non-conservation of weak charges and the finite range of weak forces. If wormhole contacts correspond to pieces of CP_2 type vacuum extremal, electro-weak gauge currents are not conserved classically unlike color and Kähler current. The non-conservation of weak isospin corresponds to the presence of pairs of right/left handed fermion and left/right handed antifermion at wormhole contacts. These wormhole contacts are excellent candidates for the TGD counterpart of Higgs boson providing the most natural mechanism for the massivation of weak bosons. The finding that that CP_2 parts of the induced gamma matrices connect different M^4 chiralities of induced spinor fields provided the original motivation for the belief that Higgs mechanism is realized in some manner in TGD Universe. This coupling must be crucial for the formation of weakly charged wormhole contacts.

There are two contributions to the mass of elementary particle corresponding to the primary and secondary topological condensation.

1. The dominant contribution to the fermion masses would be due to p-adic thermodynamics describing primary topological condensation. If weak form of Equivalence Principle holds true, inertial mass would result simply as the average of non-conserved light-like gravitational four-momentum. This contribution to the inertial mass is generated in the topological condensation of CP_2 type extremal representing elementary particle involving only single light like elementary particle horizon, say fermion, and by randomness of the zitterbewegung corresponds naturally to the contribution given by p-adic thermodynamics.

2. For gauge bosons the contribution from primary condensation should be very small or vanishing if the radius of zitterbewegung orbit is larger than the size of the space-time sheet containing the topologically condensed boson so that the motion is along a light-like geodesic in a good approximation. The space-time sheet representing massless state suffered secondary topologically condensation at a larger space-time sheet and viewed as a particle can develop mass via Higgs mechanism since wormhole contacts cannot be regarded as moving along light like geodesics in the length and time scale involved. # contacts carrying net left handed weak isospin have interpretation as TGD counterparts of neutral Higgs bosons and the formation of a coherent state involving superposition of states with varying number of wormhole contacts corresponds to the generation of a vacuum expectation value of Higgs field.

2.3 General mass formula

One of the blessings of effective 2-dimensionality is that one can treat different 2-surfaces X_i^2 as almost independent degrees of freedom. In the case of translations this is not true since independent translations lead the surfaces X^2 outside $\delta M_\pm^4 \times CP_2$. Therefore one must consider two options.

1. If one neglects the correlation between the translations and assigns to each X_i^2 independent translational degrees of freedom a separate mass formula for each X_i^2 would result:

$$M_i^2 = -\sum_i L_{0i}(SKM) + \sum_i L_{0i}(SC) \ . \tag{2}$$

   Here $L_{0i}(SKM)$ contains a CP_2 cm term giving the CP_2 contribution to the mass squared known once the spinorial partial waves associated with super generators used to construct the state are known.

2. Perhaps the only internally consistent option is based on the assignment of the mass squared with the total cm. This would give

$$M^2 = (\sum_i p_i)^2 = \sum_i M_i^2 + 2\sum_{i\neq j} p_i \cdot p_j = -\sum_i L_{0i}(SKM) + \sum_i L_{0i}(SC) \ . \tag{3}$$

Here $L_{0i}(SKM)$ contains a CP_2 cm term giving the CP_2 contribution to the mass squared known once the spinorial partial waves associated with super generators used to construct the state are known. $L_0(SC)$ term contains only leptonic or quark oscillator operators unless one allows both the lepto-quark type gamma matrices involving both D^+ and D_-^{-1} and leptonic gamma matrices involving instead of $D_\pm^{\pm 1}$ the projector P to the spinor modes with a non-vanishing eigenvalue of D.

The decomposition of the net four momentum to a sum of individual momenta can be regarded as subjective unless there is a manner to measure the individual masses. It might be that there is no unique assignment of momenta to individual partons and that this non-uniqueness is part of the gauge symmetry implied by 7–3 duality.

2.3.1 Mass squared as a thermal expectation of super Kac-Moody conformal weight

The general view about particle massivation is based on the generalized coset construction allowing to understand the p-adic thermal contribution to mass squared as a thermal expectation value of the conformal weight for super Kac-Moody Virasoro algebra ($SKMV$) or equivalently super-camonical Virasoro algebra (SCV). Conformal invariance holds true only for the generators of the differences of $SKMV$ and SCV generators. In the case of SCV and $SKMV$ only the generators L_n, $n > 0$, annihilate the physical states. Obviously the actions of super-canonical Virasoro (SCV) generators and Super Kac-Moody Virasoro generators on physical states are identical. The interpretation is in terms of Equivalence Principle.

1. Super-Kac Moody conformal weights must be negative for elementary fermions and this can be understood if the real parts of fermionic conformal weights are equal to -1/2 as required by the scaling invariance of integration measures associated with the light-like coordinate of light-cone boundary.

2. Ground state with negative confromal is generated by applying SC generators (Hamiltonians and their super counterparts) with conformal weights $-1/2 + iy$ plus SKM generators. Massless state annihilated by L_n, $n > 0$ is is obtained from this by applying Super generators.

3. Massless state is thermalized with respect to SKMV with thermal excitations created by generators L_n, $n > 0$.

2.3.2 Mass formula for bound states of partons

The coefficient of proportionality between mass squared and conformal weight can be deduced from the observation that the mass squared values for CP_2 Dirac operator correspond to definite values of conformal weight in p-adic mass calculations. It is indeed possible to assign to partonic 2-surface X^2 CP_2 partial waves correlating strongly with the net electro-weak quantum numbers of the parton so that the assignment of ground state conformal weight to CP_2 partial waves makes sense. In the case of M^4 degrees of freedom it is not possible to talk about momentum eigen states since translations take parton out of δH_+ so that momentum must be assigned with the tip of the light-cone containing the particle.

The additivity of conformal weight means additivity of mass squared at parton level and this has been indeed used in p-adic mass calculations. This implies the conditions

$$(\sum_i p_i)^2 = \sum_i m_i^2 \quad (4)$$

The assumption $p_i^2 = m_i^2$ makes sense only for massless partons moving collinearly. In the QCD based model of hadrons only longitudinal momenta and transverse momentum squared are used as labels of parton states, which would suggest that one has

$$\begin{aligned} p_{i,||}^2 &= m_i^2 \ , \\ -\sum_i p_{i,\perp}^2 + 2\sum_{i,j} p_i \cdot p_j &= 0 \ . \end{aligned} \quad (5)$$

The masses would be reduced in bound states: $m_i^2 \rightarrow m_i^2 - (p_T^2)_i$. This could explain why massive quarks can behave as nearly massless quarks inside hadrons.

2.4 Is also Higgs contribution expressible as p-adic thermal expectation?

The consideration of [A9] concerning explicit microscopic structure of dark variants of elementary particles allow to add some details to the general picture about particle massivation reducing to p-adic thermodynamics plus Higgs mechanism, both of them having description in terms of conformal weight.

2.4.1 General picture

1. The mass squared equals to the p-adic thermal average of the conformal weight. There are two contributions to this thermal average. One from the p-adic thermodynamics for super conformal representations, and one from the thermal average related to the spectrum of generalized eigenvalues λ of the modified Dirac operator D. Higgs expectation value appears in the role of a mass term in the Dirac equation just like λ in the modified Dirac equation. For the zero modes of D λ vanishes.

2. There are good motivations to believe that λ is expressible as a superposition of zeros of Riemann zeta or some more general zeta function. The problem is that λ is complex. Since Dirac operator is essentially the square root of d'Alembertian (mass squared operator), the natural interpretation of λ would be as a complex "square root" of the conformal weight.

Remark: The earlier interpretation of λ as a complex conformal weight looks rather stupid in light of this observation.

This encourages to consider the interpretation in terms of vacuum expectation of the square root of Virasoro generator, that is generators G of super Virasoro algebra, or something analogous. The super generators G of the super-conformal algebra carry fermion number in TGD framework where Majorana condition does not make sense physically. The modified Dirac operators for the two possible choices $t_\pm$ of the light-like vector appearing in the eigenvalue equation $D\Psi = \lambda t_\pm^k \Gamma_k \Psi$ could however define a bosonic algebra resembling super-conformal algebra. In fact, the operators $a_\pm = \lambda t_\pm^k \Gamma_k$ are nilpotent and anti-commute to λ so that the minimal super-algebra would be 3-dimensional.

The p-adic thermal expectation values of contractions of $t_-^k \gamma_k D_+$ and $t_+^k \gamma_k D_-$ should co-incide with the vacuum expectations of Higgs and its conjugate. Note that D_+ and D_- would be same operator but with different definition of the generalized eigenvalue and hermitian conjugation would map these two kinds of eigen modes to each other. The real contribution to the mass squared would thus come naturally as $\langle|\lambda|^2\rangle$. Of course, $\langle H\rangle = \langle\lambda\rangle$ is only a hypothesis encouraged by the internal consistency of the physical picture, not a proven mathematical fact.

2.4.2 Questions

This leaves still some questions.

1. Does the p-adic thermal expectation $\langle\lambda\rangle$ dictate $\langle H\rangle$ or vice versa? Physically it would be rather natural that the presence of a coherent state of Higgs wormhole contacts induces the mixing of the eigen modes of D. On the other hand, the quantization of the p-adic temperature T_p suggests that Higgs vacuum expectation is dictated by T_p.

2. Also the phase of $\langle\lambda\rangle$ should have physical meaning. Could the interpretation of the imaginary part of $\langle\lambda\rangle$ make possible the description of dissipation at the fundamental level?

3. Is p-adic thermodynamics consistent with the quantal description as a coherent state? The approach based on p-adic variants of finite temperature QFTs associate with the legs of generalized Feynman diagrams might resolve this question neatly since thermodynamical states would be genuine quantum states in this approach made possible by zero energy ontology.

3 Could also gauge bosons correspond to wormhole contacts?

The developments in the formulation of quantum TGD which have taken place during the period 2005-2007 [C1, C2, C3] suggest dramatic simplifications of the general picture discussed in the earlier version of this chapter. p-Adic mass calculations [F3, F4, F5] leave a lot of freedom concerning the detailed identification of elementary particles. The basic open question is whether the *theory is free at parton level* as suggested by the recent view about the construction of S-matrix and by the almost topological QFT property of quantum TGD at parton level [C2, C3]. Or more concretely: do partonic 2-surfaces carry only free many-fermion states or can they carry also bound states of fermions and anti-fermions identifiable as bosons? If the theory is free at parton level, no bi-local composites of second quantized induced spinor field would be needed in the construction of the quantum states and this would simply the theory enormously.

What is known that Higgs boson corresponds naturally to a wormhole contact. The wormhole contact connects two space-time sheets with induced metric having Minkowski signature. Wormhole contact itself has an Euclidian metric signature so that there are two wormhole throats which are light-like 3-surfaces and would carry fermion and anti-fermion number in the case of Higgs. Irrespective of the identification of the remaining elementary particles MEs (massless extremals, topological light rays) would serve as space-time correlates for elementary bosons. Higgs type wormhole contacts would connect MEs to the larger space-time sheet and the coherent state of neutral Higgs would generate gauge boson mass and could contribute also to fermion mass.

The basic question is whether this identification applies also to gauge bosons (certainly not to graviton). This identification would imply quite a dramatic simplification since the theory would be free at single parton level and the only stable parton states would be fermions and anti-fermions. As will be found this identification allows to understand the dramatic difference between graviton and other gauge bosons and the weakness of gravitational coupling, gives a connection with the string picture of gravitons, and predicts that stringy states are directly relevant for nuclear and condensed matter physics as has been proposed already earlier [F8, J1, J2]. In order to avoid confusion it must be emphasized that this picture is not consistent with the older picture discussed above.

3.1 Option I: Only Higgs as a wormhole contact

The only possibility considered hitherto has been that elementary bosons correspond to partonic 2-surfaces carrying fermion-anti-fermion pair such that either fermion or anti-fermion has a non-physical polarization. For this option CP_2 type extremals condensed on MEs and travelling with light velocity would serve as a model for both fermions and bosons. MEs are not absolutely necessary for this option. The couplings of fermions and gauge bosons to Higgs would be very similar topologically. Consider now the counter arguments.

1. This option fails if the theory at partonic level is free field theory so that anti-fermions and elementary bosons cannot be identified as bound states of fermion and anti-fermion with either of them having non-physical polarization.

2. Mathematically oriented mind could also argue that the asymmetry between Higgs and elementary gauge bosons is not plausible whereas asymmetry between fermions and gauge bosons is. Mathematician could continue by arguing that if wormhole contacts with net quantum numbers of Higgs boson are possible, also those with gauge boson quantum numbers are unavoidable.

3. Physics oriented thinker could argue that since gauge bosons do not exhibit family replication phenomenon (having topological explanation in TGD framework) there must be a profound difference between fermions and bosons.

3.2 Option II: All elementary bosons as wormhole contacts

The hypothesis that quantum TGD reduces to a free field theory at parton level is consistent with the almost topological QFT character of the theory at this level. Hence there are good motivations for studying explicitly the consequences of this hypothesis.

3.2.1 Elementary bosons must correspond to wormhole contacts if the theory is free at parton level

Also gauge bosons could correspond to wormhole contacts connecting MEs [D1] to larger space-time sheet and propagating with light velocity. For this option there would be no need to assume the presence of non-physical fermion or anti-fermion polarization since fermion and anti-fermion would reside at different wormhole throats. Only the definition of what it is to be non-physical would be different on the light-like 3-surfaces defining the throats.

The difference would naturally relate to the different time orientations of wormhole throats and make itself manifest via the definition of light-like operator $o = x^k\gamma_k$ appearing in the generalized eigenvalue equation for the modified Dirac operator [B4, C1]. For the first throat o^k would correspond to a light-like tangent vector t^k of the partonic 3-surface and for the second throat to its M^4 dual $\hat{t}^k$ in a preferred rest system in M^4 (implied by the basic construction of quantum TGD). What is nice that this picture non-asks the question whether t^k or $\hat{t}^k$ should appear in the modified Dirac operator.

Rather satisfactorily, MEs (massless extremals, topological light rays) would be necessary for the propagation of wormhole contacts so that they would naturally emerge as classical correlates of bosons. The simplest model for fermions would be as CP_2 type extremals topologically condensed on MEs and for bosons as pieces of CP_2 type extremals connecting ME to the larger space-time sheet. For fermions topological condensation is possible to either space-time sheet.

3.2.2 What about light-like boundaries and macroscopic wormhole contacts?

Light-like boundaries of the space-time sheet can have macroscopic size and can carry free many-fermion states but not elementary bosons. Number theoretic braids and anyons might be assignable to these structures. Deformations of cosmic strings to magnetic flux tubes with a light-like outer boundary are especially interesting in this respect.

If the ends of a string like object move with light velocity as implied by the usual stringy boundary conditions they indeed define light-like 3-surfaces. Many-fermion states could be assigned at the ends of string. One could also connect in pairwise manner the ends of two time-like strings having opposite time orientation using two space-like strings so that the analog of boson state consisting of two wormhole contacts and analogous to graviton would result. "Wormhole throats" could have arbitrarily long distance in M^4.

Wormhole contacts can be regarded as slightly deformed CP_2 type extremals only if the size of M^4 projection is not larger than CP_2 size. The natural question is whether one can construct macroscopic wormhole contacts at all.

1. The throats of wormhole contacts cannot belong to vacuum extremals. One might however hope that small deformations of macrosopic vacuum extremals could yield non-vacuum wormhole contacts of macroscopic size.

2. A large class of macroscopic wormhole contacts which are vacuum extremals consists of surfaces of form $X_1^2 \times X_2^2 \subset (M^1 \times Y^2) \times E^3$, where Y^2 is Lagrangian manifold of CP_2 (induced Kähler form vanishes) and $M^4 = M^1 \times E^3$ represents decomposition of M^1 to time-like and space-like sub-spaces. X_2^2 is a stationary surface of E^3. Both $X_1^2 \subset M^1 \times CP_2$ and X_2^2 have an Euclidian signature of metric except at light-like boundaries $X_a^1 \times X_2^2$ and $X_b^1 \times X_2^2$ defined by ends of X_1^2 defining the throats of the wormhole contact.

3. This kind of vacuum extremals could define an extremely general class of macroscopic wormhole contacts as their deformations. These wormhole contacts describe an interaction of wormhole throats regarded as closed strings as is clear from the fact that X^2 can be visualized as an analog of closed string world sheet X_1^2 in $M^1 \times Y^2$ describing a reaction leading from a state with a given number of incoming closed strings to a state with a given number of outgoing closed strings which correspond to wormhole throats at the two space-time sheets involved.

3.2.3 Phase conjugate states and matter- antimatter asymmetry

By fermion number conservation fermion-boson and boson-boson couplings must involve the fusion of partonic 3-surfaces along their ends identified as wormhole throats. Bosonic couplings would differ from fermionic couplings only in that the process would be $2 \rightarrow 4$ rather than $1 \rightarrow 3$ at the level of throats.

The decay of boson to an ordinary fermion pair with fermion and anti-fermion at the same space-time sheet would take place via the basic vertex at which the 2-dimensional ends of light-like 3-surfaces are identified. The sign of the boson energy would tell whether boson is ordinary boson or its phase conjugate (say phase conjugate photon of laser light) and also dictate the sign of the time orientation of fermion and anti-fermion resulting in the decay.

The two space-time sheets of opposite time orientation associated with bosons would naturally serve as space-time correlates for the positive and negative energy parts of the zero energy state and the sign of boson energy would tell whether it is phase conjugate or not. In the case of fermions second space-time sheet is not absolutely necessary and one can imagine that fermions in initial/final states correspond to single space-time sheet of positive/negative time orientation.

Also a candidate for a new kind interaction vertex emerges. The splitting of bosonic wormhole contact would generate fermion and time-reversed anti-fermion having interpretation as a phase conjugate fermion. This process cannot correspond to a decay of boson to ordinary fermion pair. The splitting process could generate matter-antimatter asymmetry in the sense that fermionic antimatter would consist dominantly of negative energy anti-fermions at space-time sheets having negative time orientation [D5, D6].

This vertex would define the fundamental interaction between matter and phase conjugate matter. Phase conjugate photons are in a key role in TGD based quantum model of living matter. This involves model for memory as communications in time reversed direction, mechanism of intentional action involving signalling to geometric past, and mechanism of remote metabolism involving sending of negative energy photons to the energy reservoir [K1]. The splitting of wormhole contacts has been considered as a candidate for a mechanism realizing Boolean cognition in terms of "cognitive neutrino pairs" resulting in the splitting of wormhole contacts with net quantum numbers of Z^0 boson [J3, M6].

3.3 Graviton and other stringy states

Fermion and anti-fermion can give rise to only single unit of spin since it is impossible to assign angular momentum with the relative motion of wormhole throats. Hence the identification of graviton as single wormhole contact is not possible. The only conclusion is that graviton must be a superposition of fermion-anti-fermion pairs and boson-anti-boson pairs with coefficients determined by the coupling of the parton to graviton. Graviton-graviton pairs might emerge in higher orders. Fermion and anti-fermion would reside at the same space-time sheet and would have a non-vanishing relative angular momentum. Also bosons could have non-vanishing relative angular momentum and Higgs bosons must indeed possess it.

Gravitons are stable if the throats of wormhole contacts carry non-vanishing gauge fluxes so that the throats of wormhole contacts are connected by flux tubes carrying the gauge flux. The mechanism producing gravitons would the splitting of partonic 2-surfaces via the basic vertex. A connection with string picture emerges with the counterpart of string identified as the flux tube connecting the wormhole throats. Gravitational constant would relate directly to the value of the string tension.

The TGD view about coupling constant evolution [C5] predicts $G \propto L_p^2$, where L_p is p-adic length scale, and that physical graviton corresponds to $p = M_{127} = 2^{127} - 1$. Thus graviton would have geometric size of order Compton length of electron which is something totally new from the point of view of usual Planck length scale dogmatism. In principle an entire p-adic hierarchy of gravitational forces is possible with increasing value of G.

The explanation for the small value of the gravitational coupling strength serves as a test for the proposed picture. The exchange of ordinary gauge boson involves the exchange of single CP_2 type extremal giving the exponent of Kähler action compensated by state normalization. In the case of graviton exchange two wormhole contacts are exchanged and this gives second power for the exponent of Kähler action which is not compensated. It would be this additional exponent that would give rise to the huge reduction of gravitational coupling strength from the naive estimate $G \sim L_p^2$.

Gravitons are obviously not the only stringy states. For instance, one obtains spin 1 states when the ends of string correspond to gauge boson and Higgs. Also non-vanishing electro-weak and color quantum numbers are possible and stringy states couple to elementary partons via standard couplings in this case. TGD based model for nuclei as nuclear strings having length of order $L(127)$ [F8] suggests that the strings with light M_{127} quark and anti-quark at their ends identifiable as companions of the ordinary graviton are responsible for the strong nuclear force instead of exchanges of ordinary mesons or color van der Waals forces.

Also the TGD based model of high T_c super-conductivity involves stringy states connecting the space-time sheets associated with the electrons of the exotic Cooper pair [J1, J2]. Thus stringy states would play a key role in nuclear and condensed matter physics, which means a profound departure from stringy wisdom, and breakdown of the standard reductionistic picture.

3.4 Spectrum of non-stringy states

The 1-throat character of fermions is consistent with the generation-genus correspondence. The 2-throat character of bosons predicts that bosons are characterized by the genera (g_1, g_2) of the wormhole throats. Note that the interpretation of fundamental fermions as wormhole contacts with second throat identified as a Fock vacuum is excluded.

The general bosonic wave-function would be expressible as a matrix M_{g_1,g_2} and ordinary gauge bosons would correspond to a diagonal matrix $M_{g_1,g_2} = \delta_{g_1,g_2}$ as required by the absence of neutral flavor changing currents (say gluons transforming quark genera to each other). 8 new gauge bosons are predicted if one allows all 3×3 matrices with complex entries orthonormalized with respect to trace meaning additional dynamical $SU(3)$ symmetry. Ordinary gauge bosons would be $SU(3)$ singlets in this sense. The existing bounds on flavor changing neutral currents give bounds on the masses of the boson octet. The 2-throat character of bosons should relate to the low value $T = 1/n \ll 1$ for the p-adic temperature of gauge bosons as contrasted to $T = 1$ for fermions.

If one forgets the complications due to the stringy states (including graviton), the spectrum of elementary fermions and bosons is amazingly simple and almost reduces to the spectrum of standard model. In the fermionic sector one would have fermions of standard model. By simple counting leptonic wormhole throat could carry $2^3 = 8$ states corresponding to 2 polarization states, 2 charge states, and sign of lepton number giving 8+8=16 states altogether. Taking into account phase conjugates gives 16+16=32 states.

In the non-stringy boson sector one would have bound states of fermions and phase conjugate fermions. Since only two polarization states are allowed for massless states, one obtains $(2+1)\times(3+1) = 12$ states plus phase conjugates giving 12+12=24 states. The addition of color singlet states for quarks gives 48 gauge bosons with vanishing fermion number and color quantum numbers. Besides 12 electro-weak bosons and their 12 phase conjugates there are 12 exotic bosons and their 12 phase conjugates. For the exotic bosons the couplings to quarks and leptons are determined by the orthogonality of the coupling matrices of ordinary and boson states. For exotic counterparts of W bosons and Higgs the sign of the coupling to quarks is opposite. For photon and Z^0 also the relative magnitudes of the couplings to quarks must change. Altogether this makes 48+16+16=80 states. Gluons would result as color octet states. Family replication would extend each elementary boson state into $SU(3)$ octet and singlet and elementary fermion states into $SU(3)$ triplets.

3.5 Higgs mechanism

Consider next the generation of mass as a vacuum expectation value of Higgs when also gauge bosons correspond to wormhole contacts. The presence of Higgs condensate should make the simple rectilinear ME curved so that the average propagation of fields would occur with a velocity less than light velocity. Field equations allow MEs of this kind as solutions [D1].

The finite range of interaction characterized by the gauge boson mass should correlate with the finite range for the free propagation of wormhole contacts representing bosons along corresponding ME. The finite range would result from the emission of Higgs like wormhole contacts from gauge boson like wormhole contact leading to the generation of coherent states of neutral Higgs particles. The emission would also induce non-rectilinearity of ME as a correlate for the recoil in the emission of Higgs.

Higgs expectation should have space-time correlate appearing in the modified Dirac operator. A good candidate is p-adic thermal average for the generalized eigenvalue λ of the modified Dirac operator vanishing for the zero modes. Thermal mass squared as opposed to Higgs contribution would correspond to the average of integer valued conformal weight. For bosons (in particular Higgs boson!) it is simply the sum of expectations for the two wormhole throats.

Both contributions are basically thermal which raises the question whether the interpretation in terms of coherent state of Higgs field (and essentially quantal notion) is really appropriate unless also thermal states can be regarded as genuine quantum states. The matrix characterizing time-like entanglement for the zero energy quantum state can be also thermal S-matrix with respect to the incoming and outgoing partons (hyper-finite factors of type III allow the analog of thermal QFT at the level of quantum states. This allows also a first principle description of p-adic thermodynamics.

4 Does the modified Dirac action define the fundamental action principle?

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. The Dirac determinant associated with the modified Dirac action is an excellent candidate in this respect.

4.1 Modified Dirac equation

In the following the problems of the ordinary Dirac action are discussed and the notion of modified Dirac action is introduced. In particular, the following problems are discussed.

1. Try to guess general formula for the spectrum of the modified Dirac operator and for super-canonical conformal weights by assuming that the eigenvalues are expressible in terms of the data assignable to the two kinds of of number theoretical braids and that the product of vacuum functional expressible as exponent of Kahler function and of the exponent of Chern-Simons action is identifiable as Dirac determinant expressible as product of M^4 and CP_2 parts. Since Kähler function is isometry invariant only the Dirac determinant defined by M^4 braid can contribute to it. Chern-Simons action is not isometry invariant and can be identified as the Dirac determinant associated with CP_2 braid.

2. Try to understand whether the zeta functions involved can be identified as Riemann Zeta or some zeta coding geometric data about partonic 2-surface. Try to understand whether the assignment of a fixed prime p to a partonic 2-surface implies that the zeta function is actually an analog for basic building block of Riemann Zeta.

4.1.1 Problems associated with the ordinary Dirac action

Minimal 2-surface represents a situation in which the representation of surface reduces to a complex-analytic map. This implies that induced metric is hermitian so that it has no diagonal components in complex coordinates $(z, \overline{z})$ and the second fundamental form has only diagonal components of type H^k_{zz}. This implies that minimal surface is in question since the trace of the second fundamental form vanishes. At first it seems that the same must happen also in the more general case with the consequence that the space-time surface is a minimal surface. Although many basic extremals of Kähler action are minimal surfaces, it seems difficult to believe that minimal surface property plus extremization of Kähler action could really boil down to the absolute minimization of Kähler action or a more general principle selecting preferred extremals as Bohr orbits [E2].

This brings in mind a similar long-standing problem associated with the Dirac equation for the induced spinors. The problem is that right-handed neutrino generates super-symmetry only provided that space-time surface and its boundary are minimal surfaces. Although one could interpret this as a geometric symmetry breaking, there is a strong feeling that something goes wrong. Induced Dirac equation and super-symmetry fix the variational principle but this variational principle is not consistent with Kähler action.

One can also question the implicit assumption that Dirac equation for the induced spinors is consistent with the super-symmetry of the configuration space geometry. Super-symmetry would obviously require that for vacuum extremals of Kähler action also induced spinor fields represent vacua. This is however not the case. This super-symmetry is however assumed in the construction of the configuration space geometry so that there is internal inconsistency.

4.1.2 Super-symmetry forces modified Dirac equation

The above described three problems have a common solution. Nothing prevents from starting directly from the hypothesis of a super-symmetry generated by covariantly constant right-handed neutrino and finding a Dirac action which is consistent with this super-symmetry. Field equations can be written as

$$\begin{aligned} D_\alpha T^\alpha_k &= 0 \ , \\ T^\alpha_k &= \frac{\partial}{\partial h^k_\alpha} L_K \ . \end{aligned} \tag{6}$$

If super-symmetry is present one can assign to this current its super-symmetric counterpart

$$\begin{aligned} J^{\alpha k} &= \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l \Psi \ , \\ D_\alpha J^{\alpha k} &= 0 \ . \end{aligned} \tag{7}$$

having a vanishing covariant divergence. The isometry currents currents and super-currents are obtained by contracting T^{α_k} and J^{α_k} with the Killing vector fields of super-symmetries. Note also that the super current

$$J^\alpha = \overline{\nu_R} T^\alpha_l \Gamma^l \Psi \tag{8}$$

has a vanishing divergence.

By using the covariant constancy of the right-handed neutrino spinor, one finds that the divergence of the super current reduces to

$$D_\alpha J^{\alpha k} = \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l D_\alpha \Psi \ . \tag{9}$$

The requirement that this current vanishes is guaranteed if one assumes that modified Dirac equation

$$\begin{aligned} \hat{\Gamma}^\alpha D_\alpha \Psi &= 0 \ , \\ \hat{\Gamma}^\alpha &= T^\alpha_l \Gamma^l \ . \end{aligned} \tag{10}$$

This equation must be derivable from a modified Dirac action. It indeed is. The action is given by

$$L = \overline{\Psi}\hat{\Gamma}^\alpha D_\alpha \Psi \ . \tag{11}$$

Thus the variational principle exists. For this variational principle induced gamma matrices are replaced with effective induced gamma matrices and the requirement

$$D_\mu \hat{\Gamma}^\mu = 0 \tag{12}$$

guaranteing that super-symmetry is identically satisfied if the bosonic field equations are satisfied. For the ordinary Dirac action this condition would lead to the minimal surface property. What sounds strange that the essentially hydrodynamical equations defined by Kähler action have fermionic counterpart: this is very far from intuitive expectations raised by ordinary Dirac equation and something which one might not guess without taking super-symmetry very seriously.

4.1.3 How can one avoid minimal surface property?

These observations suggest how to avoid the emergence of the minimal surface property as a consequence of field equations. It is not induced metric which appears in field equations. Rather, the effective n metric appearing in the field equations is defined by the anti-commutators of $\hat{\gamma}_\mu$

$$\hat{g}_{\mu\nu} = \{\hat{\Gamma}_\mu, \hat{\Gamma}_\nu\} = 2T_\mu^k T_{\nu k} \ . \tag{13}$$

Here the index raising and lowering is however performed by using the induced metric so that the problems resulting from the non-invertibility of the effective metric are avoided. It is this dynamically generated effective metric which must appear in the number theoretic formulation of the theory.

Field equations state that space-time surface is minimal surface with respect to the effective metric. Note that a priori the choice of the bosonic action principle is arbitrary. The requirement that effective metric defined by energy momentum tensor has only non-diagonal components except in the case of non-light-like coordinates, is satisfied for the known solutions of field equations.

4.1.4 Does the modified Dirac action define the fundamental action principle?

There is quite fundamental and elegant interpretation of the modified Dirac action as a fundamental action principle discussed also in [E2]. In this approach vacuum functional can be defined as the Grassmannian functional integral associated with the exponent of the modified Dirac action. This definition is invariant with respect to the scalings of the Dirac action so that theory contains no free parameters.

An alternative definition is as a Dirac determinant which might be calculated in TGD framework without applying the poorly defined functional integral. There are good reasons to expect that the Dirac determinant exponent of Kähler function for a preferred Bohr orbit like extremal of the Kähler action with the value of Kähler coupling strength coming out as a prediction. Hence the dynamics of the modified Dirac action at light-like partonic 3-surfaces X_l^3, even when restricted to almost-topological dynamics induced by Chern-Simons action, would dictate the dynamics at the interior of the space-time sheet.

The knowledge of the canonical currents and super-currents, together with the anti-commutation relations stating that the fermionic super-currents S_A and S_B associated with Hamiltonians H_A and H_B anti-commute to a bosonic current $H_{[A,B]}$, allows in principle to deduce the anti-commutation relations satisfied by the induced spinor field. In fact, these conditions replace the usual anti-commutation relations used to quantize free spinor field. Since the normal ordering of the Dirac action would give Kähler action,

Kähler coupling strength would be determined completely by the anti-commutation relations of the super-canonical algebra. Kähler coupling strength would be dynamical and the selection of preferred extremals of Kähler action would be more or less equivalent with quantum criticality because criticality corresponds to conformal invariance and the hyper-quaternionic version of the super-conformal invariance results only for the extrema of Kähler action. p-Adic (or possibly more general) coupling constant evolution and quantum criticality would come out as a prediction whereas in the case that Kähler action is introduced as primary object, the value of Kähler coupling strength must be fixed by quantum criticality hypothesis.

The mixing of the M^4 chiralities of the imbedding space spinors serves as a signal for particle massivation and breaking of super-conformal symmetry. The induced gamma matrices for the space-time surfaces which are deformations of M^4 indeed contain a small contribution from CP_2 gamma matrices: this implies a mixing of M^4 chiralities even for the modified Dirac action so that there is no need to introduce this mixing by hand.

4.2 The association of the modified Dirac action to Chern-Simons action and explicit realization of super-conformal symmetries

Super Kac-Moody symmetries should correspond to solutions of modified Dirac equation which are in some sense holomorphic. The discussion below is based on the same general ideas but differs radically from the previous picture at the level of details. The additional assumption inspired by the considerations of this section is that the action associated with the partonic 3-surfaces is non-singular and therefore Chern-Simons action for the induced Kähler gauge potential.

This means that TGD is at the fundamental level almost-topological QFT: only the light-likeness of the partonic 3-surfaces brings in the induced metric and gravitational and gauge interactions and induces the breaking of scale and super-conformal invariance. The resulting theory possesses the expected super Kac-Moody and super-canonical symmetries albeit in a more general form than suggested by the considerations of this section. A connection of the spectrum of the modified Dirac operator with the zeros or Riemann Zeta is suggestive and provides support for the earlier number theoretic speculations concerning the spectrum of super-canonical conformal weights. One can safely say, that if this formulation is correct, TGD could not differ less from a physically trivial theory.

4.2.1 Zero modes and generalized eigen modes of the modified Dirac action

Consider net the zero modes and generalized eigen modes for the modified Dirac operator.

1. The modified gamma matrices appearing in the modified Dirac equation are expressible in terms of the Lagrangian density L assignable to the light-like partonic 3-surface X^43_l as

$$\hat{\Gamma}^\alpha = \frac{\partial L}{\partial_\alpha h^k}\Gamma_k \quad , \tag{14}$$

where Γ_k denotes gamma matrices of imbedding space. The modified Dirac operator is defined as

$$D \quad = \quad \hat{\Gamma}^\alpha D_\alpha \quad , \tag{15}$$

where D_α is the covariant derivative defined by the induced spinor connection. Modified gamma matrices satisfy the condition

$$D_\alpha \hat{\Gamma}^\alpha = 0 \quad (16)$$

if the field equations associated with L are satisfied. This guarantees that one indeed obtains the analog of the massless Dirac equation. Zero modes of the modified Dirac equation should define the conformal super-symmetries.

2. The generalized eigenvalues and eigen solutions of the modified Dirac operator are defined as

$$\begin{aligned} D\Psi &= \lambda N \Psi \ , \\ N &= n^k \Gamma_k \ . \end{aligned} \quad (17)$$

Here n^k denotes a light-like vector which must satisfy the integrability condition

$$\left[D, n^k \Gamma_k\right] = 0 \ . \quad (18)$$

if the analog $D^2\Psi = 0$ for the square of massless Dirac equation is to hold true. n^k should be determined by the field equations associated with L somehow and commutativity condition could fix n more or less uniquely.

If the commutativity condition holds true then any generalized eigen mode Ψ_λ gives rise to a zero mode as $\Psi = N\Psi_\lambda$. One can add to a given non-zero mode any superposition of zero modes without affecting the generalized eigen mode property.

The commutativity condition can be satisfied if the tangent space at each point of X^4 contains preferred plane M^2 guaranteing $HO - H$ duality and having interpretation as a preferred plane of non-physical polarizations. In this case n can be chosen to be constant light-like vector in M^2.

3. The hypothesis is that Kähler function is expressible in terms of the Dirac determinant of the modified Dirac operator defined as the product of the generalized eigenvalues. The Dirac determinant must carry information about the interior of the space-time surface determined as preferred extremal of Kähler action or (as the hypothesis goes) as hyper-quaternionic or co-hyper-quaternionic 4-surface of M^8 defining unique 4-surface of $M^4 \times CP_2$. The assumption that X^3_L is light-like brings in an implicit dependence on the induced metric. The simplest but non-necessary assumption is that n^k is a light-like vector field tangential to X^3_l so that the knowledge of X^3_l fixes completely the dynamics.

4. If the action associated with the partonic light-like 3-surfaces contains induced metric, the field equations become singular and ill-defined unless one defines the field equations at X^3_l via a limiting procedure and poses additional conditions on the behavior of Ψ at X^3_l. Situation changes if the action does not contain the induced metric. The classical field equations are indeed well-defined at light-like partonic 3-surfaces for Chern-Simons action for the induced Kähler gauge potential

$$L = L_{C-S} = k\epsilon^{\alpha\beta\gamma} J_{\alpha\beta} A_\gamma \ . \qquad (19)$$

One obtains the analog of WZW model with gauge field replaced with the induced Kähler form. This action does not depend on the induced metric explicitly so that in this sense a topological field theory results. There is no dependence on M^4 gamma matrices so that local Lorentz transformations act as super-conformal symmetries of both classical field equations and modified Dirac equation and $SL(2,C)$ defines the analog of the $SU(2)$ Kac-Moody algebra for $N = 4$ SCA.

The facts that the induced metric is light-like for X^3_l, that the modified Dirac equation contains information about this and therefore about induced metric, and that Dirac determinant is the product of the non-vanishing eigen values of the modified Dirac operator, imply the failure of topological field theory property at the level of Kähler function identified as the logarithm of the Dirac determinant.

A more complicated option would be that the modified Dirac action contains also interior term corresponding to the Kähler action. This alternative would break super-conformal symmetries explicitly and almost-topological QFT property would be lost. This option is not consistent with the idea that quantum-classical correspondence relates the partonic dynamics at X^3_l with the classical dynamics in the interior of space-time providing first principle justification for the basic assumptions of the quantum measurement theory.

The classical field equations defined by L_{C-S} read as

$$\begin{aligned} D_\mu \frac{\partial L_{C-S}}{\partial_\mu h^k} &= 0 \ , \\ \frac{\partial L_{C-S}}{\partial_\mu h^k} &= \epsilon^{\mu\alpha\beta} \left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta} A_k) \right] \ . \end{aligned} \qquad (20)$$

From the explicit form of equations it is obvious that the most general solution corresponds to a X^3_l with at most 2-dimensional CP_2 projection.

Although C-S action vanishes, the color isometry currents are in general non-vanishing. One can assign currents also to super-Kac Moody and super-canonical transformations using standard formulas and the possibility that the corresponding charges define configuration space Hamiltonians and their super-counterparts must be considered seriously.

Suppose that the CP_2 projection is 2-dimensional and not a Lagrange manifold. One can introduce coordinates for which the coordinates for X^2 are same as those for CP_2 projection. For instance, complex coordinates $(z, \overline{z})$ of a geodesic sphere could be used as local coordinates for X^2. One can also assign one M^4 coordinate, call it r, with M^4 projection X^1 of X^3_l. Locally this coordinate can be taken to be one of the standard M^4 coordinates. The remaining five H-coordinates can be expressed in terms of $(r, z, \overline{z})$ and light-likeness condition boils down to the vanishing of the metric determinant:

$$det(g_3) = 0 \ . \qquad (21)$$

All diffeomorphisms of H respecting the light-likeness condition are symmetries of the solution ansatz.

Consider some special cases serve as examples.

1. The simplest situation results when X_l^4 is of form $X^1 \times X^2$, where X^1 is light-like random curve in M^4 as for CP_2 type vacuum extremals. In this case light-likeness boils down to Virasoro conditions with real parameter r playing the role analogous to that of a complex coordinate: this conformal symmetry is dynamical and must be distinguished from conformal symmetries assignable to X^2. A plausible guess is that light-likeness condition quite generally reduces to the classical Virasoro conditions.

2. A solution in which CP_2 projection is dynamical is obtained by assuming that for a given value of M^4 time coordinate CP_2- and M^4- projections are one-dimensional curves. For instance, CP_2 projection could be the circle $\Theta = \Theta(m^0 \equiv t)$ whereas M^4 projection could be the circle $\rho = \sqrt{x^2+y^2} = \rho(m^0)$. Light-likeness condition reduces to the condition $g_{tt} = 1 - R^2\partial_t\Theta^2 - \partial_t\rho^2 = 0$.

4.2.2 Classical field equations for the modified Dirac equation defined by Chern-Simons action

The modified Dirac operator is given by

$$\begin{aligned} D &= \frac{\partial L_{C-S}}{\partial_\mu h^k}\Gamma_k D_\mu \\ &= \epsilon^{\mu\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_\mu \ , \\ \hat{\epsilon}^{\alpha\beta\gamma} &= \epsilon^{\alpha\beta\gamma}\sqrt{g_3} \ . \end{aligned} \qquad (22)$$

Note $\hat{\epsilon}^{\alpha\beta\gamma}$ = does not depend on the induced metric. The operator is non-trivial only for 3-surfaces for which CP_2 projection is 2-dimensional non-Lagrangian sub-manifold. The modified Dirac operator reduces to a one-dimensional Dirac operator

$$D = \hat{\epsilon}^{r\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_r \ . \qquad (23)$$

The solutions of the modified Dirac equation are obtained as spinors which are covariantly constant with respect to the coordinate r:

$$D_r\Psi = 0 \ . \qquad (24)$$

Non-vanishing spinors $\Psi_1 = \partial_r\Psi$ satisfying $\Gamma_r\Psi_1 = 0$ are not possible. Ψ defines super-symmetry for the generalized eigen modes if the additional condition

$$\Psi = N\Psi_0 \qquad (25)$$

is satisfied. The interpretation as super-conformal symmetries makes sense if the Fourier coefficients of zero modes and their conjugates are anticommuting Grassmann numbers. The zero modes which are not of this form do not generate super-conformal symmetries and might correspond to massless particles. TGD based vision about Higgs mechanism suggest the interpretation of n^k as a non-conserved gravitational four-momentum whose time average defines inertial four-momentum of parton. The sum of the partonic four-momenta would be identified as the classical four-momentum associated with the interior of the space-time sheet.

The covariant derivatives D_α involve only CP_2 spinor connection and the metric induced from CP_2. D_r involves CP_2 spinor connection unless X_l^3 is of form $X^1 \times X^2 \subset M^4 \times CP_2$. The eigen modes of D correspond to the solutions of

$$D\Psi = \lambda N\Psi \tag{26}$$

The first guess is that $N = n^k\gamma_k$ corresponds to the tangential light-like vector $n^k = \Phi\partial_r h^k$ where Φ is a normalization factor which can depend on position.

The obvious objection is that with this assumption it is difficult to understand how Dirac determinant can correspond to an absolute extremum of Kähler action for 4-D space-time sheet containing partonic 3-surfaces as causal determinants ($\sqrt{g_4} = 0$). However, if one can select a unique M^4 time coordinate, say as that associated with the rest system for the average four-momentum defined as Chern-Simons Noether charge, then one can assign to n^k a unique dual obtained by changing the sign of its spatial components. The condition that this vector is tangential to the 4-D space-time sheet would provide information about the space-time sheet and bring in 4-dimensionality. At this stage one must however leave the question about the choice of n^k open.

One should be able to fix Φ apart from overall normalization. First of all, the requirement that zero modes defines super symmetries implies the condition $[D, n^k\Gamma_k]\Psi = 0$ for zero modes. This condition boils down to the requirement

$$D_r(\Phi\partial_r h^k\Gamma_k)\Psi = 0 \ . \tag{27}$$

This in turn boils down to a condition

$$D_r\partial_r h^k + \frac{\partial_r\Phi}{\Phi}\partial_r h^k = 0 \ . \tag{28}$$

These conditions in turn guarantee that the condition

$$D_r(h_{kl}\partial_r h^k\partial_r h^l) = 0 \tag{29}$$

implied by the light-likeness condition are satisfied. Since Φ is determined apart from a multiplicative constant from the light-likeness condition the system is internally consistent. The conditions above are not general coordinate invariant so that the coordinate r must correspond to a physically preferred coordinate perhaps defined by the conditions above.

One can express the eigenvalue equation in the form

$$\begin{aligned} \partial_r\Psi &= \lambda O\Psi \ , \\ O &= (\hat{\Gamma}^r)^{-1}N \ , \\ (\hat{\Gamma}^r)^{-1} &= \frac{\hat{\Gamma}^r}{a^k a^l h_{kl}} \ , \ \hat{\Gamma}^r \equiv a^k\Gamma_k \ . \end{aligned} \tag{30}$$

This equation defines a flow with r in the role of a time parameter. The solutions of this equation can be formally expressed as

$$\Psi(r,z,\overline{z}) = Pe^{\lambda\int O(r,z,\overline{z})dr}\Psi_0(z,\overline{z}) \ . \tag{31}$$

Here P denotes the ordered exponential needed because the operators $O(r, z\overline{z})$ need not commute for different values of r.

4.2.3 Can one allow light-like causal determinants with 3-D CP_2 projection?

The standard quantum field theory wisdom would suggest that light-like partonic 3-surfaces which are extremals of the Chern-Simons action correspond only to what stationary phase approximation gives when vacuum functional is the product of exponent of Kähler function resulting from Dirac determinant and an imaginary exponent of Chern-Simons action whose coefficient is proportional to the central charge of Kac-Moody algebras associated with CP_2 degrees of freedom.

One cannot exclude the possibility that 3-D light-like causal determinants might be required by the general consistency of the theory. The identification of the exponent of Kähler function as Dirac determinant remains a viable hypothesis for this option. "Off mass shell" breaking of super-conformal symmetries is implied since modified Dirac equation implies the conservation of super conformal currents only when CP_2 projection is at most 2-dimensional.

4.2.4 Some problems of TGD as almost-topological QFT and their resolution

There are some problems involved with the precise definition of the quantum TGD as an almost-topological QFT at the partonic level and the resolution of these problems leads to an unexpected connection between cosmology and parton level physics.

1. Three problems

The proposed view about partonic dynamics is plagued by three problems.

1. The definition of supercanonical and super-Kac-Moody charges in M^4 degrees of freedom poses a problem. These charges are simply vanishing since M^4 coordinates do not appear in field equations.

2. Classical field equations for the C-S action imply that this action vanishes identically which would suggest that the dynamics does not depend at all on the value of k. The central extension parameter k determines the over-all scaling of the eigenvalues of the modified Dirac operator. $1/k$- scaling occurs for the eigenvalues so that Dirac determinant scales by a finite power k^N if the number N of the allowed eigenvalues is finite for the algebraic extension considered. A constant $Nlog(k)$ is added to the Kähler function and its effect seems to disappear completely in the normalization of states.

3. The general picture about Jones inclusions and the possibility of separate Planck constants in M^4 and CP_2 degrees of freedom suggests a close symmetry between M^4 and CP_2 degrees of freedom at the partonic level. Also in the construction of the geometry for the world of classical worlds the symplectic and Kähler structures of both light-cone boundary and CP_2 are in a key role. This symmetry should be somehow coded by the Chern-Simons action.

2. A possible resolution of the problems

A possible cure to the above described problems is based on the modification of Kähler gauge potential by adding to it a gradient of a scalar function Φ with respect to M^4 coordinates.

1. This implies that super-canonical and super Kac-Moody charges in M^4 degrees of freedom are non-vanishing.

2. Chern-Simons action is non-vanishing if the induced CP_2 Kähler form is non-vanishing. If the imaginary exponent of C-S action multiplies the vacuum functional, the presence of the central extension parameter k is reflected in the properties of the physical states.

3. The function Φ could code for the value of $k(M^4)$ via a proportionality constant

$$\Phi = \frac{k(M^4)}{k(CP_2)}\Phi_0 \ , \tag{32}$$

Here $k(CP_2)$ is the central extension parameter multiplying the Chern-Simons action for CP_2 Kähler gauge potential. This tricks does just what is needed since it multiplies the Noether currents and super currents associated with M^4 degrees of freedom with $k(M^4)$ instead of $k(CP_2)$.

The obvious breaking of $U(1)$ gauge invariance looks strange at first but it conforms with the fact that in TGD framework the canonical transformations of CP_2 acting as $U(1)$ gauge symmetries do not give to gauge degeneracy but to spin glass degeneracy since they act as symmetries of only vacuum extremals of Kähler action.

3. How to achieve Lorentz invariance?

Lorentz invariance fixes the form of function Φ uniquely as the following argument demonstrates.

1. Poincare invariance would be broken in any case for a given light-cone in the decomposition $CH = \cup_m CH_m$ of the configuration space to sub-configuration spaces associated with light-cones at various locations of M^4 but since the functions Φ associated with various light cones would be related by a translation, translation invariance would not be lost.

2. The selection of Φ should not break Lorentz invariance. If Φ depends on the Lorentz proper time a only, this is partially achieved. Momentum currents would be proportional to m^k and become light like at the boundary of the light-cone. This fits very nicely with the interpretation that the matter emanates from the tip of the light cone in Robertson-Walker cosmology.

Lorentz invariance poses even stronger conditions on Φ.

1. Partonic four-momentum defined as Chern-Simons Noether charge is definitely not conserved and must be identified as gravitational four-momentum whose time average corresponds to the conserved inertial four-momentum assignable to the Kähler action [D3, D5]. This identification is very elegant since also gravitational four-momentum is well-defined although not conserved.

2. Lorentz invariance implies that mass squared is constant of motion. Hence it is interesting to look what expression for Φ results if the gravitational mass defined as Noether charge for C-S action is conserved. The components of the four-momentum for Chern-Simons action are given by

$$P^k = \frac{\partial L_{C-S}}{\partial(\partial_\alpha a)} m^{kl}\partial_{m^l} a \ .$$

Chern-Simons action is proportional to $A_\alpha = A_a \partial_\alpha a$ so that one has

$$P^k \propto \partial_a \Phi \partial_{m^k} a = \partial_a \Phi \frac{m^k}{a} \ .$$

The conservation of gravitational mass gives $\Phi \propto a$. Since CP_2 projection must be 2-dimensional, M^4 projection is 1-dimensional so that mass squared is indeed conserved.

Thus one could write

$$\Phi = \frac{k(M^4)}{k(CP_2)} x\theta(a)\frac{a}{R} \ , \quad (33)$$

where R the radius of geodesic sphere of CP_2 and x a numerical constant which could be fixed by quantum criticality of the theory. Chern-Simons action density does not depend on a for this choice and this independence guarantees that the earlier ansatz satisfies field equations. The presence of the step function $\theta(a)$ tells that Φ is non-vanishing only inside light-cone and gives to the gauge potential delta function term which is non-vanishing only at the light-cone boundary and makes possible massless particles.

3. If M^4 projection is 1-dimensional, only homologically charged partonic 3-surfaces can carry gravitational four-momentum. This is not a problem since M^4 projection can be 2-dimensional in the general case. For CP_2 type extremals, ends of cosmic strings, and wormhole contacts the non-vanishing of homological charge looks natural. For wormhole contacts 3-D CP_2 projection suggests itself and is possible only if one allows also quantum fluctuations around light-like extremals of Chern-Simons action. The interpretation could be that for a vanishing homological charge boundary conditions force X^4 to approach vacuum extremal at partonic 3-surfaces.

This picture does not fit completely with the picture about particle massivation provided by CP_2 type extremals. Massless partons must correspond to 3-surfaces at light-cone boundary in this picture and light-likeness allows only linear motion so that inertial mass defined as average must vanish.

5. Comment about quantum classical correspondence

The proposed general picture allows to define the notion of quantum classical correspondence more precisely. The identification of the time average of the gravitational four-momentum for C-S action as a conserved inertial four-momentum associated with the Kähler action at a given space-time sheet of a finite temporal duration (recall that we work in the zero energy ontology) is the most natural definition of the quantum classical correspondence and generalizes to all charges.

In this framework the identification of gravitational four-momentum currents as those associated with 4-D curvature scalar for the induced metric of X^4 could be seen as a phenomenological manner to approximate partonic gravitational four-momentum currents using macroscopic currents, and the challenge is to demonstrate rigorously that this description emerges from quantum TGD.

For instance, one could require that at a given moment of time the net gravitational four-momentum of $Int(X^4)$ defined by the combination of the Einstein tensor and metric tensor equals to that associated with the partonic 3-surfaces. This identification, if possible at all, would certainly fix the values of the gravitational and cosmological constants and it would not be surprising if cosmological constant would turn out to be non-vanishing.

4.2.5 The eigenvalues of D as complex square roots of conformal weight and connection with Higgs mechanism?

An alternative interpretation for the eigenvalues of D emerges from the TGD based description of particle massivation. The eigenvalues could be interpreted as complex square roots of conformal weights in the sense that $|\lambda|^2$ would have interpretation as a conformal weight. There is of course the possibility of numerical constant of proportionality.

The physical motivation for the interpretation is that λ is in the same role as the mass term in the ordinary Dirac equation and thus indeed square root of mass squared proportional to the conformal weight. The vacuum expectation of Higgs would correspond to that for λ and Higgs contribution to the mass squared would correspond to the p-adic thermodynamical expectation value $\langle|\lambda|^2\rangle$ [A9]. Additional contributions to mass squared would come from super conformal and modular degrees of freedom. The interpretation of the generalized eigenvalue as a Higgs field is also natural because the generalized eigen values of the modified Dirac operator can depend on position.

4.2.6 Super-conformal symmetries

The topological character of the solutions spectrum makes possible the expected and actually even larger conformal symmetries in X^2 degrees of freedom. Arbitrary diffeomorphisms of CP_2, including local SU(3) and its holomorphic counterpart, act as symmetries of the non-vacuum solutions. Also the canonical transformations of CP_2 inducing a U(1) gauge transformation are symmetries. More generally, the canonical transformations of $\delta M^4_\pm \times CP_2$ define configuration space symmetries.

Diffeomorphisms of M^4 respecting the light-likeness condition define Kac-Moody symmetries. In particular, holomorphic deformations of X^3_l defined in E^2 factor of $M^2 \times E^2$ compensated by a hyper-analytic deformation in M^2 degrees taking care that light-likeness is not lost, act as symmetry transformations. This requires that M^2 and E^2 contributions of the deformation to the induced metric compensate each other.

The fact that the modified Dirac equation reduces to a one-dimensional Dirac equation allows the action of Kac-Moody algebra as a symmetry algebra of spinor fields. In M^4 degrees of freedom X^2-local SL(2,C) acts as super-conformal symmetries and extends the SU(2) Kac-Moody algebra of $N = 4$ super-conformal algebra to $SL(2,C)$. The reduction to SU(2) occurs naturally. These symmetries act on all spinor components rather than on the second spinor chirality or right handed neutrinos only. Also electro-weak $U(2)$ acts as X^2-local Kac-Moody algebra of symmetries. Hence all the desired Kac-Moody symmetries are realized.

The action of Super Kac-Moody symmetries corresponds to the addition of a linear combination of zero modes of D to a given eigen mode. This defines a symmetry if zero modes satisfy the additional condition $N\Psi = 0$ implied by $\Psi = N\Psi_0$ in turn guaranteed by the already described conditions. These symmetries are super-conformal symmetries with respect to z and $\overline{z}$.

The radial conformal symmetries generalize the dynamical conformal symmetries characterizing CP_2 type vacuum extremals and could be regarded as dynamical conformal symmetries defining the spectrum of super-canonical conformal weights assigned originally to the radial light-like coordinate of $\delta M^4_\pm$. It deserves to be emphasized that the topological QFT character of TGD at fundamental level broken only by the light-likeness of X^3_l carrying information about H metric makes possible these symmetries.

$N = 4$ super-conformal symmetry corresponding to the maximal representation with the group $SU(2)\times SU(2)\times U(1)$ acting as rotations and electro-weak symmetries on imbedding space spinors is in question. This symmetry is broken for light-like 3-surfaces not satisfying field equations. It seems that rotational SU(2) can be extended to the full Lorentz group.

4.2.7 How the super-conformal symmetries of TGD relate to the conventional ones?

The representation of super-symmetries as an addition of anticommuting zero modes to the second quantized spinor field defined by the superposition of non-zero modes of the modified Dirac equation differs radically from the standard realization based on the replacement of the world sheet or target space coordinates with super-coordinates. Also the fundamental role of the generalized eigen modes of the modified Dirac operator is something new and absolutely essential for the understanding of how super-conformal invariance is broken: the breaking of super-symmetries is indeed the basic problem of the super-string theories.

Since the spinor fields in question are not Majorana spinors the standard super-field formalism cannot work in TGD context. It is however interesting to look to what extent this formalism generalizes and whether it allows some natural modification allowing to formally integrate the notions of the bosonic action and corresponding modified Dirac action.

1. One can consider the formal introduction of super fields by replacing of X^3_l coordinates by super-coordinates requiring the introduction of anti-commuting parameters θ and $\overline{\theta}$ transforming as H-spinors of definite chirality, which is not consistent with Majorana condition. Using real coordinates x^α for X^3_l, one would have

$$x^\alpha \rightarrow X^\alpha = x^\alpha + \overline{\theta}\hat{\Gamma}^\alpha \Psi + \overline{\Psi}\hat{\Gamma}^\alpha \theta \ ,$$

Super-conformal symmetries would add to θ a zero mode with Grassmann number valued coefficient. The replacement $z^\alpha \rightarrow X^\alpha$ for the arguments of CP_2 and M^4 coordinates would super-symmetrize the field C-S action density. As a matter fact, the super-symmetrization is non-trivial only in radial degree of freedom since only $\hat{\Gamma}^r$ is non-vanishing.

2. Also imbedding space coordinates could be formally replaced with super-fields using a similar recipe and super-symmetries would act on them. The topological character of Chern-Simons action would allow the super-symmetries induced by the translation of θ by an anticommuting zero mode as formal symmetries at the level of the imbedding space. In both cases it is however far from clear whether the formal super-symmetrization has any real physical meaning.

3. The notion of super-surface suggests itself and would mean that imbedding space Θ parameters are functions of single θ parameter assignable with X^3_l. A possible representation of super-part of the imbedding is a generalization of ordinary imbedding in terms of constraints $H_{i)}(h^k) = 0$, $i = 1, 2....$ Symmetries allow only linear functions so that one would have

$$c^\alpha_{i)}(r, z, \overline{z})\Theta_\alpha = 0 \ .$$

A hyper-plane in the space of theta parameters is obtained. Since only single theta parameter is possible in integral the number of constraints is seven and one obtains the modified Dirac action from the super-space imbedding.

Consider next the basic difficulty and its resolution.

1. The super-conformal symmetries do not generalize to the level of action principle in the standard sense of the word and the reason is the failure of the Majorana property forced by the separate conservation of quark and lepton numbers so that the standard super-space formalism remains empty of physical content.

2. One can however consider the modification of the integration measure $\prod_i d\theta_i d\overline{\theta}_i$ over Grassmann parameters by replacing the product of bilinears with

$$d\overline{\theta}\gamma_1 d\theta d\overline{\theta}\gamma_2 d\theta ...$$

analogous to the product $dx^1 \wedge dx^2$... (where γ^k would be gamma matrices of the imbedding space) transforming like a pseudoscalar. It seems that the replacement of product with wedge product leads to a trivial theory. This formalism could work for super fields obeying Weyl condition instead of Majorana condition and it would be interesting to find what kind of super-symmetric field theories it would give rise to.

The requirement that the number of Grassmann parameters given by $2D$ is the number of spinor components of definite chirality (counting also conjugates) given by $2 \times 2^{D/2-1}$ gives critical dimension $D = 8$, which suggest that this kind of quantum field theory might exist. As found, the zero modes which are not of form $\Psi = N\Psi_0$ do not generate super-conformal symmetries in the strict sense of the word and might correspond to light particles. One could ask whether chiral SUSY in $M^4 \times CP_2$ might describe the low energy dynamics of corresponding light parton states. General arguments do not however support space-time super-symmetry.

3. Because of the light-likeness the super-symmetric variant of C-S action should involve the modified gamma matrices $\hat{\Gamma}^\alpha$ instead of the ordinary ones. Since only $\hat{\Gamma}^r$ is non-vanishing for the extremals of C-S action and since super-symmetrization takes place for the light-like coordinate r only, the integration measure must be defined as $d\overline{\theta}\hat{\Gamma}_r d\theta$, with θ perhaps assignable to a fixed covariantly constant right-handed neutrino spinor and $\hat{\Gamma}_r$ the inverse of $\hat{\Gamma}^r$. This action gives rise to the modified Dirac action with the modified gamma matrices emerging naturally from the Taylor expansion of the C-S action in powers of super-coordinate.

4.3 Why the cutoff in the number superconformal weights and modes of D is needed?

Two kinds of cutoffs are necessary in the number theoretic approach involving a hierarchy of algebraic extensions of rationals with increasing algebraic dimension.

4.3.1 Spatial cutoff realized in terms of number theoretical braids

The first cutoff corresponds to a spatial discretization selecting a subset of algebraic points of the partonic 2-surface X^2 as a subset of the points common to the real and p-adic variants of X^2 obeying the same algebraic equations. Almost topological field theory property allows to assume algebraic equations and also quantum criticality and generalization of the imbedding space concept are crucial for achieving the cutoff as a completely inherent property of X^2.

4.3.2 Cutoff in the number of super-canonical conformal weights

It is not quite clear whether the number of radial conformal weights should be finite or not. The assumption HFF property is realized also in configuration space degrees of freedom would motivate finiteness for the number of conformal weights and would effectively replace the world of the classical worlds with a finite-D space. Also super-symmetry suggests the same. Finiteness would be guaranteed if the ζ function involved characterizes partonic 2-surface and is labelled by p-adic prime: this would also guarantee that zeros of ζ are algebraic numbers. If the zeta function in question characterizes the spectrum of modified Dirac operator and the number of eigenvalues is finite then this goal is achieved. In the case of Riemann Zeta one would be forced to use cutoff due related to the algebraic extension of p-adic numbers used and to assume that zeros and even more general arguments are algebraic numbers.

4.3.3 Cutoff in the number of generalized eigenvalues of the modified Dirac operator

Second cutoff corresponds to a cutoff in the number of generalized eigenvalues of the modified Dirac operator and also now almost TQFT provides the needed flexibility.

1. If the generalized eigenvalues are interpreted as Higgs field then the number of eigenvalues is just one and also orthogonality condition for the modes is achieved without posing ad hoc correlations between longitudinal and transversal degrees of freedom.

2. A priori the dependence of the eigenmodes on transversal degrees of freedom of X^2 is arbitrary. This looks strange on basis of experience with quantum field theory and would imply non-stringy anti-commutation relations. Holomorphic dependence however leads to stringy anti-commutations.

3. Anti-commutativity at braid points only would be highly satisfactory since it would allow to avoid delta functions but would require that the transverse degrees of freedom reduce to a finite number of modes. The reduction of this cutoff to inherent properties of X^2 remains to be understood. What is clear is that the number of conformal modes in transversal degrees of freedom corresponds essentially to the number of points in the braid and the precise realization of this cutoff remains to be understood. Since this cutoff relates to finite measurement resolution, the idea that non-commutative S^2_{II} coordinates provides an elegant manner to realize the anti-commutativity at finite number of points.

It is natural to choose the modes to be S^2_{II} partial waves with a well defined color isospin quantum numbers I, I_3. The Abelianity of the color holonomy group of induced spinor connection suggests also color confinement in weak sense meaning vanishing of I_3 and Y for the physical states.

Since cutoff hierarchy must relate closely to the hierarchy of quantum phases, it seems natural to assume that for given value of $q = exp(i2\pi/n_b)$ only the angular momentum values $I \leq n_b$ are allowed. Here n_b is the order of the maximal cyclic subgroup of G_b involved with the Jones inclusion. In the similar manner one can introduce cutoff for S^2 partial waves in $\delta M^4_\pm$ as cutoff $l \leq n_b$ for angular momentum. Both cutoffs are needed in the definition of configuration space Hamiltonians and super-Hamiltonians allowing to approximate configuration space with a finite-dimensional space which is obviously in spirit with the hyper-finiteness.

Cutoffs imply that n-point functions are finite and non-trivial since the anticommutators of second quantized induced spinor fields are non-local and delta function singularity is smoothed out. Non-locality implies that vertices are non-trivial and pair creation becomes possible. It is of course essential that the dynamics of the space-time interior induces correlations between different partonic 2-surfaces.

That this picture can give rise to the basic vertices of quantum theory seems clear. For instance, suppose that bosons can be assigned to the fermionic representation of Hamiltonians and fermions by super Hamiltonians. The idea would be that right handed neutrino represents vacuum state to which imbedding space gamma matrices act like creation operators. The vertex for the emission of boson would involve sum of vacuum expectation values for the product of the operators $\overline{\Psi} J_A \Psi(x)$, $\overline{\nu} J_B \Psi(y)$ $\overline{\Psi} J_C \nu(z)$, $J_A = j^k_A \Gamma_k$ with various choices of arguments. Anticommutation relations would give sum over the values of the quantity $\overline{\nu} J_A(x) J_B(y) J_C(z) \nu$ multiplied by "wave functions" coming the modes of Ψ. Summation would be over the discrete set of points of the number theoretical braid. A discretized version of stringy scattering amplitude would be in question.

4.3.4 Attempt to form an overall view

This approach leads to both a hierarchy of discretized theories and continuum theory. Continuum theory indeed seems to be completely well defined and would correspond to string theory with free fermions with $N = 4$ super-conformal symmetry as far vertices are considered.

The interpretation encouraged by Jones inclusion hierarchy is that the limit $n \to \infty$ for quantum phase $q = exp(i2\pi/n)$ is not equivalent with the exact real theory based on stringy amplitudes defined using 1-D integrals over the inverse image of the image of the critical line. The natural interpretation for the stringy option without discretization could be in terms of Jones inclusions with group $SU(2)$ and classified by extended ADE diagrams relating to the monodromies of the theory. This interpretation would also conform with the full Kac-Moody invariance whereas for quantum version infinite-dimensional symmetries are reduced to finite-dimensional ones. Note that quantum trace should be equivalent with the condition that the trace of the unit matrix is unity for hyper-finite factors of type II_1.

The number theoretic cutoff hierarchy for the allowed zeros of ζ relates closely to the hierarchy of finite-dimensional extensions of p-adic numbers and to the quantum criticality realized in terms of the generalized imbedding space. This hierarchy of extensions defines a hierarchy of number theoretic braids with an increasing number of strands since the number of points in the intersection between real and corresponding p-adic surface increases and does also the number of allowed zeros. Also the hierarchy of finite-dimensional approximations for the inclusions of hyperfinite factors of type II_1 can be visualized in terms of a hierarchy of braid inclusions with increasing number of braids and is described in terms of Temperley-Lieb algebras. This hierarchy of approximate representations of the inclusion means the replacement of the Beraha number $B_n = 4cos^2(\pi/n)$ by a rational number defining the ratio of dimensions of two subsequent finite-dimensional algebras in the hierarchy. Hence the number theoretic braid hierarchy could provide a concrete representation for the hierarchy of approximations for the hyper-finite factors of type II_1 and their Jones inclusions in terms of inclusions of Temperley Lieb algebras assignable to the number theoretic braids. Physics itself would define this sequence of approximations via p-adicization which basically means space-time realization of cognitive representations.

4.4 The spectrum of Dirac operator and radial conformal weights from physical and geometric arguments

The identification of the generalized eigenvalues of the modified Dirac operator as Higgs field suggests the possibility of understanding the spectrum of D purely geometrically by combining physical and geometric constraints.

The standard zeta function associated with the eigenvalues of the modified Dirac action is the best candidate concerning the interpretation of super-canonical conformal weights as zeros of ζ. This ζ should have very concrete geometric and physical interpretation related to the quantum criticality if these eigenvalues have geometric meaning based on geometrization of Higgs field.

Before continuing it its convenient to introduce some notations. Denote the complex coordinate of a point of X^2 w, its $H = M^4 \times CP_2$ coordinates by $h = (m, s)$, and the H coordinates of its $R_+ \times S^2_{II}$ projection by $h_c = (r_+, s_{II})$.

4.4.1 Generalized eigenvalues

The generalized eigenvalue equation defined by the modified Dirac equation is a differential equation involving only the derivative with respect to r. Hence the eigenvalues λ can depend on X^2 coordinate w and on the coordinates of the critical manifold $R_+ \times S^2_{II}$ via the dependence of w these. As a function of $R_+ \times S^2_{II}$ coordinates they would be many-valued functions of these coordinates since several points of X^2 can project at given point of $R_+ \times S^2_{II}$.

The replacement of the ordinary eigenvalues with continuous functions would be a space-time analog for generalized eigenvalues identified as Hermitian operators (or equivalently, their spectra) inspired by the quantum measurement theory based on inclusions of hyper-finite factors of type II_1 [A8]. The replacement of these functions with their values in a discrete set defined by number theoretic braid would in turn be the counterpart for the finite measurement resolution.

The interpretation of eigenvalue as a complex Higgs field gives the most concrete interpretation for the generalized eigenvalues. Of course, only single eigenvalue would be realized in this kind of situation. Also the requirement that different modes are orthogonal with respect to the inner product at the partonic 2-surface allows only single generalized eigenvalue. Hence the modes in transversal degrees of freedom would code for physics as in the usual QFT.

This interpretation does not kill the idea about eigenvalues as inverses of zeta function $\lambda = \zeta^{-1}(z)$, S^2_{II}. The point is that one can regard X^2 as a covering of S^2 and assign different branches of ζ^{-1} to the different sheets of covering. Different branches of $\zeta^{-1}(z)$, call them $\zeta_k^{-1}(z)$, would combine to single function of the coordinate w of X^2. In the case of Riemann zeta the corresponding construction would replaced complex plane with its infinite-fold covering.

4.4.2 General definition of Dirac determinant

The first guess is that Dirac determinant can defined as a product of determinants assignable to the points $z = z_k$ of the number theoretic braids:

$$det(D) = \prod_{z_k)} det(D(z_k)) \ . \tag{34}$$

The determinant $det(D(z))$ at point z of S^2 would be defined as the product of the eigenvalues $\lambda(z)$ at points associated with the number theoretic braids.

$$det(D)(z_k) = \left[\prod_i \zeta_i^{-1}(z_k)\right]^{n(z_k)} \ , \tag{35}$$

$n(z_k)$ is the number of strands in the number theoretical braid of associated with z_k. Higgs interpretation would imply that only single value of Higgs contributes for a given point of X^2. Dirac determinant must be an algebraic number. This is the case if the total number of points of number theoretic braids involved is finite. It turns out that this guess is quite not general enough: it turns out that actual Dirac determinant must be identified as a ratio of two determinants.

4.4.3 Interpretation of eigenvalues of D as Higgs field

The eigenvalues of the modified Dirac operator have a natural interpretation as Higgs field which vanishes for the unstable extrema of Higgs potential. These unstable extrema correspond naturally to quantum critical points resulting as intersection of M^4 *resp.* CP_2 projection of the partonic 2-surface X^2 with R_+ *resp.* S^2_{II}.

Quantum criticality suggests that the counterpart of Higgs potential could be identified as the modulus square of ζ:

$$V(H(s)) = -|H(s)|^2 \ . \tag{36}$$

which indeed has the points s with $V(H(s)) = 0$ as extrema which would be unstable in accordance with quantum criticality. The fact that for ordinary Higgs mechanism minima of V are the important ones raises the question whether number theoretic braids might more naturally correspond to the minima of V rather than intersection points with S^2. This turns out to be the case. It will also turn out that the detailed form of Higgs potential does not matter: the only thing that matters is that $|V|$ is monotonically decreasing function of the distance from the critical manifold.

4.4.4 Purely geometric interpretation of Higgs

Geometric interpretation of Higgs field suggests that critical points with vanishing Higgs correspond to the maximally quantum critical manifold $R_+ \times S^2_{II}$. The value of H should be determined once $h(w)$ and $R_+ \times S^2_{II}$ projection $h_c(w)$ are known. $|H|$ should increase with the distance between these points. The question is whether one can assign to a given point pair $(h(w), h_c(w))$ naturally a value of H. The first guess is that value of H is most determined by the shortest piece of the geodesic line connecting the points which is a product of geodesics of δM^4_+ and CP_2.

This guess need not be quite correct. An alternative guess is that M^4 projection is indeed geodesic but that CP_2 projection extremizes its length subject to the constraint that the absolute value of the phase defined by the one-dimensional Kähler action $\int A_\mu dx^\mu$ is minimized: this point will be discussed below.

The value should be in general complex and invariant under the isometries of δH affecting h and h_c. The minimal distance $d(h, h_c)$ between the two points constrained by extremal property of phase would define the first candidate for the modulus of H.

The phase factor should relate close to the Kähler structure of CP_2 and one possibility would be the non-integrable phase factor $U(s, s_{II})$ defined as the integral of the induced Kähler gauge potential along the geodesic line in question. Hence the first guess for the Higgs would be as

$$\begin{aligned} H(w) &= d(h, h_c) \times U(s, s_{II}) \ , \\ d(h, h_c) &= \int_h^{h_c} ds \ , \ U(s, s_{II}) = exp\left[i \int_s^{s^1} A_k ds^k\right] \ . \end{aligned} \tag{37}$$

This gives rise to a holomorphic function in X^2 the local complex coordinate of X^2 is identified as $w = d(h, h_s)U(s, s_{II})$ so that one would have $H(w) = w$ locally. This view about H would be purely geometric.

One can ask whether one should include to the phase factor also the phase obtained using the Kähler gauge potential associated with S^2_r having expression $(A_\theta, A_\phi) = (k, cos(\theta))$ with k even integer from the requirement that the non-integral phase factor at equator has the same value irrespective of whether it is calculated with respect to North or South pole. For $k = 0$ the contribution would be vanishing. The value of k might correlate directly with the value of quantum phase. The objection against inclusion of this term is that Kähler action defining Kähler function should contain also M^4 part if this term is included. If this inclusion is allowed then internal consistency requires also the extremization of $\int A_\mu dx^\mu$ so that geodesic lines are not allowed.

In each coordinate patch Higgs potential could be simply the quadratic function $V = -w\overline{w}$. Negative sign is required by quantum criticality. As noticed any monotonically increasing function of V works as well since the minima of the potential remain unaffected. Potential could indeed have minima as minimal distance of X^2 point from $R_+ \times S^2_{II}$. Earth's surface with zeros as tops of mountains and bottoms of valleys as minima would be a rather precise visualization of the situation for given value of r_+. Mountains would have a shape of inverted rotationally symmetry parabola in each local coordinate system.

4.4.5 Consistency with the vacuum degeneracy of Kähler action and explicit construction of preferred extremals

An important constraint comes from the condition that the vacuum degeneracy of Kähler action should be understood from the properties of the Dirac determinant. In the case of vacuum extremals Dirac determinant should have unit modulus.

Suppose that the space-time sheet associated with the vacuum parton X^2 is indeed vacuum extremal. This requires that also X^3_l is a vacuum extremal: in this case Dirac determinant must be real although it need not be equal to unity. The CP_2 projection of the vacuum extremal belongs to some Lagrangian sub-manifold Y^2 of CP_2. For this kind of vacuum partons the ratio of the product of minimal H distances to corresponding $M^4_\pm$ distances must be equal to unity, in other words minima of Higgs potential must belong to the intersection $X^2 \cap S^2_{II}$ or to the intersection $X^2 \cap R_+$ so that distance reduces to M^4 or CP_2 distance and Dirac determinant to a phase factor. Also this phase factor should be trivial.

It seems however difficult to understand how to obtain non-trivial phase in the generic case for all points if the phase is evaluated along geodesic line in CP_2 degrees of freedom. There is however no deep reason to do this and the way out of difficulty could be based on the requirement that the phase defined by the Kähler gauge potential is evaluated along a curve either minimizing the absolute value of the phase modulo 2π.

One must add the condition that curve is not shorter than the geodesic line between points. For a given curve length s_0 the action must contain as a Lagrange multiplier the curve length so that the action using curve length s as a coordinate reads as

$$S = \int A_s ds + \lambda(\int ds - s_0) \ . \quad (38)$$

This gives for the extremum the equation of motion for a charged particle with Kähler charge $Q_K = 1/\lambda$:

$$\begin{aligned} \frac{D^2 s^k}{ds^2} + \frac{1}{\lambda} \times J^k_{\ l} \frac{ds^l}{ds} &= 0 \ , \\ \frac{D^2 m^k}{ds^2} &= 0 \ . \end{aligned} \quad (39)$$

The magnitude of the phase must be further minimized as a function of curve length s.

If the extremum curve in CP_2 consists of two parts, first belonging to X^2_{II} and second to Y^2, the condition is certainly satisfied. Hence if $X^2_{CP_2} \times Y^2$ is not empty, the phases are trivial. In the generic case 2-D sub-manifolds of CP_2 have intersection consisting of discrete points (note again the fundamental role of 4-dimensionality of CP_2). Since S^2_{II} itself is a Lagrangian sub-manifold, it has especially high probably to have intersection points with S^2_{II}. If this is not the case one can argue that X^3_l cannot be vacuum extremal anymore.

Radial conformal invariance of $\delta M^4_\pm$ raises the question whether $\delta M^4_\pm$ geodesics should be defined by allowing $r_M(s)$ to be arbitrary rather than constant. The minimization of $\delta M^4_\pm$ distance would favor geodesics for which $r_M(s)$ decreases as fast as possible so that one has a light-like geodesics going directly to the tip of $\delta M^4_\pm$. Therefore this option does not seem to work.

The construction gives also a concrete idea about how the 4-D space-time sheet $X^4(X^3_l)$ becomes assigned with X^3_l. The point is that the construction extends X^2 to 3-D surface by connecting points of X^2 to points of S^2_{II} using the proposed curves. This process can be carried out in each intersection of X^3_l and M^4_+ shifted to the direction of future. The natural conjecture is that the resulting space-time sheet defines the 4-D preferred extremum of Kähler action.

The most obvious objection is that this construction might not work for cosmic strings of form $X^2 \times S^2_I$, where S^2_I is a homologically non-trivial geodesic sphere of CP_2. In this case X^2 would correspond to string ends, copies of S^2_I at different points of $\delta M^4_\pm$. There seems to be however no real problem. If $S^2_I \cap S^2_{II}$ is not empty, the orbits representing motion in the induced Kähler gauge field could simply define a flow at S^2_I connecting the points of S^2_I to one of the intersection points. Since geodesic manifold is in question one expects that the orbits indeed belong to S^2_I and cosmic string is obtained. Also a flow with several sources and sinks is possible. Situation should be the same for complex 2-sub-manifolds of CP_2. The 3-D character of the resulting surface would be due to the fact that $\delta M^4_\pm$ projections of the orbits are not points. If the second end of the string is at R_+ string and has the same value of r_M coordinate, single string would result. Otherwise one would obtain two strings with second end point at R_+ with the same value of r_M.

4.4.6 About the definition of the Dirac determinant and number theoretic braids

The definition of Dirac determinant should be independent of the choice of complex coordinate for X^2 and local complex coordinate implied by the definition of Higgs is a unique choice for this coordinate. The physical intuition based on Higgs mechanism suggests that apart from normalization factor the Dirac determinant should be defined simply as product of the eigenvalues of D, that is those of Higgs field, associated with the number theoretic braids.

If only single kind of braid is allowed then the minima of Higgs field define the points of the braid very naturally. The points in $R_+ \times S^2_{II}$ cannot contribute to the Dirac determinant since Higgs vanishes at the critical manifold. Note that at S^2_{II} criticality Higgs values become real and the exponent of Kähler action should become equal to one. This is guaranteed if Dirac determinant is normalized by dividing it with the product of $\delta M^4_\pm$ distances of the extrema from R_+. The value of the determinant would equal to one also at the limit $R_+ \times S^2_{II}$.

One would define the Dirac determinant as the product of the values of Higgs field over all minima of local Higgs potential

$$det(D) = \frac{\prod_k H_(w_k)}{\prod_k H_0(w_k)} = \prod_k \frac{w_k}{w_k^0} \quad . \tag{40}$$

Here w_k^0 are M^4 distances of extrema from R_+. Equivalently: one can identify the values of Higgs field as dimensionless numbers w_k/w_k^0. The modulus of Higgs field would be the ratio of H and $M^4_\pm$ distances from the critical sub-manifold. The modulus of the Dirac determinant would be the product of the ratios of H and M^4 depths of the valleys.

This definition would be general coordinate invariant and independent of the topology of X^2. It would also introduce a unique conformal structure in X^2 which should be consistent with that defined by the induced metric. Since the construction used relies on the induced metric this looks natural. The number of eigen modes of D would be automatically finite and eigenvalues would have purely geometric interpretation as ratios of distances on one hand and as masses on the other hand. The inverse of CP_2 length defines the natural unit of mass. The determinant is invariant under the scalings of H metric as are also Kähler action and Chern-Simons action. This excludes the possibility that Dirac determinant could also give rise to the exponent of the area of X^2.

Number theoretical constraints require that the numbers w_k are algebraic numbers and this poses some conditions on the allowed partonic 2-surfaces unless one drops from consideration the points which do not belong to the algebraic extension used.

4.4.7 About the detailed definition of number theoretic braids

Consider now the detailed definition of number theoretic braids. One can define a pile X^2_t of cross sections of $X^3_l \cap (\delta M^4_{\pm,t} \times CP_2)$, where $\delta M^4_{\pm,t}$ represents $\delta M^4_\pm$ shifted by t in a preferred time direction defined by M^2. In the same manner one can decompose M^2 to a pile of light-like geodesics $R_{+,t}$ defining the quantization axis of angular momentum. For each value of t one obtains a collection of minima of the "Higgs field" λ_t in 3-dimensional space $R_{+,t} \times S^2_{II}$. The minima define orbits $\gamma(t)$: $(r_{+,i}(t), s_{II}(t))$ in $M^2 \times S^2_{II}$ space.

One can consider braidings (or more generally tangles, two minima can disappear in collision or can be created from vacuum) both in X^3_l and at the level of imbedding space.

1. Braids in X^3_l

A braid in X_l^3 is obtained by considering the fate of points of $X^2 t = 0$ in X_l^3 and by assigning a braiding to the minima of Higgs field in X_l^3. Also the field lines of Kähler magnetic field or of Kähler gauge potential on X_l^3 going through the initial positions of Higgs minima can be considered. Since the construction of the Higgs field involves induced Kähler gauge potential in an essential manner, the braiding defined by the Kähler gauge potential could be consistent with the time evolution for the positions of the minima of Higgs.

Recall that only topological rather than point-wise equivalence of the braids is required. It is not clear how much these definition depend on the coordinates used for X_l^3. For instance, could one trivialize the braid by making a time dependent coordinate change for X^2? This requires that it is possible to define global time coordinate whose coordinate lines correspond to field lines. This is possible only if the flow satisfies additional integrability conditions [D1].

2. Braidings defined by imbedding space projections

One can define braidings also by the projections to the heavenly spheres S^2_{II} of CP_2 and S^2_r of $\delta M^4_\pm$. A linear braid like structure is also obtained by considering the projections of Higgs minima in M^2.

1. The simplest option is the identification of the braid as the projections of the orbits of the minima of Higgs field to S^2_{II} or S^2_r (for various values of t). This seems to be the most elegant choice. One could decompose the braid to sub-braids such that each initial value $r_{+,i}(0)$ would define its own braid in S^2_{II} or S^2_r. Also each point of S^2_{II} or S^2_r could define its own sub-braid.
2. Factoring quantum field theories defined in M^2 [25, 26] suggest a further definition of a braid like structure based on the projections of Higgs minima to M^2. The braid like structure would result from the motion of braid points with different velocities so that they would pass by each other. This kind of pattern with constant velocities of particles describes scattering in factoring quantum field theories defined in M^2. The M^2 velocities of particles would not be constant now. S-matrix is almost trivial inducing only a permutation of the initial state momenta and S-matrix elements are mere phases. The interpretation is that each pass-by process induces a time lag. At the limit when the velocities approach to zero or infinity such that their ratios remain constant, S-matrix reduces to a braiding S-matrix.

The Higgs minima contributing to the elements of S-matrix (or at least U-matrix) should correspond to algebraic points of braids. This suggests that the information about the braids comes from the minima of Higgs in X_l^3 rather than X_t^2 so that only some values of t at each strand $\gamma(t)$ give rise to physically relevant braid points. The condition that the resulting numbers are algebraic poses restrictions on X_l^3 as does also the condition that X_l^3 have also p-adic counterparts. This does not of course mean the loss of braids. Note that the discretization allows to assign Dirac determinant and zeta function to any 3-surface X_l^3 rather than only those corresponding to the maxima of Kähler function.

4.4.8 The identification of zeta function

The proposed picture supports the identification of the eigenvalues of D in terms of a Higgs fields having purely geometric meaning. It also seems that number theoretic braids must be identified as minima of Higgs potential in X^2. Furthermore, the braiding operation could be defined for all intersections of X_l^3 defined by shifts $M^4_\pm$ as orbits of minima of Higgs potential. Second option is braiding by Kähler magnetic flux lines.

The question is how to understand super-canonical conformal weights for which the identification as zeros of a zeta function of some kind is highly suggestive. The natural answer would be that the normalized eigenvalues of D defines this zeta function as

$$\zeta(s) = \sum_k (\frac{H(w_k)}{H_0(w_k)})^{-s} . \quad (41)$$

The number of eigenvalues contributing to this function would be finite and $H(w_k)/H_0(w_k)$ should be rational or algebraic at most. ζ function would have a precise meaning consistent with the usual assignment of zeta function to Dirac determinant.

The case of Riemann Zeta inspires the question whether one should allow only the moduli of the eigenvalues in the zeta or allow only real and positive eigenvalues. The moduli of eigenvalues are not smaller than unity as is the case also for Riemann Zeta. Real eigenvalues correspond to vanishing phase and thus vanishing Chern-Simons action and unit eigenvalues to the quantum critical points of S^2_{II}.

The ζ function would directly code the basic geometric properties of X^2 since the moduli of the eigenvalues characterize the depths of the valleys of the landscape defined by X^2 and the associated non-integrable phase factors. The degeneracies of eigenvalues would in turn code for the number of points with same distance from a given zero intersection point.

The zeros of the ζ function in turn define natural candidates for the super-canonical conformal weights and their number would thus be finite in accordance with the idea about inherent cutoff present also in configuration space degrees of freedom. Super-canonical conformal weights would be functionals of X^2. The scaling of λ by a constant depending on p-adic prime factors out from the zeta so that zeros are not affected: this is in accordance with the renormalization group invariance of both super-canonical conformal weights and Dirac determinant.

The zeta function should exist also in p-adic sense. This requires that the numbers λ^s at the points s of S^2_{II} which corresponds to the number theoretic braid are algebraic numbers. The freedom to scale λ could help to achieve this.

The conformal weights defined by the zeros of zeta would be constant. One could however consider also the generalization of the super-canonical conformal weights to functions of S^2_{II} or S^2_r coordinate although this is not necessary and would spoil the simple group theoretical properties of the δH Hamiltonians. The coordinate s appearing as the argument of ζ could be formally identified as S^2_{II} or S^2_r coordinate so that generalized super-canonical conformal weights could be interpreted geometrically as inverses of $\zeta^{-1}(s)$ defined as a function in S^2_{II} or S^2_r.

In this case also the notion of number theoretic braids defined as sets of points for which $X^2_{M^4}$ projection intersects R_+ at same point could make sense for super-canonical conformal weights. This would require that the number for the branches of ζ^{-1} is same as the number of points of braid.

4.4.9 The relationship between λ and Higgs field

The generalized eigenvalue $\lambda(w)$ is only proportional to the vacuum expectation value of Higgs, not equal to it. Indeed, Higgs and gauge bosons as elementary particles correspond to wormhole contacts carrying fermion and antifermion at the two wormhole throats and must be distinguished from the space-time correlate of its vacuum expectation as something proportional to λ. In the fermionic case the vacuum expectation value of Higgs does not seem to be even possible since fermions do not correspond to wormhole contacts between two space-time sheets but possess only single wormhole throat (p-adic mass calculations are consistent with this). Gauge bosons can have Higgs expectation proportional to λ. The proportionality must be of form $\langle H\rangle \propto \lambda/p^{n/2}$ if gauge boson mass squared is of order $1/p^n$.

4.4.10 Possible objections related to the interpretation of Dirac determinant

Suppose that that Dirac determinant is defined as a product of determinants associated with various points z_k of number theoretical braids and that these determinants are defined as products of corresponding eigenvalues.

Since Dirac determinant is not real and is not invariant under isometries of CP_2 and of $\delta M^4_{\pm}$, it cannot give only the exponent of Kähler function which is real and $SU(3) \times SO(3,1)$ invariant. The natural guess is that Dirac determinant gives also the Chern-Simons exponential and possible phase factors depending on quantum numbers of parton.

1. The first manner to circumvent this objection is to restrict the consideration to maxima of Kähler function which select preferred light-like 3-surfaces X^3_l. The basic conjecture forced by the number theoretic universality and allowed by TGD based view about coupling constant evolution indeed is that perturbation theory at the level of configuration space can be restricted to the maxima of Kähler function and even more: the radiative corrections given by this perturbative series vanish being already coded by Kähler function having interpretation as analog of effective action.

2. There is also an alternative way out of the difficulty: define the Dirac determinant and zeta function using the minima of the modulus of the generalized Higgs as a function of coordinates of X^3_l so that continuous strands of braids are replaced by a discrete set of points in the generic case.

The fact that general Poincare transformations fail to be symmetries of Dirac determinant is not in conflict with Poincare invariance of Kähler action since preferred extremals of Kähler action are in question and must contain the fixed partonic 2-surfaces at $\delta M^4_{\pm}$ so that these symmetries are broken by boundary conditions which does not require that the variational principle selecting the preferred extremals breaks these symmetries.

One can exclude the possibility that the exponent of the stringy action defined by the area of X^2 emerges also from the Dirac determinant. The point is that Dirac determinant is invariant under the scalings of H metric whereas the area action is not.

The condition that the number of eigenvalues is finite is most naturally satisfied if generalized ζ coding information about the properties of partonic 2-surface and expressible as a rational function for which the inverse has a finite number of branches is in question.

4.4.11 How unique the construction of Higgs field is?

Is the construction of space-time correlate of Higgs as λ really unique? The replacement of H with its power H^r, $r > 0$, leaves the minima of H invariant as points of X^2 so that number theoretic braid is not affected. As a matter fact, the group of monotonically increasing maps real-analytic maps applied to H leaves number theoretic braids invariant.

The map $H \rightarrow H^r$ scales Kähler function to its r-multiple, which could be interpreted in terms of $1/r$- scaling of the Kähler coupling strength. Also super-canonical conformal weights identified as zeros of ζ are scaled as $h \rightarrow h/r$ and Chern-Simons charge k is replaced with k/r so that at least $r = 1/n$ might be allowed.

One can therefore ask whether the powers of H could define a hierarchy of quantum phases labelled by differen values of k and α_K. The interpretation as separate phases would conform with the idea that D in some sense has entire spectrum of generalized eigenvalues.

4.5 Quantization of the modified Dirac action

The quantization of the modified Dirac action involves a fusion of various number theoretical ideas. Stringy picture need not be correct with string being replaced number theoretic braids.

1. The first question is how M^4 and CP_2 braids relate. Since one assumes that the data associated with both braids are independent, it seems necessary to assume anti-commutativity between all points of X^2 belonging to some number theoretic braid.

2. There is no correlation between λ and eigenvalues associated with transverse degrees of freedom as in the case of d'Alember operator. Therefore an infinite number of eigen-modes of D for a given eigenvalue λ can be considered unless one poses some additional conditions. This would mean that one could have anti-commutativity for different points of X^2 and anti-commutators of Ψ and conjugate at same point would be proportional to delta function. This would not conform with the stringy picture.

3. How could one obtain stringy anticommutations? The assumption that modes are holomorphic or antiholomorphic would guarantee this since formally only single coordinate variable would appear in Ψ. Anti-commutativity along string requires that in a given sector of configuration space isometries commute with the selection of quantization axes for the isometry algebra of the imbedding space. This might be justified by quantum classical correspondence. The unitarity for Yang-Baxter matrices and unitarity of the inner product for the radial modes r^{Δ}, $\Delta = 1/2+iy$, is consistent with the stringy option where y would now label those points of R_+ which do not correspond to $z = 0$. String corresponds to the ζ-image of the critical line containing non-trivial zeros of zeta at the geodesic sphere of S^2_r.

4. One could ask whether number theoretic braids might have deeper meaning in terms of anticommutativity. This would be the case if the modes in transversal degrees of freedom reduce to a finite number and are actually labelled by λ. This could be achieved if there is no other dependence on transverse degrees of freedom than that coming through $\lambda(z)$. Anti-commutativity would hold true only at finite number of points and that anti-commutators would be finite in general. This outcome would be very nice.

5. An interesting question is whether the number theoretic braid could be also described by introducing a non-commutativity of the complex coordinate of X^2 provided by S^2_r or S^2_{II}. This should replace anti-commutativity in X^2 with anti-commutativity for different points of the number theoretic braid. The nice outcome would be the finiteness of anti-commutators at same point.

The following is an attempt to formulate this general vision in a more detail manner.

4.5.1 Fermionic anticommutation relations: non-stringy option

The fermionic anti-commutation relations must be consistent with the vacuum degeneracy and with the anti-commutation relations of configuration space gamma matrices defining the matrix elements of configuration space metric between complexified Hamiltonians.

1. The bosonic representation of configuration space Hamiltonians is naturally as Noether charges associated with Chern-Simons action:

$$\begin{aligned} H_A &= \int d^2x \pi^0_k j^k_A \ , \\ \pi^\alpha &= \frac{\partial L_{C-S}}{\partial_\alpha h^k} \ . \end{aligned} \tag{42}$$

   π^0_k denotes bosonic canonical momentum density. Note that also fermionic dynamics allows definition of Hamiltonians as fermionic charges) and this would give rise to fermionic representation of super-canonical algebra. Same applies to the super Kac-Moody algebra generators which super Kac-Moody generators realized as X^3-local isometries of the imbedding space.

2. Super Hamiltonians identifiable as contractions of configuration space gamma matrices with Killing vector fields of symplectic transformations in CH can be defined as matrix elements of $j^k_A\Gamma_k$ between $\overline{\nu}_R$ and Ψ:

$$J^K_A\Gamma_K \quad\equiv\quad \Gamma_A = H_{S,A} = \int d^2x \overline{\nu}_R j^k_A \Gamma_k \Psi \quad . \tag{43}$$

$H^{\dagger}_{S,A}$ is obtained by Hermitian conjugation.

3. The anti-commutation relations read as

$$\{\overline{\Psi}(x), \Gamma_k\Psi(y)\} \quad = \quad \pi^0_k J^{rs}\Sigma_{rs}\delta^2(x,y) \quad . \tag{44}$$

Here J^{rs} denotes the degenerate Kähler form of $\delta M^4_+ \times CP_2$. What makes these anti-commutation relations non-stringy is that anti-commutator is proportional to 2-D delta function rather than 1-D delta function at 1-D sub-manifold of X^2 as in the case of conformal field theories. Hence one would have 3-D quantum field theory with one light-like direction.

4. The matrix elements of configuration space metric for the complexified Killing vector fields of symplectic transformations give the elements of configuration space Kähler form and metric as

$$\{\Gamma^{\dagger}_A, \Gamma_B\} \quad = \quad iG_{\overline{A},B} = J_{\overline{A},B} = \{\overline{H_A}, H_B\} = H_{[\overline{A},B]} \quad . \tag{45}$$

4.5.2 Fermionic anti-commutation relations: stringy option

As already noticed, 2-dimensional delta function in the anti-commutation relations implies that spinor field is 2-D Euclidian free field rather than conformal field. The usual stringy picture would require anti-commutativity only along circle and nonlocal commutators outside this circle.

Also the original argument based on the observation that the points of CP_2 parameterize a large class of solutions of Yang-Baxter equation suggests the stringy option. The subset of commuting Yang-Baxter matrices corresponds to a geodesic sphere S^2 of CP_2 and the subset of unitary Yang-Baxter matrices to a geodesic circle of S^2 identifiable as real line plane compactified to S^2. Physical intuition strongly favors unitarity.

Stringy choice is consistent with the identification of the configuration space Hamiltonians as bosonic Noether charges only if Noether charges correspond to closed but in general not exact 2-forms and thus reduce to integrals of a 1-form over 1-dimensional manifold representing the discontinuity of the associated vector potential. That Noether charges would reduce to cohomology would conform with almost TQFT property. This is indeed the case under conditions which will be idenfied below.

1. The canonical momentum density associated with C-S action has the expression

$$\pi_k \quad = \quad \epsilon_{\alpha\beta 0}(\partial_\beta\left[A_\alpha A_k\right) - \partial_\alpha(A_\beta A_k\right] \quad , \tag{46}$$

and is thus a closed two-form. Note that the discontinuity of the monopole like vector potential implies that the form in question is not exact.

2. Also the Hamiltonian densities

$$H_A = j_A^k \pi_k = J^{kl}\partial_l H_A \epsilon_{\alpha\beta 0}\left[\partial_\beta(A_\alpha A_k) - \partial_\alpha(A_\beta A_k)\right] \quad (47)$$

should define closed forms

$$H_A = j_A^k \pi_k = \epsilon_{\alpha\beta 0}\left[\partial_\beta(A_\alpha A_k J^{kl}\partial_l H_A) - \partial_\alpha(A_\beta A_k \partial_l J^{kl} H_A\right] \ . \quad (48)$$

3. This is not the case in general since the derivatives coming from j_A^k give the term

$$\epsilon_{\alpha\beta 0} A_\alpha A_k J^{kl} D_r(\partial_l H_A)\partial_\beta h^r - A_\beta A_k J^{kl} D_r(\partial_l H_A)\partial_\alpha h^r \ . \quad (49)$$

which does not vanish unless the condition

$$A_k J^{kl} D_r(\partial_l H_A) = \partial_r \Phi \quad (50)$$

holds true.

The condition is equivalent with the vanishing of the Poisson bracket between Hamiltonian and components of the Kähler potential:

$$\partial_k H_A J^{kl}\partial_l A_r = 0 \ . \quad (51)$$

This poses a restriction on the group of isometries of configuration space. The restriction of Kähler potential to A_r is given by $(A_\theta, A_\phi) = (0, cos(\theta))$ and A_ϕ generates rotations in z-direction. Hence only the Hamiltonians commuting with Kähler gauge potential of $\delta M^4_\pm \times CP_2$ at X^2 would have vanishing color isospin and presumably also vanishing color hyper charge in the case of CP_2 and vanishing net spin in case of δM^4_+.

4. The discontinuity of Φ would result from the topological magnetic monopole character of the Kähler potential A_k in $\delta M^4_\pm \times CP_2$.

5. Quantum classical correspondence suggests that quantum measurement theory is realized at the level of the configuration space and induces a decomposition of the configuration space to a union of sub-configuration spaces corresponding to different choices of quantization axes of angular momentum and color quantum numbers. Hence the interpretation of configuration space isometries in terms of a maximal set of commuting observables would make sense. Of course, also the canonical transformations for which Hamiltonians do not reduce to 1-D integrals act as symmetries although they do not possess super counterparts. They play same role as Lorentz boosts whereas the super-symmetrizable part of the algebra is analogous to the little group of Lorentz group leaving momentum invariant. This means that complete reduction to string model type theory does not occur even at the level of quantum states.

Consider now the basic formulas for the stringy option.

1. Hamiltonians can be expressed as

$$H_A \quad = \quad \int dx A A_k J^{kl} \partial_l H_A \quad . \tag{52}$$

where A denotes the projection of Kähler gauge potential to the 1-dimensional manifold in question.

2. The fermionic super-currents defining super-Hamiltonians and configuration space gamma matrices would be given by

$$J_A^K \Gamma_K \quad \equiv \quad \Gamma_A = H_{S,A} = \int dx \overline{\nu}_R j_A^k \Gamma_k \Psi \quad . \tag{53}$$

$H_{S,A}^{\dagger}$ is obtained by Hermitian conjugation.

3. The anti-commutation relations would read as

$$\{\overline{\Psi}(x), \Gamma_k \Psi(y)\} \quad = \quad A A_k J^{kl} \partial_l H_A J^{rs} \Sigma_{rs} \delta(x,y) \quad . \tag{54}$$

The general formulas for the matrix elements of the configuration space metric and Kähler form are as for the non-stringy option.

4.5.3 String as the inverse image for image of critical line for zeros of zeta

Number theoretical argument suggests that 1-D dimensional delta function corresponds to the point set for which δM^4_+ projection corresponds to the line of non-trivial zeros for ζ: $z = \zeta(1/2 + iy)$ that is intersection of X^2 with R_+. Thus stringy anti-commutation would be along R_+. In CP_2 the discrete set of points along which anticommutations would be given would be subset in S^2_{II}. Anti-commutativity on quantum critical set which corresponds to vacuum extremals would be indeed very natural.

In case of Riemann zeta one must consider also trivial zeros at $x = -2n$, $n = 1, 2....$ These would correspond to the integer powers of r^n for which the definition of inner product is problematic. Note however that for negative powers $-2n$ corresponding to zeros of ζ there are no problems if there is cutoff $r > r_0$.

The number theoretic counterpart of string would be most naturally a curve whose S^2_r projection belongs to the image of the critical line consisting of points $\zeta(1/2 + iy)$. This image consist of the real axis of S^2 interpreted as compactified plane since ζ is real at the critical line. Note that in case of Riemann zeta also real axis is mapped to the real line so that it gives nothing new. Also this has a number theoretical justification since the basis $r^{1/2+iy}$, where r could correspond to the light-like coordinate of both $\delta M^4_\pm$ and partonic 3-surface, forms an orthogonal basis with respect to the inner product defined by the scaling invariant integration measure dx/x.

For number theoretical reasons which should be already clear, the values of y would be restricted to $y = \sum_k n_k y_k$ of imaginary parts of zeros of ζ. In the case of partonic 3-surface this would mean that eigenvalues of the modified Dirac operator would be of form $1/2 + i \sum_k n_k y_k$ and the number theoretical cutoff regularizing the Dirac determinant would emerge naturally. The important implication would be that not only q^{iy_k} but also y_k must be algebraic numbers. Note that the zeros of Riemann zeta at this line correspond to quantum criticality against phase transitions changing Planck constant meaning geometrically a leakage between different sectors of the imbedding space.

4.6 Number theoretic braids and global view about anti-commutations of induced spinor fields

The anti-commutations of induced spinor fields are reasonably well understood locally. The basic objects are 3-dimensional light-like 3-surfaces. These surfaces can be however seen as random light-like orbits of partonic 2-surfaces taking which would thus seem to take the role of fundamental dynamical objects. Conformal invariance in turn seems to make the 2-D partons 1-D objects and number theoretical braids in turn discretizes strings. And it also seems that the strands of number theoretic braid can in turn be discretized by considering the minima of Higgs potential in 3-D sense.

Somehow these apparently contradictory views should be unifiable in a more global view about the situation allowing to understand the reduction of effective dimension of the system as one goes to short scales. The notions of measurement resolution and number theoretic braid indeed provide the needed insights in this respect.

4.6.1 Anti-commutations of the induced spinor fields and number theoretical braids

The understanding of the number theoretic braids in terms of Higgs minima and maxima allows to gain a global view about anti-commutations. The coordinate patches inside which Higgs modulus is monotonically increasing function define a division of partonic 2-surfaces $X^2_t = X^3_l \cap \delta M^4_{\pm,t}$ to 2-D patches as a function of time coordinate of X^3_l as light-cone boundary is shifted in preferred time direction defined by the quantum critical sub-manifold $M^2 \times CP_2$. This induces similar division of the light-like 3-surfaces X^3_l to 3-D patches and there is a close analogy with the dynamics of ordinary 2-D landscape.

In both 2-D and 3-D case one can ask what happens at the common boundaries of the patches. Do the induced spinor fields associated with different patches anti-commute so that they would represent independent dynamical degrees of freedom? This seems to be a natural assumption both in 2-D and 3-D case and correspond to the idea that the basic objects are 2- *resp.* 3-dimensional in the resolution considered but this in a discretized sense due to finite measurement resolution, which is coded by the patch structure of X^3_l. A dimensional hierarchy results with the effective dimension of the basic objects increasing as the resolution scale increases when one proceeds from braids to the level of X^3_l.

If the induced spinor fields associated with different patches anti-commute, patches indeed define independent fermionic degrees of freedom at braid points and one has effective 2-dimensionality in discrete sense. In this picture the fundamental stringy curves for X^2_t correspond to the boundaries of 2-D patches and anti-commutation relations for the induced spinor fields can be formulated at these curves. Formally the conformal time evolution scaled down the boundaries of these patches. If anti-commutativity holds true at the boundaries of patches for spinor fields of neighboring patches, the patches would indeed represent independent degrees of freedom at stringy level.

The cutoff in transversal degrees of freedom for the induced spinor fields means cutoff $n \leq n_{max}$ for the conformal weight assignable to the holomorphic dependence of the induced spinor field on the complex coordinate. The dropping of higher conformal weights should imply the loss of the anti-commutativity of the induced spinor fields and its conjugate except at the points of the number theoretical braid. Thus the number theoretic braid should code for the value of n_{max}: the naive expectation is that for a given stringy curve the number of braid points equals to n_{max}.

4.6.2 The decomposition into 3-D patches and QFT description of particle reactions at the level of number theoretic braids

What is the physical meaning of the decomposition of 3-D light-like surface to patches? It would be very desirable to keep the picture in which number theoretic braid connects the incoming positive/negative energy state to the partonic 2-surfaces defining reaction vertices. This is not obvious if X_l^3 decomposes into causally independent patches. One can however argue that although each patch can define its own fermion state it has a vanishing net quantum numbers in zero energy ontology, and can be interpreted as an intermediate virtual state for the evolution of incoming/outgoing partonic state.

Another problem - actually only apparent problem - has been whether it is possible to have a generalization of the braid dynamics able to describe particle reactions in terms of the fusion and decay of braid strands. For some strange reason I had not realized that number theoretic braids naturally allow fusion and decay. Indeed, cusp catastrophe is a canonical representation for the fusion process: cusp region contains two minima (plus maximum between them) and the complement of cusp region single minimum. The crucial control parameter of cusp catastrophe corresponds to the time parameter of X_l^3. More concretely, two valleys with a mountain between them fuse to form a single valley as the two real roots of a polynomial become complex conjugate roots. The continuation of light-like surface to slicing of X^4 to light-like 3-surfaces would give the full cusp catastrophe.

In the catastrophe theoretic setting the time parameter of X_l^3 appears as a control variable on which the roots of the polynomial equation defining minimum of Higgs depend: the dependence would be given by a rational function with rational coefficients.

This picture means that particle reactions occur at several levels which brings in mind a kind of universal mimicry inspired by Universe as a Universal Computer hypothesis. Particle reactions in QFT sense correspond to the reactions for the number theoretic braids inside partons. This level seems to be the simplest one to describe mathematically. At parton level particle reactions correspond to generalized Feynman diagrams obtained by gluing partonic 3-surfaces along their ends at vertices. Particle reactions are realized also at the level of 4-D space-time surfaces. One might hope that this multiple realization could code the dynamics already at the simple level of single partonic 3-surface.

4.6.3 About 3-D minima of Higgs potential

The dominating contribution to the modulus of the Higgs field comes from $\delta M^4_\pm$ distance to the axis R_+ defining quantization axis. Hence in scales much larger than CP_2 size the geometric picture is quite simple. The orbit for the 2-D minimum of Higgs corresponds to a particle moving in the vicinity of R_+ and minimal distances from R_+ would certainly give a contribution to the Dirac determinant. Of course also the motion in CP_2 degrees of freedom can generate local minima and if this motion is very complex, one expects large number of minima with almost same modulus of eigenvalues coding a lot of information about X_l^3.

It would seem that only the most essential information about surface is coded: the knowledge of minima and maxima of height function indeed provides the most important general coordinate invariant information about landscape. In the rational category where X_l^3 can be characterized by a finite set of rational numbers, this might be enough to deduce the representation of the surface.

What if the situation is stationary in the sense that the minimum value of Higgs remains constant for some time interval? Formally the Dirac determinant would become a continuous product having an infinite value. This can be avoided by assuming that the contribution of a continuous range with fixed value of Higgs minimum is given by the contribution of its initial point: this is natural if one thinks the situation information theoretically. Physical intuition suggests that the minima remain constant for the maxima of Kähler function so that the initial partonic 2-surface would determine the entire contribution to the Dirac determinant.

4.6.4 How generalized braid diagrams relate to the perturbation theory?

The association of generalized braid diagrams to incoming and outgoing partonic legs and possibly also vertices of the generalized Feynman diagrams forces to ask whether the generalized braid diagrams could give rise to a counterpart of perturbation theoretical formalism via the functional integral over configuration space degrees of freedom.

The question is how the functional integral over configuration space degrees of freedom relates to the generalized braid diagrams. The basic conjecture motivated also number theoretically is that radiative corrections in this sense sum up to zero for critical values of Kähler coupling strength and Kähler function codes radiative corrections to classical physics via the dependence of the scale of M^4 metric on Planck constant. Cancellation occurs only for critical values of Kähler coupling strength α_K: for general values of α_K cancellation would require separate vanishing of each term in the sum and does not occur.

This would mean following.

1. One would not have perturbation theory around a given maximum of Kähler function but as a sum over increasingly complex maxima of Kähler function. Radiative corrections in the sense of perturbative functional integral around a given maximum would vanish (so that the expansion in terms of braid topologies would not make sense around single maximum). Radiative corrections would not vanish in the sense of a sum over 3-topologies obtained by adding radiative corrections as zero energy states in shorter time scale.

2. Connes tensor product with a given measurement resolution would correspond to a restriction on the number of maxima of Kähler function labelled by the braid diagrams. For zero energy states in a given time scale the maxima of Kähler function could be assigned to braids of minimal complexity with braid vertices interpreted in terms of an addition of radiative corrections. Hence a connection with QFT type Feyman diagram expansion would be obtained and the Connes tensor product would have a practical computational realization.

3. The cutoff in the number of topologies (maxima of Kähler function contributing in a given resolution defining Connes tensor product) would be always finite in accordance with the algebraic universality.

4. The time scale resolution defined by the temporal distance between the tips of the causal diamond defined by the future and past light-cones applies to the addition of zero energy sub-states and one obtains a direct connection with p-adic length scale evolution of coupling constants since the time scales in question naturally come as negative powers of two. More precisely, p-adic primes near power of two are very natural since the coupling constant evolution comes in powers of two of fundamental 2-adic length scale.

There are still some questions. Radiative corrections around given 3-topology vanish. Could radiative corrections sum up to zero in an ideal measurement resolution also in 2-D sense so that the initial and final partonic 2-surfaces associated with a partonic 3-surface of minimal complexity would determine the outcome completely? Could the 3-surface of minimal complexity correspond to a trivial diagram so that free theory would result in accordance with asymptotic freedom as measurement resolution becomes ideal?

The answer to these questions seems to be 'No'. In the p-adic sense the ideal limit would correspond to the limit $p \rightarrow 0$ and since only $p \rightarrow 2$ is possible in the discrete length scale evolution defined by primes, the limit is not a free theory. This conforms with the view that CP_2 length scale defines the ultimate UV cutoff.

4.6.5 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

One can wonder how this picture relates to the earlier hypothesis that p-adic length coupling constant evolution is coded to the hypothesized $log(p)$ normalization of the eigenvalues of the modified Dirac operator D. There are objections against this normalization. $log(p)$ factors are not number theoretically favored and one could consider also other dependencies on p. Since the eigenvalue spectrum of D corresponds to the values of Higgs expectation at points of partonic 2-surface defining number theoretic braids, Higgs expectation would have $log(p)$ multiplicative dependence on p-adic length scale, which does not look attractive.

Is there really any need to assume this kind of normalization? Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5$ μm (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3.

4.6.6 How quantum classical correspondence is realized at parton level?

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or a boundary term of YM action associated with a particle carrying gauge charges of the quantum state. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography. The difference between Chern-Simons action characterizing quantum state and the fundamental Chern-Simons type factor associated with the Kähler form would be that the latter emerges as the phase of the Dirac determinant.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

5 Super-symmetries at space-time and configuration space level

The first difference between TGD and standard conformal field theories and string models is that super-symmetry generators acting as configuration space gamma matrices acting as super generators carry either lepton or quark number. Only the anti-commutators of quark like generators expressible in terms of Hamiltonians H_A of $X^3_l \times CP_2$ can contribute to the super-symmetrization of the Poisson algebra and thus to CH metric via Poisson central extension, whereas leptonic generators, which are proportional to $j^{Ak}\Gamma_k$ can contribute to the super-symmetrization of the function algebra of CH. Quarks correspond to N-S type representations and kappa symmetry of string models whereas leptons correspond to Ramond type representations and ordinary super-symmetry.

Also Super Kac-Moody invariance allows lepton-quark dichotomy. What forces to assign leptons with Ramond representation is that covariantly constant neutrino must correspond to one conformal mode $(z^n, n = 0)$. The p-adic mass calculations [6] carried for more than decade ago led to the same assignment on physical grounds: p-adic mass calculations also forced to include $SO(3,1)$ besides M^4 a tensor factor to super-conformal representations, which in recent context suggests that causal determinants $X^3_l \times CP_2$, $X^3_l \subset M^4$ an arbitrary light like 3-surface rather than just a translate of δM^4_+, must be allowed. Also now the lepton-quark, Ramond-NS and SUSY-kappa dichotomies correspond to one and same dichotomy so that the general structure looks quite satisfactory although it must be admitted that it is based on heuristic guess work.

Second deep difference is the appearance of the zeros of Riemann Zeta as conformal weights of the generating elements of the super-canonical algebra and the expected action of conformal algebra associated with 3-D CDS as a spectral flow in the space of super-canonical conformal weights inducing a mere gauge transformation infinitesimally and a braiding action in topological degrees of freedom.

In this section the relationship of Super Kac-Moody invariance to ordinary super-conformal symmetry and the interaction between Super-Kac Moody and super-canonical symmetries are discussed. For years the role of quaternions and octonions in TGD has been under an active speculation. These aspects are considered in [E2], where the number theoretic equivalent of spontaneous compactification is proposed. The conjecture states that space-time surfaces can be regarded either as 4-surfaces in $M^4 \times CP_2$ or as hyper-quaternionic 4-surfaces in the space $HO = M^8$ possessing hyper-octonionic structure (the attribute 'hyper' means that imaginary units are multiplied by $\sqrt{-1}$ in order to achieve number theoretic norm with Minkowskian signature).

5.1 Super-canonical and Super Kac-Moody symmetries

The proper understanding of super symmetries has turned out to be crucial for the understanding of quantum TGD and it seems that the mis-interpreted super-symmetries are one of the basic reasons for the difficulties of super string models too. At this moment one can fairly say that the construction of the configuration space spinor structure reduces to a purely group theoretical problem of constructing representations for the super generators of the super-canonical algebra of CP_2 localized with respect to $\delta M^4_\pm$ in terms of second quantized induced spinor fields.

5.1.1 Super canonical symmetries

One can imagine two kinds of causal determinants besides $\delta M^4_+ \times CP_2$. In principle all surfaces $X^3_l \times CP_2$, where X^3_l is a light like 3-surface of M^4, could act as effective causal determinants: the reason is that the creation of pairs of positive and negative energy space-time sheets is possible at these surfaces. There are good hopes that the super-canonical and super-conformal symmetries associated with δX^3_l allow to generalize the construction of the configuration space geometry performed at $\delta M^4_+ \times CP_2$. If X^3_l can be restricted to be unions of future and past light cone boundaries, the generalization is more or less trivial: one just forms a union of configuration spaces associated with unions of translates of δM^4_+ and δM^4_-.

As explained in the previous chapter, one can understand how the causal determinants $X^3_l \times CP_2$ emerge from the facts that space-time sheets with negative time orientation carry negative energy and that the most elegant theory results when the net quantum numbers and conserved classical quantities vanish for the entire Universe. Crossing symmetry allows consistency with elementary particle physics and the identification of gravitational 4-momentum as difference of conserved inertial (Poincare) 4-momenta for positive and negative energy matter provides consistency with macroscopic physics.

The emergence of these additional causal determinants means that super-canonical symmetries become microscopic, rather than only cosmological, symmetries commuting with Poincare transformations exactly for $M^4 \times CP_2$ and apart from small quantum gravitational effects for $M^4_+ \times CP_2$. Super-canonical symmetry differs in many respects from Kac-Moody symmetries of particle physics, which in fact correspond to the conformal invariance associated with the modified Dirac action and correspond to the product of Poincare, electro-weak and color groups. It seems that these symmetries are dually related.

5.1.2 Super Kac-Moody symmetries associated with the light like causal determinants

Also the light like 3-surfaces X^3_l of H defining elementary particle horizons at which Minkowskian signature of the metric is changed to Euclidian and boundaries of space-time sheets can act as causal determinants, and thus contribute to the configuration space metric. In this case the symmetries correspond to the isometries of the imbedding space localized with respect to the complex coordinate of the 2-surface X^2 determining the light like 3-surface X^3_l so that Kac-Moody type symmetry results. Also the condition $\sqrt(g_3) = 0$ for the determinant of the induced metric seems to define a conformal symmetry associated with the light like direction. This conforms with duality since also the 7-D causal determinants $X^3_l \times CP_2$ allow both radial and transversal conformal symmetry.

Good candidate for the counterpart of this symmetry in the interior of space-time surface is hyper-quaternion conformal invariance [E2]. All that is needed for these symmetries to be equivalent that the spaces of super-gauge degrees of freedom defined by them are equivalent. Kac Moody generators and their super counterparts can be associated with the 3-D light like CDs.

If is enough to localize only the H-isometries with respect to X^3_l, the purely bosonic part of the Kac-Moody algebra corresponds to the isometry group $M^4 \times SO(3,1) \times SU(3)$. The physical interpretation of these symmetries is not so obvious as one might think. The point is that one can generalize the formulas characterizing the action of infinitesimal isometries on spinor fields of finite-dimensional Kähler manifold to the level of the configuration space. This gives rise to bosonic generators containing also a sigma-matrix term bilinear in fermionic oscillator operators. This representation is not equivalent with the purely fermionic representations provided by induced Dirac action. Thus one has two groups of local color charges and the challenge is to find a physical interpretation for them. The following arguments fix the identification.

1. The hint comes from the fact that $U(2)$ in the decomposition $CP_2 = SU(3)/U(2)$ corresponds in a well-defined sense electro-weak algebra identified as a holonomy algebra of the spinor connection. Hence one could argue that the $U(2)$ generators of either $SU(3)$ algebra might be identifiable as generators of local $U(2)_{ew}$ gauge transformations whereas non-diagonal generators would correspond to Higgs field. This interpretation would conform with the idea that Higgs field is a genuine scalar field rather than a composite of fermions.

2. Since X^3_l-local $SU(3)$ transformations represented by fermionic currents are characterized by central extension they would naturally correspond to the electro-weak gauge algebra and Higgs bosons. This is also consistent with the fact that both leptons and quarks define fermionic Kac Moody currents.

3. The fact that only quarks appear in the gamma matrices of the configuration space supports the view that action of the generators of X^3_l-local color transformations on configuration space spinor fields represents local color transformations. If the action of X^3_l-local $SU(3)$ transformations on configuration space spinor fields has trivial central extension term the identification as a representation of local color symmetries is possible.

The topological explanation of the family replication phenomenon is based on an assignment of 2-dimensional boundary to a 3-surface characterizing the elementary particle. The precise identification of this surface has remained open and one possibility is that the 2-surface X^2 defining the light light-like surface associated with an elementary particle horizon is in question. This assumption would conform with the notion of elementary particle vacuum functionals defined in the zero modes characterizing different conformal equivalences classes for X^2.

5.1.3 The relationship of the Super-Kac Moody symmetry to the standard super-conformal invariance

Super-Kac Moody symmetry can be regarded as $N = 4$ complex super-symmetry with complex H-spinor modes of H representing the 4 physical helicities of 8-component leptonic and quark like spinors acting as generators of complex dynamical super-symmetries. The super-symmetries generated by the covariantly constant right handed neutrino appear with *both* M^4 helicities: it however seems that covariantly constant neutrino does not generate any global super-symmetry in the sense of particle-sparticle mass degeneracy. Only righthanded neutrino spinor modes (apart from covariantly constant mode) appear in the expressions of configuration space gamma matrices forming a subalgebra of the full super-algebra.

$N=2$ real super-conformal algebra is generated by the energy momentum tensor $T(z)$, $U(1)$ current $J(z)$, and super generators $G^{\pm}(z)$ carrying $U(1)$ charge. Now $U(1)$ current would correspond to right-handed neutrino number and super generators would involve contraction of covariantly constant neutrino spinor with second quantized induced spinor field. The further facts that $N=2$ algebra is associated naturally with Kähler geometry, that the partition functions associated with $N=2$ super-conformal representations are modular invariant, and that $N=2$ algebra defines so called chiral ring defining a topological quantum field theory [25], lend a further support for the belief that $N=2$ super-conformal algebra acts in super-canonical degrees of freedom.

The values of c and conformal weights for $N=2$ super-conformal field theories are given by

$$\begin{aligned} c &= \frac{3k}{k+2} \ , \\ \Delta_{l,m}(NS) &= \frac{l(l+2)-m^2}{4(k+2)} \ , \ l=0,1,...,k \ , \\ q_m &= \frac{m}{k+2} \ , \ m=-l,-l+2,....,l-2,l \ . \end{aligned} \quad (55)$$

q_m is the fractional value of the $U(1)$ charge, which would now correspond to a fractional fermion number. For $k=1$ one would have $q=0,1/3,-1/3$, which brings in mind anyons. $\Delta_{l=0,m=0}=0$ state would correspond to a massless state with a vanishing fermion number. Note that $SU(2)_k$ Wess-Zumino model has the same value of c but different conformal weights. More information about conformal algebras can be found from the appendix of [25].

For Ramond representation $L_0-c/24$ or equivalently G_0 must annihilate the massless states. This occurs for $\Delta=c/24$ giving the condition $k=2\left[l(l+2)-m^2\right]$ (note that k must be even and that $(k,l,m)=(4,1,1)$ is the simplest non-trivial solution to the condition). Note the appearance of a fractional vacuum fermion number $q_{vac}=\pm c/12=\pm k/4(k+2)$. I have proposed that NS and Ramond algebras could combine to a larger algebra containing also lepto-quark type generators but this not necessary.

The conformal algebra defined as a direct sum of Ramond and NS $N=4$ complex sub-algebras associated with quarks and leptons might further extend to a larger algebra if lepto-quark generators acting effectively as half odd-integer Virasoro generators can be allowed. The algebra would contain spin and electro-weak spin as fermionic indices. Poincare and color Kac-Moody generators would act as symplectically extended isometry generators on configuration space Hamiltonians expressible in terms of Hamiltonians of $X^3_l\times CP_2$. Electro-weak and color Kac-Moody currents have conformal weight $h=1$ whereas T and G have conformal weights $h=2$ and $h=3/2$.

The experience with $N=4$ complex super-conformal invariance suggests that the extended algebra requires the inclusion of also second quantized induced spinor fields with $h=1/2$ and their super-partners with $h=0$ and realized as fermion-antifermion bilinears. Since G and Ψ are labelled by 2×4 spinor indices, super-partners would correspond to $2\times(3+1)=8$ massless electro-weak gauge boson states with polarization included. Their inclusion would make the theory highly predictive since induced spinor and electro-weak fields are the fundamental fields in TGD.

5.1.4 How could conformal symmetries of light like 3-D CDs act on super-canonical degrees of freedom?

An important challenge is to understand the action of super-conformal symmetries associated with the light like 3-D CDs on super-canonical degrees of freedom. The breakthrough in this respect via the algebraic formulation for the vision about vanishing loop corrections of ordinary Feynman diagrams in terms of equivalence of generalized Feynman diagrams with loops with tree diagrams [C7]. The formulation involves Yang-Baxter equations, braid groups, Hopf algebras, and so called ribbon categories and led to the following vision. The original formulation to be discussed in this sub-subsection is very heuristic and a more quantitative formulation follows in the next subsection.

1. Quantum classical correspondence suggests that the complex conformal weights of super-canonical algebra generators have space-time counterparts. The proposal is that the weights are mapped to the points of the homologically non-trivial geodesic sphere S^2 of CP_2 corresponds to the super-canonical conformal weights, and corresponds to a discrete set of points at the space-time surface. These points would also label mutually commutating R-matrices. The map is completely analogous to the map of momenta of quantum particles to the points of celestial sphere. These points would belong to a "time=constant" section of 2-dimensional "space-time", presumably circle, defining physical states of a two-dimensional conformal field theory for which the scaling operator L_0 takes the role of Hamiltonian.

2. One could thus regard super-generators as conformal fields in space-time or complex plane having super-canonical conformal weights as punctures. The action of super-conformal algebra and braid group on these points realizing monodromies of conformal field theories [25] would induce by a pull-back a braid group action on the super-canonical conformal weights of configuration space gamma matrices (super generators) and corresponding isometry generators.

At the first sight the explicit realization of super-canonical and Kac Moody generators seems however to be in conflict with this vision. The interaction of the conformal algebra of X_l^3 on super-canonical algebra is a pure gauge interaction since the definition of super canonical generators is not changed by the action of conformal transformations of X_l^3. This is however consistent with the assumption that the action defined by the quantum-classical correspondence is also a pure gauge interaction locally. The braiding action would be analogous to the holonomies encountered in the case of non-Abelian gauge fields with a vanishing curvature in spaces possessing non-trivial first homotopy group.

Quantum classical correspondence would allow to map abstract configuration space level to space-time level.

1. The complex argument z of Kac Moody and Virasoro algebra generators $T(z) = \sum T_n z^n$ would be discretized so that it would have values on the set of supercanonical conformal weights corresponding to the space t in the Cartan decomposition $g = t + h$ of the tangent space of the configuration space. These points could be interpreted as punctures of the complex plane restricted to the lines $Re(z) = \pm 1/2$ and positive real axis if zeros of Riemann zeta define the conformal weights.

2. The vacuum expectation values of the enveloping algebra of the super-canonical algebra would reduce to n-point functions of a super-conformal quantum field theory in the complex plane containing infinite number of punctures defined by the super-canonical conformal weights, for which primary fields correspond to the representations of $SO(3) \times SU(3)$. These representations would combine to form infinite-dimensional representations of super-canonical algebra. The presence of the gigantic super-canonical symmetries raises the hope that quantum TGD could be solvable to a very high degree.

3. The Super Virasoro algebra and Super Kac Moody algebra associated with 3-D light like CDs would act as symmetries of this theory and the S-matrix of TGD would involve the n-point functions of this field theory. By 7–3 duality this indeed makes sense. The situation would reduce to that encountered in WZW theory in the sense that one would have space-like

3-surfaces X^3 containing two-dimensional closed surfaces carrying representations of Super Kac-Moody algebra.

This picture also justifies the earlier proposal that configuration space Clifford algebra defined by the gamma matrices acting as super generators defines an infinite-dimensional von Neumann algebra possessing hierarchies of type II_1 factors [27] having a close connection with the non-trivial representations of braid group and quantum groups. The sequence of non-trivial zeros of Riemann Zeta along the line $Re(s) = 1/2$ in the plane of conformal weights could be regarded an an infinite braid behind the von Neumann algebra [27]. Contrary to the expectations, also trivial zeros seem to be important. The finite braids defined by subsets of zeros, and also superpositions of non-trivial zeros of form $1/2 + \sum_i y_i$, could be seen as a hierarchy of completely integrable 1-dimensional spin chains leading to quantum groups and braid groups [24, 25] naturally.

It seems that not only Riemann's zeta but also polyzetas [28, 29, 30, 32] could play a fundamental role in TGD Universe. The super-canonical conformal weights of interacting particles, in particular of those forming bound states, are expected to have "off mass shell" values. An attractive hypothesis is that they correspond to zeros of Riemann's polyzetas. Interaction would allow quite concretely the realization of braiding operations dynamically. The physical justification for the hypothesis would be quantum criticality. Indeed, it has been found that the loop corrections of quantum field theory are expressible in terms of polyzetas [31]. If the arguments of polyzetas correspond to conformal weights of particles of many-particle bound state, loop corrections vanish when the super-canonical conformal weights correspond to the zeros of polyzetas including zeta.

5.2 The relationship between super-canonical and Super Kac-Moody algebras, Equivalence Principle, and justification of p-adic thermodynamics

The relationship between super-canonical algebra (SC) acting at light-cone boundary and Super Kac-Moody algebra (SKM) acting on light-like 3-surfaces has remained somewhat enigmatic due to the lack of physical insights. This is not the only problem. The question to precisely what extent Equivalence Principle (EP) remains true in TGD framework and what might be the precise mathematical realization of EP is waiting for an answer. Also the justification of p-adic thermodynamics for the scaling generator L_0 of Virasoro algebra -in obvious conflict with the basic wisdom that this generator should annihilate physical states- is lacking. It seems that these three problems could have a common solution.

Before going to describe the proposed solution, some background is necessary. The latest proposal for $SC - SKM$ relationship relies on non-standard and therefore somewhat questionable assumptions.

1. SKM Virasoro algebra (SKMV) and SC Virasoro algebra (SCV) (anti)commute for physical states.

2. SC algebra generates states of negative conformal weight annihilated by SCV generators L_n, $n < 0$, and serving as ground states from which SKM generators create states with non-negative conformal weight.

This picture could make sense for elementary particles. On other hand, the recent model for hadrons [F4] assumes that SC degrees of freedom contribute about 70 per cent to the mass of hadron but at space-time sheet different from those assignable to quarks. The contribution of SC degrees of freedom to the thermal average of the conformal weight would be positive. A contradiction results unless one assumes that there exists also SCV ground states with positive conformal weight annihilated by SCV elements L_n, $n < 0$, but also this seems implausible.

5.2.1 New vision about the relationship between SCV and $SKMV$

Consider now the new vision about the relationship between SCV and $SKMV$.

1. The isometries of H assignable with SKM are also symplectic transformations [B3] (note that I have used the term canonical instead of symplectic previously). Hence might consider the possibility that SKM could be identified as a subalgebra of SC. If this makes sense, a generalization of the coset construction obtained by replacing finite-dimensional Lie group with infinite-dimensional symplectic group suggests itself. The differences of SCV and $SKMV$ elements would annihilate physical states and (anti)commute with $SKMV$. Also the generators O_n, $n > 0$, for both algebras would annihilate the physical states so that the differences of the elements would annihilate automatically physical states for $n > 0$.

2. The super-generator G_0 contains the Dirac operator D of H. If the action of SCV and $SKMV$ Dirac operators on physical states are identical then cm of degrees of freedom disappear from the differences $G_0(SCV) - G_0(SKMV)$ and $L_0(SCV) - L_0(SKMV)$. One could interpret the identical action of the Dirac operators as the long sought-for precise realization of Equivalence Principle (EP) in TGD framework. EP would state that the total inertial four-momentum and color quantum numbers assignable to SC (imbedding space level) are equal to the gravitational four-momentum and color quantum numbers assignable to SKM (space-time level). Note that since super-canonical transformations correspond to the isometries of the "world of classical worlds" the assignment of the attribute "inertial" to them is natural.

3. The analog of coset construction applies also to SKM and SC algebras which means that physical states can be thought of as being created by an operator of SKM carrying the conformal weight and by a genuine SC operator with vanishing conformal weight. Therefore the situation does not reduce to that encountered in super-string models.

4. The reader can recognize $SC - SKM$ as a precise formulation for $7 - 3$ duality discussed in the section *About dualities and conformal symmetries in TGD framework* stating that 3-D light-like causal determinants and 7-D causal determinants $\delta M^4_\pm \times CP_2$ are equivalent.

5.2.2 Consistency with p-adic thermodynamics

The consistency with p-adic thermodynamics provides a strong reality test and has been already used as a constraint in attempts to understand the super-conformal symmetries in partonic level.

1. In physical states the p-adic thermal expectation value of the SKM and SC conformal weights would be non-vanishing and identical and mass squared could be identified to the expectation value of SKM scaling generator L_0. There would be no need to give up Super Virasoro conditions for $SCV - SKMV$.

2. There is consistency with p-adic mass calculations for hadrons [F4] since the non-perturbative SC contributions and perturbative SKM contributions to the mass correspond to space-time sheets labeled by different p-adic primes. The earlier statement that SC is responsible for the dominating non-perturbative contributions to the hadron mass transforms to a statement reflecting $SC - SKM$ duality. The perturbative quark contributions to hadron masses can be calculated most conveniently by using p-adic thermodynamics for SKM whereas non-perturbative contributions to hadron masses can be calculated most conveniently by using p-adic thermodynamics for SC. Also the proposal that the exotic analogs of baryons resulting when baryon looses its valence quarks [F5] remains intact in this framework.

3. The results of p-adic mass calculations depend crucially on the number N of tensor factors contributing to the Super-Virasoro algebra. The required number is $N = 5$ and during years I have proposed several explanations for this number. It seems that holonomic contributions that is electro-weak and spin contributions must be regarded as contributions separate from those coming from isometries. SKM algebras in electro-weak degrees and spin degrees of of freedom, would give 2+1=3 tensor factors corresponding to $U(2)_{ew} \times SU(2)$. $SU(3)$ and $SO(3)$ (or $SO(2) \subset SO(3)$ leaving the intersection of light-like ray with S^2 invariant) would give 2 additional tensor factors. Altogether one would indeed have 5 tensor factors.

There are some further questions which pop up in mind immediately.

1. Why mass squared corresponds to the thermal expectation value of the net conformal weight? This option is forced among other things by Lorentz invariance but it is not possible to provide a really satisfactory answer to this question yet. In the coset construction there is no reason to require that the mass squared equals to the integer value conformal weight for SKM algebra. This allows the possibility that mass squared has same value for states with different values of SKM conformal weights appearing in the thermal state and equals to the average of the conformal weight.

   The coefficient of proportionality can be however deduced from the observation that the mass squared values for CP_2 Dirac operator correspond to definite values of conformal weight in p-adic mass calculations. It is indeed possible to assign to partonic 2-surface X^2 CP_2 partial waves correlating strongly with the net electro-weak quantum numbers of the parton so that the assignment of ground state conformal weight to CP_2 partial waves makes sense. In the case of M^4 degrees of freedom it is not possible to talk about momentum eigen states since translations take parton out of δH_+ so that momentum must be assigned with the tip of the light-cone containing the particle.

2. The additivity of conformal weight means additivity of mass squared at parton level and this has been indeed used in p-adic mass calculations. This implies the conditions

$$(\sum_i p_i)^2 = \sum_i m_i^2 \tag{56}$$

   The assumption $p_i^2 = m_i^2$ makes sense only for massless partons moving collinearly. In the QCD based model of hadrons only longitudinal momenta and transverse momentum squared are used as labels of parton states, which together with the presence of preferred plane M^2 would suggest that one has

$$\begin{aligned} p_{i,||}^2 &= m_i^2 \ , \\ -\sum_i p_{i,\perp}^2 + 2\sum_{i,j} p_i \cdot p_j &= 0 \ . \end{aligned} \tag{57}$$

   The masses would be reduced in bound states: $m_i^2 \to m_i^2 - (p_T^2)_i$. This could explain why massive quarks can behave as nearly massless quarks inside hadrons.

3. Single particle super-canonical conformal weights can have also imaginary part, call it y. The question is what complex mass squared means physically. Complex conformal weights have been assigned with an inherent time orientation distinguishing positive energy particle from negative energy antiparticle (in particular, phase conjugate photons from ordinary photons). This suggests an interpretation of y in terms of a decay width. p-Adic thermodynamics suggest that y vanishes for states with vanishing conformal weight (mass square

4. and that the measured value of y is a p-adic thermal average with non-vanishing contributions from states with mass of order CP_2 mass. This makes sense if y_k are algebraic or perhaps even rational numbers.

   For instance, if a massless state characterized by p-adic prime p has p-adic thermal average $y = psy_k$, where s is the denominator of rational valued $y_k = r/s$, the lowest order contribution to the decay width is proportional to $1/p$ by the basic rules of p-adic mass calculations and the decay rate is of same order of magnitude as mass. If the p-adic thermal average of y is of form $p^n y_k$ for massless state then a decay width of order $\Gamma \sim p^{-(n-1)/2}m$ results. For electron n should be rather large. This argument generalizes trivially to the case in which massless state has vanishing value of y.

5.2.3 Can SKM be lifted to a sub-algebra of SC?

A picture introducing only a generalization of coset construction as a new element, realizing mathematically Equivalence Principle, and justifying p-adic thermodynamics is highly attractive but there is a problem. SKM is defined at light-like 3-surfaces X^3 whereas SC acts at light-cone boundary $\delta H_\pm = \delta M^4_\pm \times CP_2$. One should be able to lift SKM to imbedding space level somehow. Also SC should be lifted to entire H. This problem was the reason why I gave up the idea about coset construction and $SC - SKM$ duality as it appeared for the first time.

A possible solution of the lifting problem comes from the observation making possible a more rigorous formulation of $HO - H$ duality stating that one can regard space-time surfaces either as surfaces in hyper-octonionic space $HO = M^8$ or in $H = M^4 \times CP_2$ [C1, E2]. Consider first the formulation of $HO - H$ duality.

1. Associativity also in the number theoretical sense becomes the fundamental dynamical principle if $HO - H$ duality holds true [E2]. For a space-time surface $X^4 \subset HO = M^8$ associativity is satisfied at space-time level if the tangent space at each point of X^4 is some hyper-quaternionic sub-space $HQ = M^4 \subset M^8$. Also partonic 2-surfaces at the boundaries of causal diamonds formed by pairs of future and past directed light-cones defining the basic imbedding space correlate of zero energy state in zero energy ontology and light-like 3-surfaces are assumed to belong to $HQ = M^4 \subset HO$.

2. $HO - H$ duality requires something more. If the tangent spaces contain the same preferred commutative and thus hyper-complex plane $HC = M^2$, the tangent spaces of X^4 are parameterized by the points s of CP_2 and $X^4 \subset HO$ can be mapped to $X^4 \subset M^4 \times CP_2$ by assigning to a point of X^4 regarded as point (m, e) of $M^4_0 \times E^4 = M^8$ the point (m, s). Note that one must also fix a preferred global hyper-quaternionic subspace $M^4_0 \subset M^8$ containing M^2 to be not confused with the local tangent planes M^4.

3. The preferred plane M^2 can be interpreted as the plane of non-physical polarizations so that the interpretation as a number theoretic analog of gauge conditions posed in both quantum field theories and string models is possible.

4. An open question is whether the resulting surface in H is a preferred extremal of Kähler action. This is possible since the tangent spaces at light-like partonic 3-surfaces are fixed to contain M^2 so that the boundary values of the normal derivatives of H coordinates are fixed and field equations fix in the ideal case X^4 uniquely and one obtains space-time surface as the analog of Bohr orbit.

5. The light-like "Higgs term" proportional to $O = \gamma_k t^k$ appearing in the generalized eigenvalue equation for the modified Dirac operator is an essential element of TGD based description of Higgs mechanism. This term can cause complications unless t is a covariantly constant

light-like vector. Covariant constancy is achieved if t is constant light-like vector in M^2. The interpretation as a space-time correlate for the light-like 4-momentum assignable to the parton might be considered.

6. Associativity requires that the hyper-octonionic arguments of N-point functions in HO description are restricted to a hyperquaternionic plane $HQ = M^4 \subset HO$ required also by the $HO-H$ correspondence. The intersection $M^4 \cap int(X^4)$ consists of a discrete set of points in the generic case. Partonic 3-surfaces are assumed to be associative and belong to M^4. The set of commutative points at the partonic 2-surface X^2 is discrete in the generic case whereas the intersection $X^3 \cap M^2$ consists of 1-D curves so that the notion of number theoretical braid crucial for the p-adicization of the theory as almost topological QFT is uniquely defined.

7. The preferred plane $M^2 \subset M^4 \subset HO$ can be assigned also to the definition of N-point functions in HO picture. It is not clear whether it must be same as the preferred planes assigned to the partonic 2-surfaces. If not, the interpretation would be that it corresponds to a plane containing the over all cm four-momentum whereas partonic planes M_i^2 would contain the partonic four-momenta. M^2 is expected to change at wormhole contacts having Euclidian signature of the induced metric representing horizons and connecting space-time sheets with Minkowskian signature of the induced metric.

The presence of globally defined plane M^2 and the flexibility provided by the hyper-complex conformal invariance raise the hopes of achieving the lifting of SC and SKM to H. At the light-cone boundary the light-like radial coordinate can be lifted to a hyper-complex coordinate defining coordinate for M^2. At X^3 one can fix the light-like coordinate varying along the braid strands can be lifted to some hyper-complex coordinate of M^2 defined in the interior of X^4. The total four-momenta and color quantum numbers assignable to the SC and SKM degrees of freedom are naturally identical since they can be identified as the four-momentum of the partonic 2-surface $X^2 \subset X^3 \cap \delta M^4_\pm \times CP_2$. Equivalence Principle would emerge as an identity.

5.2.4 Questions about conformal weights

One can pose several non-trivial questions about conformal weights.

1. The negative SKM conformal weights of ground states for elementary particles [F3] remain to be understood in this framework. In the case of light-cone boundary the natural value for ground state conformal weight of a scalar field is $-1/2$ since this implies a complete analogy with a plane wave with respect to the radial light-like coordinate r_M with inner product defined by a scale invariant integration measure dr_M/r_M. If the coset construction works same should hold true for SKM degrees of freedom for a proper choice of the light-like radial coordinate. There are thus good hopes that negative ground state conformal weights could be understood.

2. Further questions relate to the imaginary parts of ground state conformal weights, which can be vanishing in principle. Do the ground state conformal weights correspond to the zeros of some zeta function- most naturally the zeta function defined by generalized eigenvalues of the modified Dirac operator and satisfying Riemann hypothesis so that ground state conformal weight would have real part -1/2? Do SC and SKM have same spectrum of complex conformal weights as the coset construction suggests? Does the imaginary part of the conformal weight bring in a new degree of freedom having interpretation in terms of space-time correlate for the arrow of time with the generalization of the phase conjugation of laser physics representing the reversal of the arrow of geometric time?

3. The opposite light-cone boundaries of the causal diamond bring in mind the hemispheres of S^2 in ordinary conformal theory. In ordinary conformal theory positive/negative powers of z correspond to these hemispheres. Could it be that the radial conformal weights are of opposite sign and of same magnitude for the positive and negative energy parts of zero energy state?

5.2.5 Further questions

There are still several open questions.

1. Is it possible to define hyper-quaternionic variants of the superconformal algebras in both H and HO or perhaps only in HO. A positive answer to this question would conform with the conjecture that the geometry of "world of classical worlds" allows Hyper-Kähler property in either or both pictures [B3].

2. How this picture relates to what is known about the extremals of field equations [D1] characterized by generalized Hamilton-Jacobi structure bringing in mind the selection of preferred M^2?

3. Is this picture consistent with the views about Equivalence Principle and its possible breaking based on the identification of gravitational four-momentum in terms of Einstein tensor is interesting [D3]?

5.3 Brief summary of super-conformal symmetries in partonic picture

The notion of conformal super-symmetry is very rich and involves several non-trivial aspects, and as the following discussions shows, one could assign the attribute super-conformal to several symmetries. In the following I try to sum up what I see as important. What is new is that it is now possible to tie everything to the fundamental description in terms of the parton level action principle and provide a rigorous justification and precise realization for the claimed super-conformal symmetries.

5.3.1 Super-canonical symmetries

Super-canonical symmetries correspond to the isometries of the configuration space CH (the world of classical worlds) and are induced from the corresponding symmetries of $\delta H_\pm \equiv \delta M^4_\pm \times CP_2$. The explicit representations have been constructed for both 2-D and stringy options. The most stringent option having strong support from various considerations is that single particle conformal weights are of form $1/2+i\sum_k n_k y_k$, where $s_k = 1/2+iy_k$ is zero of Riemann zeta. The construction of many particle conformally bound states for poly-zetas leads to the same spectrum for bound states and predicts that only 2- and 3-parton bound states are irreducible. On the other hand, conformal weights are additive for the (anti)commutators of (super)Hamiltonians and gives thus all weights of form $s = n + i\sum_k n_k y_k$.

The interpretation of this picture is not obvious.

1. The first interpretation would be that also other conformal weights are possible but that the commutator and anti-commutator algebras of super-canonical algebra containing conformal weights $Re(s) = k/2$, $k > 1$, represent gauge degrees of freedom. The sub-Virasoro algebra generated by L_n, $n > 0$, would generate these conformal weights which would suggest that L_n, $n > 0$, but not L_0, must annihilate the physical states. The problem is that this makes p-adic thermodynamics impossible.

2. p-Adic mass calculations would suggest that Super Kac-Moody Virasoro (SKMV) generators L_n, $n>0$, do not correspond to pure gauge degrees of freedom, and a more general interpretation would be that all these conformal weights are possible and represent genuine physical degrees of freedom. The extension of the algebra using the standard assumption $L_{-n}=L_n^\dagger$ would bring in also the conformal weights $Re(s)=-k/2$, $k\geq 1$. p-adic mass calculations would encourage to think that it is super-canonical (SC) generators L_{-n}, $n>0$, which annihilate tachyonic ground states and stabilize them against tachyonic p-adic thermodynamics. The physical ground state with a vanishing conformal weight would be constructed from this tachyonic ground state and p-adic thermodynamics for $SKMV$ generators L_n, $n>0$, would apply to it.

3. In the discrete variant of theory required by number theoretic universality all stringy submanifolds of X^2 corresponding to the inverse images of $z=\zeta(n/2+i\sum_k n_k y_k)\in S^2\subset CP_2$ would be realized so that one would have probability amplitude in the discrete set of these number theoretic strings. $SKMV$ generators L_n and G_r would excite $n>0$ "shells" in this structure whereas SC generators would generate $n<0$ shells.

4. Also the trivial zeros $s_n=-2n$, $n>0$, of Riemann Zeta could correspond to physically interesting conformal weights for the super-canonical algebra (at least). In the region $r\geq r_0$ the function r^{-2n} approaches zero and these powers are square integrable in this region. The orthogonality with other states could be achieved by arranging things suitably in other degrees of freedom [B2]. Since ζ is real also along real line, the set of even integers $\sum_k n_k s_k$, $n_k\in Z$ is mapped by ζ to the same real line of $S^2\subset CP_2$ as non-trivial zeros of ζ. p-Adic mass calculations would suggest that states with conformal weight $s_{min}=-2n_{max}$ (at least these) could represent null states annihilated by L_{-n}, $n>0$.

5.3.2 Bosonic super Kac-Moody algebra

The generators of bosonic super Kac-Moody algebra leave the light-likeness condition $\sqrt{g_3}=0$ invariant. This gives the condition

$$\delta g_{\alpha\beta}Cof(g^{\alpha\beta}) \quad = \quad 0 \ , \tag{58}$$

Here Cof refers to matrix cofactor of $g_{\alpha\beta}$ and summation over indices is understood. The conditions can be satisfied if the symmetries act as combinations of infinitesimal diffeomorphisms $x^\mu\rightarrow x^\mu+\xi^\mu$ of X^3 and of infinitesimal conformal symmetries of the induced metric

$$\delta g_{\alpha\beta} \quad = \quad \lambda(x)g_{\alpha\beta}+\partial_\mu g_{\alpha\beta}\xi^\mu+g_{\mu\beta}\partial_\alpha\xi^\mu+g_{\alpha\mu}\partial_\beta\xi^\mu \ . \tag{59}$$

1. Ansatz as an X^3-local conformal transformation of imbedding space

Write δh^k as a super-position of X^3-local infinitesimal diffeomorphisms of the imbedding space generated by vector fields $J^A=j^{A,k}\partial_k$:

$$\delta h^k \quad = \quad c_A(x)j^{A,k} \ . \tag{60}$$

This gives

$$\begin{aligned} c_A(x) \quad & \left[D_kj^A_l+D_lj^A_k\right]\partial_\alpha h^k\partial_\beta h^l+2\partial_\alpha c_A h_{kl}j^{A,k}\partial_\beta h^l \\ = \quad & \lambda(x)g_{\alpha\beta}+\partial_\mu g_{\alpha\beta}\xi^\mu+g_{\mu\beta}\partial_\alpha\xi^\mu+g_{\alpha\mu}\partial_\beta\xi^\mu \ . \end{aligned} \tag{61}$$

If an X^3-local variant of a conformal transformation of the imbedding space is in question, the first term is proportional to the metric since one has

$$D_k j_l^A + D_l j_k^A = 2h_{kl} \ . \tag{62}$$

The transformations in question includes conformal transformations of $H_\pm$ and isometries of the imbedding space H.

The contribution of the second term must correspond to an infinitesimal diffeomorphism of X^3 reducible to infinitesimal conformal transformation ψ^μ:

$$2\partial_\alpha c_A h_{kl} j^{A,k} \partial_\beta h^l = \xi^\mu \partial_\mu g_{\alpha\beta} + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta \xi^\mu \ . \tag{63}$$

2. A rough analysis of the conditions

One could consider a strategy of fixing c_A and solving solving ξ^μ from the differential equations. In order to simplify the situation one could assume that $g_{ir} = g_{rr} = 0$. The possibility to cast the metric in this form is plausible since generic 3-manifold allows coordinates in which the metric is diagonal.

1. The equation for g_{rr} gives

   $$\partial_r c_A h_{kl} j^{Ak} \partial_r h^k = 0 \ . \tag{64}$$

   The radial derivative of the transformation is orthogonal to X^3. No condition on ξ^α results. If c_A has common multiplicative dependence on $c_A = f(r) d_A$ by a one obtains

   $$d_A h_{kl} j^{Ak} \partial_r h^k = 0 \ . \tag{65}$$

   so that J^A is orthogonal to the light-like tangent vector $\partial_r h^k$ X^3 which is the counterpart for the condition that Kac-Moody algebra acts in the transversal degrees of freedom only. The condition also states that the components g_{ri} is not changed in the infinitesimal transformation.

   It is possible to choose $f(r)$ freely so that one can perform the choice $f(r) = r^n$ and the notion of radial conformal weight makes sense. The dependence of c_A on transversal coordinates is constrained by the transversality condition only. In particular, a common scale factor having free dependence on the transversal coordinates is possible meaning that X^3- local conformal transformations of H are in question.

2. The equation for g_{ri} gives

   $$\partial_r \xi^i = \partial_r c_A h_{kl} j^{Ak} h^{ij} \partial_j h^k \ . \tag{66}$$

   The equation states that g_{ri} are not affected by the symmetry. The radial dependence of ξ^i is fixed by this differential equation. No condition on ξ^r results. These conditions imply that the local gauge transformations are dynamical with the light-like radial coordinate r playing the role of the time variable. One should be able to fix the transformation more or less arbitrarily at the partonic 2-surface X^2.

3. The three independent equations for g_{ij} give

$$\xi^\alpha \partial_\alpha g_{ij} + g_{kj}\partial_i \xi^k + g_{ki}\partial_j \xi^k \quad = \quad \partial_i c_A h_{kl} j^{Ak} \partial_j h^l \ . \tag{67}$$

These are 3 differential equations for 3 functions ξ^α on 2 independent variables x^i with r appearing as a parameter. Note however that the derivatives of ξ^r do not appear in the equation. At least formally equations are not over-determined so that solutions should exist for arbitrary choices of c_A as functions of X^3 coordinates satisfying the orthogonality conditions. If this is the case, the Kac-Moody algebra can be regarded as a local algebra in X^3 subject to the orthogonality constraint.

This algebra contains as a subalgebra the analog of Kac-Moody algebra for which all c_A except the one associated with time translation and fixed by the orthogonality condition depends on the radial coordinate r only. The larger algebra decomposes into a direct sum of representations of this algebra.

3. Commutators of infinitesimal symmetries

The commutators of infinitesimal symmetries need not be what one might expect since the vector fields ξ^μ are functionals c_A and of the induced metric and also c_A depends on induced metric via the orthogonality condition. What this means that $j^{A,k}$ in principle acts also to ϕ_B in the commutator $[c_A J^A, c_B J^B]$.

$$\left[c_A J^A, c_B J^B\right] \quad = \quad c_A c_B J^{[A,B]} + J^A \circ c_B J^B - J^B \circ c_A J^A \ , \tag{68}$$

where $\circ$ is a short hand notation for the change of c_B induced by the effect of the conformal transformation J^A on the induced metric.

Luckily, the conditions in the case $g_{rr} = g_{ir} = 0$ state that the components g_{rr} and g_{ir} of the induced metric are unchanged in the transformation so that the condition for c_A resulting from g_{rr} component of the metric is not affected. Also the conditions coming from $g_{ir} = 0$ remain unchanged. Therefore the commutation relations of local algebra apart from constraint from transversality result.

The commutator algebra of infinitesimal symmetries should also close in some sense. The orthogonality to the light-like tangent vector creates here a problem since the commutator does not obviously satisfy this condition automatically. The problem can be solved by following the recipes of non-covariant quantization of string model.

1. Make a choice of gauge by choosing time translation P^0 in a preferred M^4 coordinate frame to be the preferred generator $J^{A_0} \equiv P^0$, whose coefficient $\Phi_{A_0} \equiv \Psi(P^0)$ is solved from the orthogonality condition. This assumption is analogous with the assumption that time coordinate is non-dynamical in the quantization of strings. The natural basis for the algebra is obtained by allowing only a single generator J^A besides P^0 and putting $d_A = 1$.

2. This prescription must be consistent with the well-defined radial conformal weight for the $J^A \neq P^0$ in the sense that the proportionality of d_A to r^n for $J^A \neq P^0$ must be consistent with commutators. SU(3) part of the algebra is of course not a problem. From the Lorentz vector property of P^k it is clear that the commutators resulting in a repeated commutation have well-defined radial conformal weights only if one restricts $SO(3,1)$ to $SO(3)$ commuting with P^0. Also D could be allowed without losing well-defined radial conformal weights but the argument below excludes it. This picture conforms with the earlier identification of the Kac-Moody algebra.

Conformal algebra contains besides Poincare algebra and the dilation $D = m^k\partial_{m^k}$ the mutually commuting generators $K^k = (m^r m_r \partial_{m^k} - 2m^k m^l \partial_{m^l})/2$. The commutators involving added generators are

$$\begin{array}{ll} \left[D, K^k\right] = -K^k \ , & \left[D, P^k\right] = P^k \ , \\ \left[K^k, K^l\right] = 0 \ , & \left[K^k, P^l\right] = m^{kl}D - M^{kl} \ . \end{array} \tag{69}$$

From the last commutation relation it is clear that the inclusion of K^k would mean loss of well-defined radial conformal weights.

3. The coefficient dm^0/dr of $\Psi(P^0)$ in the equation

$$\Psi(P^0)\frac{dm^0}{dr} = -J^{Ak}h_{kl}\partial_r h^l$$

is always non-vanishing due to the light-likeness of r. Since P^0 commutes with generators of $SO(3)$ (but not with D so that it is excluded!), one can *define* the commutator of two generators as a commutator of the remaining part and identify $\Psi(P^0)$ from the condition above.

4. Of course, also the more general transformations act as Kac-Moody type symmetries but the interpretation would be that the sub-algebra plays the same role as $SO(3)$ in the case of Lorentz group: that is gives rise to generalized spin degrees of freedom whereas the entire algebra divided by this sub-algebra would define the coset space playing the role of orbital degrees of freedom. In fact, also the Kac-Moody type symmetries for which c_A depends on the transversal coordinates of X^3 would correspond to orbital degrees of freedom. The presence of these orbital degrees of freedom arranging super Kac-Moody representations into infinite multiplets labelled by function basis for X^2 means that the number of degrees of freedom is much larger than in string models.

5. It is possible to replace the preferred time coordinate m^0 with a preferred light-like coordinate. There are good reasons to believe that orbifold singularity for phases of matter involving non-standard value of Planck constant corresponds to a preferred light-ray going through the tip of $\delta M^4_\pm$. Thus it would be natural to assume that the preferred M^4 coordinate varies along this light ray or its dual. The Kac-Moody group $SO(3) \times E^3$ respecting the radial conformal weights would reduce to $SO(2) \times E^2$ as in string models. E^2 would act in tangent plane of $S^2_\pm$ along this ray defining also $SO(2)$ rotation axis.

4. Hamiltonians

The action of these transformations on Chern-Simons action is well-defined and one can deduce the conserved quantities having identification as configuration space Hamiltonians. Hamiltonians also correspond to closed 2-forms. The condition that the Hamiltonian reduces to a dual of closed 2-form is satisfied because X^2-local conformal transformations of $M^4_\pm \times CP_2$ are in question (X^2-locality does not imply any additional conditions).

5. Action on spinors

One can imagine two interpretations for the action of generalized Kac-Moody transformations on spinors.

1. Both SO(3) and SU(3) rotations have a standard action as spin rotation and electro-weak rotation allowing to define the action of the Kac-Moody algebra J^A on spinors. This action is not consistent with the generalized eigenvalue equation unless one restricts it to X^2 at $\delta H_{\pm}$.

2. Since Kac-Moody generator performs a local spinor rotation and increases the conformal weight by n units, the simplest possibility is that the action of transformation adds to Ψ_λ with $\lambda = 1/2 + i\sum_k n_k y_k$, a term with eigenvalue $\lambda + n$ and having $J^A\Psi_\lambda$ as initial values at X^2. This would make natural the interpretation as a gauge transformation apart from the effects caused by the possible central extension term.

6. How central extension term could emerge?

The central extension term of Kac-Moody algebra could correspond to a symplectic extension which can emerge from the freedom to add a constant term to Hamiltonians as in the case of super-canonical algebra. The expression of the Hamiltonians as closed forms could allow to understand how the central extension term emerges.

In principle one can construct a representation for the action of Kac-Moody algebra on fermions a representations as a fermionic bilinear and the central extension of Kac-Moody algebra could emerge in this construction just as it appears in Sugawara construction.

5.3.3 Fermionic Kac-Moody algebra in spin and electro-weak degrees of freedom

The action of spin rotations and electro-weak rotations can be identified in terms of the group $SU(2) \times SU(2) \times U(1)$ associated inherently with $N = 4$ super-conformal symmetry. The action on zero modes and eigen modes Ψ is straightforward to write as multiplication on the initial values at X^2 and assuming that λ in the generalized eigenvalue equation is replaced by $\lambda + n$.

Fermionic super-generators correspond naturally to zero modes and eigen modes of the modified Dirac operator labelled by the radial conformal weights $\lambda = 1/2 + i\sum_k n_k yk$ and by the quantum numbers labelling the dependence on transversal degrees of freedom. The real part of the conformal weight would corresponds for $D\Psi = 0$ to ground state conformal weight $h = 0$(Ramond) and to $h = 1/2$ for $\lambda \neq 0$ (N-S). That also bosonic super-canonical Hamiltonians can have half odd integer conformal weight is however in conflict with the intuition that half-odd integer conformal weights correspond to states with odd fermion number.

For Ramond representations the lines $\zeta(Re(s) = n) \subset S^2$, $n \geq 0$, would represent the conformal weights at space-time level and for N-S representations the lines would correspond to $\zeta(Re(s) = n + 1/2) \subset S^2$. If also trivial zeros are possible they would correspond to the lines $\zeta(Re(s) = n - 2k) \subset S^2$, $k = 1, 2, ...$

5.3.4 Radial Super Virasoro algebras

The radial Super Virasoro transformations act on both $\delta H_{\pm}$ and partonic 3-surface X^3 and are consistent with the freedom to choose the basis of $H_{\pm}$ Hamiltonians and the eigenmode basis of the modified Dirac operator by a re-scaling the light-like vector (t^k or more plausibly, its dual n^k) appearing in the definition of the generalized eigenvalue equation.

In the partonic sector a possible interpretation is as local diffeomorphisms of X^3. These transformations do not however leave X^3 invariant as a whole, which brings in some delicacies. In the case of $\delta H_{\pm}$ the tip of the future light-cone remains invariant only for $n \geq 0$ and $r = \infty$ only for $n \leq 0$. These facts could explain why only the generators L_n, $n < 0$ (or $n < 0$ depending on whether positive or negative energy component of zero energy state is in question) annihilate the ground states.

One can assign to the Virasoro algebra of $H_\pm$ Hamiltonians as Noether charges defined by current $\Pi^0_k j^{Ak}$ which reduces to a dual of a closed 2-form in the case of $H_\pm$ because its symplectic form annihilates j^{Ak}. The transformations associated with X^3 correspond to a unique shift of X^2 in the light-like direction by $\delta h^k = r^n \partial_r h^k$ so that the Hamiltonian is well-defined and reduces to a value of a closed 2-form so that the stringy picture emerges.

The corresponding fermionic super Hamiltonians $G_r = \overline{\nu} r^n \Gamma_r \Psi$ anti-commute to these as is easy to see by noticing that the light-like radial gamma matrices Γ_r appear in the combination $\Gamma_r \gamma^0 \Gamma_r = \gamma_0$ in the anti-commutator so that it does not vanish. One can consider also more general fermionic generators obtained by replacing right-handed neutrino spinor with an arbitrary solution of $D\Psi = 0$ which is eigen spinor of $J^{kl}\Sigma_{kl}$ appearing in the fermionic anti-commutation relations. This would give rise to a full $N = 4$ super-conformal symmetry of Ramond type but having infinite degeneracy due to the dependence on transversal coordinates of X^3. If one allows also the solutions of $D\Psi = \lambda\Psi$ one obtains counterparts of N-S type representations with a similar degeneracy.

It must be emphasized that four-momentum does not appear neither in the representations of Super Virasoro generators as it does in string models and this is consistent with the Lorentz invariant identification of mass squared as vacuum expectation value of the net conformal weight. Also the problems with tachyons are avoided. Four-momentum could creep in if one had Sugawara type representation of Super Virasoro generators in terms of Kac-Moody generators which indeed contain also translation generators now. Note also that the stringy conformal weight would be associated with partonic 2-surface, whereas radial conformal weight is associated with its light-like orbit. Furthermore, the origin of the radial super-conformal symmetries is light-likeness rather than stringy character. It is not clear whether it is useful to assign the usual conformal weights with the conformal fields at X^2 and whether the stringy anti-commutation relations for Ψ force this kind of assignment.

5.3.5 Gauge super-symmetries associated with the generalized eigenvalue equation for D

Zero modes which are annihilated by the operator $T = t^k\gamma_k$ or $N = n^k\gamma_k$. t^k (n^k) is the light-like appearing in the generalized eigenvalue equation for the modified Dirac operator. t^k is parallel to X^3 and n^k, which corresponds to the more plausible option, is obtained by changing the direction of the spatial part of t^k in the preferred M^4 coordinate frame associated with the space-time sheet (the rest system or number theoretically determined M^4 time). n^k defines inwards directed tangent vector to the space-time sheet containing X^3. The zero modes of the modified Dirac operator annihilated by T (N) act as super gauge symmetries for the generalized eigen modes of the generalized Dirac operator. They do not depend on r and thus have a vanishing conformal weight.

The freedom to choose the scaling of t^k (n^k) rather freely gives rise to a further symmetry which does not affect the eigenvalue spectrum but modifies the eigen modes. This symmetry is definitely a pure gauge symmetry.

5.3.6 What about ordinary conformal symmetries?

Ordinary conformal symmetries acting on the complex coordinate of X^2 have not yet been discussed. These symmetries involve the dependence on the induced metric through the moduli of characterizing the conformal structure of X^2. Stringy picture would suggest in the case of a spherical topology that the zero modes and eigen modes of Ψ are proportional to z^n at X^2. Only $n \geq 0$ mode would be non-singular at the northern hemisphere and $n \leq 0$ at the southern hemisphere and the eigen modes are non-normalizable.

One cannot glue these modes together at equator unless one assumes the behavior z^n, $n \geq 0$, on the northern hemisphere and $\overline{z}^{-n}$, $n \geq 0$, on the southern hemisphere. The identification $\Psi_+(z) = \Psi_-^\dagger(z)$ ($z \rightarrow \overline{z}$ in Hermitian conjugation) at equator would state that "positive energy" particle at the northern hemisphere corresponds to a negative energy antiparticle at the southern hemisphere. The assumption that energy momentum generators $T_+(z)$ and $T_-(z)$ are related in the same manner at equator gives $L_n = L_{-n}^\dagger$ as required. Second candidate for the basis are spherical harmonics which are eigenstates of $L_0 - \overline{L_0}$ defining angular momentum operator L_z but they do not possess well defined conformal weights.

The radial time evolution for the Kac-Moody generators does not commute with L_0 whereas well-defined radial conformal weights are possible. This would support the view that the conformal weight associated with X^2 degrees of freedom does not contribute to the mass squared. If this picture is correct, L_0 would label different SKM representations and play a role similar to that in conformal field theories for critical systems.

5.3.7 How to interpret the overall sign of conformal weight?

The overall sign of conformal weight can be changed by replacing r with $1/r$ and the region $r > r_0$ with $r < r_0$ of $\delta H_\pm$ or of partonic 3-surface. The earlier idea that the conformal weights associated with the super-conformal algebras assignable to $\delta H_\pm$ and to light-like partonic 3-surfaces have opposite signs would allow to construct representations of super-canonical algebra by constructing a tachyonic ground state using super-canonical generators and its excitations using super Super-Kac Moody generators as in super string models.

There is however an objection against this idea. The partons at $\delta H_\pm$ would have a finite distance from the tip of the light cone at all points where they correspond to non-vacuum extremals, so that the phase transitions changing the value of Planck constant should always occur via vacuum extremals. This would not allow the leakage of Kähler magnetic flux between different sectors of imbedding space. The cautious conclusion is that at least in the super-canonical sector both $r > r_0$ and $r < r_0$ sectors related by the conformal transformation $r \rightarrow 1/r$ must be allowed and correspond to positive and negative values for the radial super-conformal weights.

In zero energy ontology particle reactions correspond to zero energy states which at space-time level carry positive energy particles at the end of world in geometric past and negative energy particles at the end of world in the geometric future. Also conformal weights are of opposite sign so that vanishing of the net conformal weights holds true only for zero energy states in accordance with the spirit of p-adic mass calculations. If the states of geometric past correspond to positive (negative) super Kac-Moody (super-canonical) conformal weights, the scattering could be regarded as a process leading from the region $r > r_0$ at δM^4_+0 to the region $r < r_0$ at δM^4_-. At partonic level the incoming partons would correspond to the region $r < r_0$ and outgoing partons to the region $r > r_0$, which conforms with the idea that the final state can partons can be arbitrary far in the geometric future.

In certain sense this picture would reproduce big ban-big crush picture at the level of super-canonical algebra. $r < r_0$ means that partons can be arbitrarily near to the tip of δM^4_- representing the final singularity whereas $r > r_0$ for δM^4_+ would be the counterpart for big bang.

5.3.8 Absolute extremum property for Kähler action implies dynamical Kac-Moody and super conformal symmetries

The identification of the criterion selecting the preferred extremal of Kähler action defining space-time surface as a counterpart of Bohr orbit has been a long standing challenge. The first guess was that an absolute minimum is in question. The number theoretic theoretic picture, in particular $HO - H$ duality [E2] resolves the problem by assigning to each point of X^4 a preferred plane M^2, which also fixes the boundary conditions for the field equations at light-like partonic 3-surfaces. The still open questions are whether the H images of hyper-quaternionic 4-surfaces of $HO = M^8$ are indeed extremals of Kähler action and whether these preferred extremals satisfy absolute extremum property. Be as it may, the following argument suggests that absolute extremum property gives rise to additional symmetries.

The extremal property for Kähler action with respect to variations of time derivatives of initial values keeping h^k fixed at X^3 implies the existence of an infinite number of conserved charges assignable to the small deformations of the extremum and to H isometries. Also infinite number of local conserved super currents assignable to second variations and to covariantly constant right handed neutrino are implied. The corresponding conserved charges vanish so that the interpretation as dynamical gauge symmetries is appropriate. This result provides strong support that the local extremal property is indeed consistent with the almost-topological QFT property at parton level.

The starting point are field equations for the second variations. If the action contain only derivatives of field variables one obtains for the small deformations δh^k of a given extremal

$$\begin{aligned} \partial_\alpha J_k^\alpha &= 0 \ , \\ J_k^\alpha &= \frac{\partial^2 L}{\partial h_\alpha^k \partial h_\beta^l} \delta h_\beta^l \ , \end{aligned} \tag{70}$$

where h_α^k denotes the partial derivative $\partial_\alpha h^k$. A simple example is the action for massless scalar field in which case conservation law reduces to the conservation of the current defined by the gradient of the scalar field. The addition of mass term spoils this conservation law.

If the action is general coordinate invariant, the field equations read as

$$D_\alpha J^{\alpha,k} = 0 \tag{71}$$

where D_α is now covariant derivative and index raising is achieved using the metric of the imbedding space.

The field equations for the second variation state the vanishing of a covariant divergence and one obtains conserved currents by the contraction this equation with covariantly constant Killing vector fields j_A^k of M^4 translations which means that second variations define the analog of a local gauge algebra in M^4 degrees of freedom.

$$\begin{aligned} \partial_\alpha J_n^{A,\alpha} &= 0 \ , \\ J_n^{A,\alpha} &= J_n^{\alpha,k} j_k^A \ . \end{aligned} \tag{72}$$

Conservation for Killing vector fields reduces to the contraction of a symmetric tensor with $D_k j_l$ which vanishes. The reason is that action depends on induced metric and Kähler form only.

Also covariantly constant right handed neutrino spinors Ψ_R define a collection of conserved super currents associated with small deformations at extremum

$$\begin{aligned} J_n^\alpha &= J_n^{\alpha,k} \gamma_k \Psi_R \ , \\ &. \end{aligned} \tag{73}$$

Second variation gives also a total divergence term which gives contributions at two 3-dimensional ends of the space-time sheet as the difference

$$
\begin{aligned}
Q_n(X^3_f) - Q_n(X^3) &= 0 \ , \\
Q_n(Y^3) &= \int_{Y^3} d^3x J_n \ , \quad J_n = J^{tk} h_{kl} \delta h^l_n \ .
\end{aligned}
\tag{74}
$$

The contribution of the fixed end X^3 vanishes. For the extremum with respect to the variations of the time derivatives $\partial_t h^k$ at X^3 the total variation must vanish. This implies that the charges Q_n defined by second variations are identically vanishing

$$
Q_n(X^3_f) = \int_{X^3_f} J_n = 0 \ . \tag{75}
$$

Since the second end can be chosen arbitrarily, one obtains an infinite number of conditions analogous to the Virasoro conditions. The analogs of unbroken loop group symmetry for H isometries and unbroken local super symmetry generated by right handed neutrino result. Thus extremal property is a necessary condition for the realization of the gauge symmetries present at partonic level also at the level of the space-time surface. The breaking of super-symmetries could perhaps be understood in terms of the breaking of these symmetries for light-like partonic 3-surfaces which are not extremals of Chern-Simons action.

5.4 Large $N = 4$ SCA is the natural option

The arguments below support the view that "large" $N = 4$ SCA is the natural algebra in TGD framework.

5.4.1 How $N = 4$ super-conformal invariance emerges from the parton level formulation of quantum TGD?

The discovery of the formulation of TGD as a $N = 4$ almost topological super-conformal QFT with light-like partonic 3-surfaces identified as basic dynamical objects led to the final understanding of super-conformal symmetries and their breaking. $N = 4$ super-conformal algebra corresponds to the maximal algebra with $SU(2) \times U(2)$ Kac-Moody algebra as inherent fermionic Kac-Moody algebra having interpretation in terms of rotations and electro-weak gauge group.

5.4.2 Large $N = 4$ SCA algebra

Large $N = 4$ super-conformal symmetry with $SU(2)_+ \times SU(2)_- \times U(1)$ inherent Kac-Moody symmetry seems to define the fundamental partonic super-conformal symmetry in TGD framework. In the case of SKM algebra the groups would act on induced spinors with $SU(2)_+$ representing spin rotations and $SU(2)_- \times U(1) = U(2)_{ew}$ electro-weak rotations. In super-canonical sector the action would be geometric: $SU(2)_+$ would act as rotations on light-cone boundary and $U(2)$ as color rotations leaving invariant a preferred CP_2 point.

A concise discussion of this symmetry with explicit expressions of commutation and anticommutation relations can be found in [20]. The representations of SCA are characterized by three central extension parameters for Kac-Moody algebras but only two of them are independent and given by

$$
\begin{aligned}
k_\pm &\equiv k(SU(2)_\pm) \ , \\
k_1 &\equiv k(U(1)) = k_+ + k_- \ .
\end{aligned}
\tag{76}
$$

The central extension parameter c is given as

$$c = \frac{6k_+k_-}{k_+ + k_-} . \quad (77)$$

and is rational valued as required.

A much studied $N = 4$ SCA corresponds to the special case

$$\begin{aligned} k_- &= 1 \ , \ k_+ = k+1 \ , \ k_1 = k+2 \ , \\ c &= \frac{6(k+1)}{k+2} . \end{aligned} \quad (78)$$

$c = 0$ would correspond to $k_+ = 0, k_- = 1, k_1 = 1$. Central extension would be trivial in rotational degrees of freedom but non-trivial in $U(2)_{ew}$. For $k_+ > 0$ one has $k_1 = k_+ + k_- \neq k_+$. A possible interpretation is in terms of electro-weak symmetry breaking with $k_+ > 0$ signalling for the massivation of electro-weak gauge bosons.

An interpretation consistent with the general vision about the quantization of Planck constants is that k_+ and k_- relate directly to the integers n_a and n_b characterizing the values of $M^4_\pm$ and CP_2 Planck constants via the formulas $n_a = k_+ + 2$ and $n_b = k_- + 2$. This would require $k_\pm \geq 1$ for G_i a finite subgroup of $SU(2)$ ("anyonic" phases). In stringy phases with $G_i = SU(2)$ for $i = a$ or $i = b$ or for both, k_i could also vanish so that also $n_i = 2$ corresponding to A_2 ADE diagram and $SU(2)$ Kac-Moody algebra becomes possible. In the super-canonical sector $k_+ = 0$ would mean massless gluons and $k_- = k_1$ that $U(2) \subset SU(3)$ and possibly entire $SU(3)$ represents an unbroken symmetry.

5.4.3 About breaking of large $N = 4$ SCA

Partonic formulation predicts that large $N = 4$ SCA is a broken symmetry, and the first guess is that breaking could be thought to occur via several steps. First a "small" $N = 4$ SCA with Kac-Moody group $SU(2) \times U(1)$ would result. The next step would lead to $N = 2$ SCA and the final step to $N = 0$ SCA. Several symmetry breaking scenarios are possible.

1. $SU(2) \times U(1)$ could correspond to electro-weak gauge group such that rotational degrees of freedom are frozen dynamically by the massivation of the corresponding excitations. This interpretation could apply in stringy phase: for cosmic strings rotational excitations are indeed hyper-massive.

2. The interpretation of $SU(2)$ as spin rotation group and $U(1)$ as electromagnetic gauge group conforms with the general vision about electroweak symmetry breaking in non-stringy phase. The interpretation certainly makes sense for covariantly constant right handed neutrinos for which spin direction is free.

The next step in the symmetry breaking sequence would be $N = 2$ SCA with $U(1) \subset SU(2) \times U(2)$ sub-algebra. The interpretation could be as electro-weak symmetry breaking in the stringy sector (cosmic strings) so that $U(1)$ would correspond to em charge or possibly weak isospin.

5.4.4 Relationship to super-strings and M-theory

The (4,4) signature characterizing $N = 4$ SCA topological field theory is not a problem since in TGD framework the target space becomes a fictive concept defined by the Cartan algebra. Both $M^4 \times CP_2$ decomposition of the imbedding space and space-time dimension are crucial for the

$2+2+2+2$ structure of the Cartan algebra, which together with the notions of the configuration space and generalized coset representation formed from super Kac-Moody and super-canonical algebras guarantees $N = 4$ super-conformal invariance.

Including the 2 gauge degrees of freedom associated with M^2 factor of $M^4 = M^2 \times E^2$ the critical dimension becomes $D = 10$ and and including the radial degree of light-cone boundary the critical dimension becomes $D = 11$ of M-theory. Hence the fictive target space associated with the vertex operator construction corresponds to a flat background of super-string theory and flat background of M-theory with one light-like direction. From TGD point view the difficulties of these approaches are due to the un-necessary assumption that the fictive target space defined by the Cartan algebra corresponds to the physical imbedding space. The flatness of the fictive target space forces to introduce the notion of spontaneous compactification and dynamical imbedding space and this in turn leads to the notion of landscape.

5.4.5 Questions

A priori one can consider 3 different options concerning the identification of quarks and leptons.

1. Could also quarks define $N = 4$ superconformal symmetry?

One can ask, whether the construction could be extended by allowing H-spinors of opposite chirality to have leptonic quantum numbers so that free quarks would have integer charge. The construction does not work. The direct sum of $N = 4$ SCAs can be realized but $N = 8$ algebra would require SO(7) rotations mixing states with different fermion numbers: for $N = 4$ SCA this is not needed. Furthermore, only $N < 4$ super-conformal algebras allow an associative realization and $N = 8$ non-associative realization discovered first by Englert exists only at the limit when Kac-Moody central extension parameter k becomes infinite (this corresponds to a critical phase formally and $q = 1$ Jones inclusion). This is not enough for the purposes of TGD and number theoretic vision strongly supports "small" $N = 4$ SCA.

2. Integer charged leptons and fractionally charged quarks?

Second option would be leptons and fractionally charged quarks with $N = 4$ SCA in leptonic sector. It is indeed possible to realize both quark and lepton spinors as super generators of super affinized quaternion algebras (a generalization of super-Kac Moody algebras) so that the fundamental spectrum generating algebra is obtained. Quarks with their natural charges can appear only in $n = 3, k = 1$ phase together with fractionally charged leptons. Leptons in this phase would have strong interactions with quarks. The penetration of lepton into hadron would give rise to this kind of situation. Leptons can indeed move in triality 1 states since 3-fold covering of CP_2 points by M^4 points means that 3 full rotations for the phase angle of CP_2 complex coordinate corresponds to single 2π rotation for M^4 point.

Hadron like states would correspond to the lowest possible Jones inclusion characterized by n=3 and the subgroup A_2 (Z_3) of SU(2). The work with quantization of Planck constant had already earlier led to the realization that ADE Dynkin diagrams assignable to Jones inclusions indeed correspond to gauge groups [A8]: in particular, A_2 corresponds to color group SU(3). Infinite hierarchy of hadron like states with $n = 3, 4, 5...$ quarks or leptons is predicted corresponding to the hierarchy of Jones inclusions, and I have already earlier proposed that this hierarchy should be crucial for the understanding of living matter [M3]. For states containing quarks n would be multiple of 3.

One can understand color confinement of quarks as absolute if one accepts the generalization of the notion of imbedding space forced by the quantization of Planck constant. Ordinary gauge bosons come in two varieties depending on whether their couplings are H-vectorial or H-axial. Strong interactions inside hadrons could be also interpreted as H-axial electro-weak interactions which have become strong (presumably because corresponding gauge bosons are massless) as is clear from the fact that arbitrary high n-point functions are non-vanishing in the phases with $q \neq 1$. Already earlier the so called HO-H duality inspired by the number theoretical vision [E2] led to the same proposal but for ordinary electro-weak interactions which can be also imagined in the scenario in which only leptons are fundamental fermions.

3. Quarks as fractionally charged leptons?

For the third option only leptons would appear as free fermions. The dramatic prediction would be that quarks would be fractionally charged leptons. It is however not clear whether proton can decay to positron plus something (recall the original erratic interpretation of positron as proton by Dirac!): lepton number fractionization meaning that baryon consists of three positrons with fermion number 1/3 might allow this. If not, then only the interactions mediated by the exchanges of gauge bosons (vanishing lepton number is essential) between worlds corresponding to different Jones inclusions are possible and proton would be stable.

There are however also objections. In particular, the resulting states are not identical with color partial waves assignable to quarks and the nice predictions of p-adic mass calculations for quark and hadron masses might be lost. Hence the cautious conclusion is that the original scenario with integer charged quarks predicting confinement automatically is the correct one.

6 Color degrees of freedom

The ground states for the Super Virasoro representations correspond to spinor harmonics in $M^4 \times CP_2$ characterized by momentum and color quantum numbers. The correlation between color and electro-weak quantum numbers is wrong for the spinor harmonics and these states would be also hyper-massive. The super-canonical generators allow to build color triplet states having negative vacuum conformal weights, and their values are such that p-adic massivation is consistent with the predictions of the earlier model differing from the recent one in the quark sector. In the following the construction and the properties of the color partial waves for fermions and bosons are considered. The discussion follows closely to the discussion of [18].

6.1 SKM algebra and counterpart of Super Virasoro conditions

The geometric part of SKM algebra is defined as an algebra respecting the light-likeness of the partonic 3-surface. It consists of X^3-local conformal transformations of $M^4_\pm$ and SU(3)-local SU(3) rotations. The requirement that generators have well defined radial conformal weight with respect to the lightlike coordinate r of X^3 restricts M^4 conformal transformations to the group $SO(3) \times E^3$. This involves choice of preferred time coordinate. If the preferred M^4 coordinate is chosen to correspond to a preferred light-like direction in $\delta M^4_\pm$ characterizing the theory, a reduction to $SO(2) \times E^2$ more familiar from string models occurs. The algebra decomposes into a direct sum of sub-algebras mapped to themselves by the Kac-Moody algebra generated by functions depending on r only. SKM algebra contains also $U(2)_{ew}$ Kac-Moody algebra acting as holonomies of CP_2 and having no bosonic counterpart.

p-Adic mass calculations require $N = 5$ sectors of super-conformal algebra. These sectors correspond to the 5 tensor factors for the $SO(3) \times E^3 \times SU(3) \times U(2)_{ew}$ (or $SO(2) \times E^2 \times SU(3) \times U(2)_{ew}$) decomposition of the SKM algebra to gauge symmetries of gravitation, color and electro-weak interactions. These symmetries act on the intersections $X^2 = X^3_l \cap X^7$ of 3-D light like causal determinants (CDs) X^3_l and 7-D light like CDs $X^7 = \delta M^4_+ \times CP_2$. This constraint leaves only the 2 transversal M^4 degrees of freedom since the translations in light like directions associated with X^3_l and δM^4_+ are eliminated.

The algebra differs from the standard one in that super generators $G(z)$ carry lepton and quark numbers are not Hermitian as in super-string models (Majorana conditions are not satisfied). The counterparts of Ramond representations correspond to zero modes of a second quantized spinor field with vanishing radial conformal weight. Non-zero modes with generalized eigenvalues $\lambda = 1/2+iy$, $y = \sum_k n_k y_k$, $n_k \geq 0$, of the modified Dirac operator with $s_k = 1/2 + iy_k$ a zero or Rieman Zeta, define ground states of N-S type super Virasoro representations.

What is new is the imaginary part of conformal weight which means that the arrow of geometric time manifests itself via the sign of the imaginary part y already at elementary particle level. More concretely, positive energy particle propagating to the geometric future is not equivalent with negative energy particle propagating to the geometric past. The strange properties of the phase conjugate provide concrete physical demonstration of this difference. p-Adic mass calculations suggest the interpretation of y in terms of a decay width of the particle.

The Ramond or N-S type Virasoro conditions satisfied by the physical states in string model approach are replaced by the formulas expressing mass squared as a conformal weight. The condition is not equivalent with super Virasoro conditions since four-momentum does not appear in super Virasoro generators. It seems possible to assume that the commutator algebra $[SKM, SC]$ and the commutator of $[SKMV, SCV]$ of corresponding Super Virasoro algebras annihilate physical states. This would give rise to the analog of Super Virasoro conditions which could be seen as a Dirac equation in the world of classical worlds.

6.1.1 CP_2 CM degrees of freedom

Important element in the discussion are center of mass degrees of freedom parameterized by imbedding space coordinates. By the effective 2-dimensionality it is indeed possible to assign to partons momenta and color partial waves and they behave effectively as free particles. In fact, the technical problem of the earlier scenario was that it was not possible to assign symmetry transformations acting only on on the boundary components of 3-surface.

One can assign to each eigen state of color quantum numbers a color partial wave in CP_2 degrees of freedom. Thus color quantum numbers are not spin like quantum numbers in TGD framework except effectively in the length scales much longer than CP_2 length scale. The correlation between color partial waves and electro-weak quantum numbers is not physical in general: only the covariantly constant right handed neutrino has vanishing color.

6.1.2 Mass formula, and condition determining the effective string tension

Mass squared eigenvalues are given by

$$M^2 \quad = \quad m^2_{CP_2} + kL_0 \ . \tag{79}$$

The contribution of CP_2 spinor Laplacian to the mass squared operator is in general not integer valued.

The requirement that mass squared spectrum is integer valued for color partial waves possibly representing light states fixes the possible values of k determining the effective string tension modulo integer. The value $k = 1$ is the only possible choice. The earlier choice $k_L = 1$ and $k_q = 2/3$, $k_B = 1$ gave integer conformal weights for the lowest possible color partial waves. The assumption that the total vacuum weight h_{vac} is conserved in particle vertices implied $k_B = 1$.

6.2 General construction of solutions of Dirac operator of H

The construction of the solutions of massless spinor and other d'Alembertians in $M^4_+ \times CP_2$ is based on the following observations.

1. d'Alembertian corresponds to a massless wave equation $M^4 \times CP_2$ and thus Kaluza-Klein picture applies, that is M^4_+ mass is generated from the momentum in CP_2 degrees of freedom. This implies mass quantization:

$$M^2 = M_n^2 ~,$$

where M_n^2 are eigenvalues of CP_2 Laplacian. Here of course, ordinary field theory is considered. In TGD the vacuum weight changes mass squared spectrum.

2. In order to get a respectable spinor structure in CP_2 one must couple CP_2 spinors to an odd integer multiple of the Kähler potential. Leptons and quarks correspond to $n = 3$ and $n = 1$ couplings respectively. The spectrum of the electromagnetic charge comes out correctly for leptons and quarks.

3. Right handed neutrino is covariantly constant solution of CP_2 Laplacian for $n = 3$ coupling to Kähler potential whereas right handed 'electron' corresponds to the covariantly constant solution for $n = -3$. From the covariant constancy it follows that all solutions of the spinor Laplacian are obtained from these two basic solutions by multiplying with an appropriate solution of the scalar Laplacian coupled to Kähler potential with such a coupling that a correct total Kähler charge results. Left handed solutions of spinor Laplacian are obtained simply by multiplying right handed solutions with CP_2 Dirac operator: in this operation the eigenvalues of the mass squared operator are obviously preserved.

4. The remaining task is to solve scalar Laplacian coupled to an arbitrary integer multiple of Kähler potential. This can be achieved by noticing that the solutions of the massive CP_2 Laplacian can be regarded as solutions of S^5 scalar Laplacian. S^5 can indeed be regarded as a circle bundle over CP_2 and massive solutions of CP_2 Laplacian correspond to the solutions of S^5 Laplacian with $exp(is\tau)$ dependence on S^1 coordinate such that s corresponds to the coupling to the Kähler potential:

$$s = n/2 ~.$$

Thus one obtains

$$D_5^2 = (D_\mu - iA_\mu\partial_\tau)(D^\mu - iA^\mu\partial_\tau) + \partial_\tau^2 \qquad (80)$$

so that the eigen values of CP_2 scalar Laplacian are

$$m^2(s) = m_5^2 + s^2 \qquad (81)$$

for the assumed dependence on τ.

5. What remains to do, is to find the spectrum of S^5 Laplacian and this is an easy task. All solutions of S^5 Laplacian can be written as homogenous polynomial functions of C^3 complex coordinates Z^k and their complex conjugates and have a decomposition into the representations of $SU(3)$ acting in natural manner in C^3.

6. The solutions of the scalar Laplacian belong to the representations $(p, p+s)$ for $s \geq 0$ and to the representations $(p+|s|, p)$ of $SU(3)$ for $s \leq 0$. The eigenvalues $m^2(s)$ and degeneracies d are

$$m^2(s) = \frac{2\Lambda}{3}[p^2 + (|s|+2)p + |s|] \ , \ \ p > 0 \ ,$$
$$d = \frac{1}{2}(p+1)(p+|s|+1)(2p+|s|+2) \ . \qquad (82)$$

Λ denotes the 'cosmological constant' of CP_2 ($R_{ij} = \Lambda s_{ij}$).

6.3 Solutions of the leptonic spinor Laplacian

Right handed solutions of the leptonic spinor Laplacian are obtained from the asatz of form

$$\nu_R = \Phi_{s=0}\nu_R^0 \ ,$$

where u_R is covariantly constant right handed neutrino and Φ scalar with vanishing Kähler charge. Right handed 'electron' is obtained from the ansats

$$e_R = \Phi_{s=3}e_R^0 \ ,$$

where e_R^0 is covariantly constant for $n = -3$ coupling to Kähler potential so that scalar function must have Kähler coupling $s = n/2 = 3$ a in order to get a correct Kähler charge. The d'Alembert equation reduces to

$$(D_\mu D^\mu - (1-\epsilon)\Lambda)\Phi = -m^2\Phi \ ,$$
$$\epsilon(\nu) = 1 \ , \quad \epsilon(e) = -1 \ . \qquad (83)$$

The two additional terms correspond to the curvature scalar term and $J_{kl}\Sigma^{kl}$ terms in spinor Laplacian. The latter term is proportional to Kähler coupling and of different sign for ν and e, which explains the presence of the sign factor ϵ in the formula.

Right handed neutrinos correspond to (p,p) states with $p \geq 0$ with mass spectrum

$$m^2(\nu) = \frac{m_1^2}{3}\left[p^2 + 2p\right] \ , \ \ p \geq 0 \ ,$$
$$m_1^2 \equiv 2\Lambda \ . \qquad (84)$$

Right handed 'electrons' correspond to $(p, p+3)$ states with mass spectrum

$$m^2(e) = \frac{m_1^2}{3}\left[p^2 + 5p + 6\right] \ , \ \ p \geq 0 \ . \qquad (85)$$

Left handed solutions are obtained by operating with CP_2 Dirac operator on right handed solutions and have the same mass spectrum and representational content as right handed leptons with one exception: the action of the Dirac operator on the covariantly constant right handed neutrino ($(p = 0, p = 0)$ state) annihilates it.

6.4 Quark spectrum

Quarks correspond to the second conserved H-chirality of H-spinors. The construction of the color partial waves for quarks proceeds along similar lines as for leptons. The Kähler coupling corresponds to $n=1$ (and $s=1/2$) and right handed U type quark corresponds to a right handed neutrino. U quark type solutions are constructed as solutions of form

$$U_R = u_R \Phi_{s==1} \ \ ,$$

where u_R possesses the quantum numbers of covariantly constant right handed neutrino with Kähler charge $n=3$ ($s=3/2$). Hence Φ_s has $s=-1$. For D_R one has

$$D_R = d_r \Phi_{s=2} \ \ .$$

d_R has $s=-3/2$ so that one must have $s=2$. For U_R the representations $(p+1,p)$ with triality one are obtained and $p=0$ corresponds to color triplet. For D_R the representations $(p,p+2)$ are obtained and color triplet is missing from the spectrum ($p=0$ corresponds to $\overline{6}$).

The CP_2 contributions to masses are given by the formula

$$\begin{aligned} m^2(U,p) &= \frac{m_1^2}{3}\left[p^2+3p+2\right] \ \ , \ \ p\geq 0 \ \ , \\ m^2(D,p) &= \frac{m_1^2}{3}\left[p^2+4p+4\right] \ \ , \ \ p\geq 0 \ \ . \end{aligned} \tag{86}$$

Left handed quarks are obtained by applying Dirac operator to right handed quark states and mass formulas and color partial wave spectrum are the same as for right handed quarks.

The color contributions to p-adic mass squared are integer valued if $m_0^2/3$ is taken as a fundamental p-adic unit of mass squared. This choice has an obvious relevance for p-adic mass calculations since canonical identification does not commute with a division by integer. More precisely, the images of number xp in canonical identification has a value of order 1 when x is a non-trivial rational whereas for $x=np$ the value is n/p and extremely is small for physically interesting primes. This choice does not however affect the spectrum of massless states but can affect the spectrum of light states in case of electro-weak gauge bosons.

7 Exotic states

The possibility of exotic states poses a serious problem. The assumption that only free many fermion states are possible eliminates a huge number of exotics and only the degrees of freedom associated with ground states remain. Coset construction implying duality between SCV and $SKMV$ algebras removes a huge number of exotic states but genuinely SC contributions with a vanishing conformal weight are possible. Also other kinds of exotic states are predicted.

7.1 What kind of exotic states one expects

The physical consequences of the exotic light leptons, quarks, and bosons are considered in the chapter devoted to the New Physics [F5]. Here it only suffices to make a short summary. Consider first what kind of exotic particles extended conformal symmetries predict.

1. Massless states are expected to become massive by p-adic thermodynamics meaning that one has superposition of states with Super Kac-Moody conformal weight equal to Super Virasoro conformal weight and annihilated by SKMV and SCV generators G_n,L_n, $n>0$. This condition allows degeneracy since there are many manners to create a ground state with a given angular momentum and color quantum numbers and conformal weight n and annihilated by L_n, $n<0$, by using super-canonical generators. The combinations of super-canonical generators which do not belong to SKM algebra and create singlets in color and

rotational degrees of freedom would be responsible for this degeneracy. The condition that the states in the superposition are annihilated by G_n, L_n, $n > 0$, reduces the number of the massless states.

2. The original expectation that the spectrum has $N = 1$ space-time super-symmetry seems to be wrong. The understanding of the super-conformal symmetries as at parton level allowed to identify partonic super-conformal symmetries in terms of a generalization of large $N = 4$ SCA with Kac-Moody group extended to contain also canonical transformations of $\delta H_{\pm}$. Thus an immense generalization of string model conformal symmetries is in question. This allows to conclude that sparticles in the sense of super Poincare symmetry are certainly absent. This does not affect the mass calculations in any manner and dramatically reduces the number of exotic states.

3. If elementary particles correspond to CP_2 type extremals, one can argue that all massless exotic massless particles can be constructed using colored generators and by color confinement cannot induce macroscopic long range interactions.

4. The possibility that conformal weights have imaginary part expressible as linear combination of imaginary parts of zeros of ζ function associated with the modified Dirac operator satisfying Riemann hypothesis brings in additional richness of structure. A possible interpretation is that the non-vanishing imaginary part allows to distinguish between positive energy particle propagating into geometric future and negative energy propagating to the geometric past. Phase conjugate photons for which dissipation occurs in time reversed direction would be basic examples of this. Dissipation would be visible already in the mathematical description of partons. The imaginary part of the conformal weight might relate directly to the decay rate of the particle or to the length of the time interval separating positive energy particle and corresponding negative energy particle in zero energy ontology where all physical states have vanishing net quantum numbers [C2].

These exotic particles relate to the extended conformal symmetries. There are also other kinds of exotic particles.

1. The existence of fermionic families suggests the existence of higher bosonic families too. If gauge bosons correspond to wormhole contacts, three families would mean that bosons are labelled by pairs (g_i, g_j) of genera associated with wormhole contacts and $U(3)$ dynamical gauge symmetry emerges naturally. The observed gauge bosons would correspond to SU(3) singlets which do not induced genus changing transitions. The new view about particle decay as a branching of partonic 2-surface is consistent with this picture but not the earlier stringy view. Only three fermion families are predicted if $g > 2$ topologies for partonic 2-surfaces correspond to free many-handle states rather than bound states as for $g < 3$ topologies: who this could happen is discussed in [F1].

2. Also p-adically scaled up copies of various particles are possible as well as scaled-up/scaled-down versions of QCD associated with both quarks [F8] and colored leptons [F7]. There is now quite a lot of evidence that neutrino masses depend on environment [37]: this dependence could have an explanation in terms of topological condensation occurring in several p-adic length scales.

3. Dark matter hierarchy based on the spectrum of Planck constants [A9] infinite number of zoomed up copies of ordinary elementary particles with same mass spectrum.

4. Electro-weak doublet Higgs particle would be present in the spectrum and be identifiable as wormhole contact, contrary to the long held beliefs. Also $q - \overline{q}$ bound states of M_{89} hadron

physics such that quark and anti-quark have parallel spins and relative angular momentum $L = 1$ could mimic scalar mesons. The effective couplings of these states to leptons and quarks could mimic the couplings of Higgs boson to some degree. Scalar bound states of heavy quarks are also present in ordinary hadron physics.

7.2 Are S^2 degrees frozen for elementary particles?

As the system approaches CP_2 type extremal, radial waves in $\delta M^4_\pm$ for 2-D partonic surface having 0-dimensional $\delta M^4_\pm$ projection become constant. Hence one might argue that the radial conformal weights vanish for SC. This would however lead to a contradiction since radial conformal weights are absolutely essential for p-adic mass calculations. Parton picture allows to understand what really happens. artons correspond to light-like 3-surfaces correspond to wormhole throats resulting when CP_2 type extremal is glued to the space-time sheet with Minkowskian signature of induced metric so that M^4 projection is necessarily 3-dimensional although metrically 2-D.

One can however consider the possibility that the S^2 degrees of freedom associated with δM^4_+ are essentially frozen at elementary particle level with graviton forming a possible exception. The reason would be simply the extremely small size of wormhole contacts implying that the super-canonical generators are essentially constant in S^2 degrees of freedom. Only color Hamiltonians would generate tachyonic ground states as null states.

7.3 More detailed considerations

The exotic states can emerge both from super-canonical and super Kac-Moody sectors. The tachyonic ground states correspond to null states of super-canonical Super Virasoro representations having negative conformal weight $h < 0$ and satisfying the condition $L_n|h\rangle = 0$, $n < 0$. Massless state is obtained by applying super Hamiltonians and SKM generators to this state. Null state conditions certainly reduce dramatically the number of ground states since this kind of states are possible only for special values of c and h. For instance, in $N = 2$ super-conformal theories only very special rational values of c and h are possible and the number of null states is finite.

7.3.1 First vision

If one assumes that elementary particles correspond to CP_2 type extremals, and that $SO(3)$ Hamiltonians with vanishing conformal weight are "frozen" to a constant at this limit, the predicted exotic massless states would be generated by color Hamiltonians only. This justifies the hope that new macroscopic long range forces are absent in TGD Universe. It will be found that this assumption is not necessary and fails at hadronic space-time sheets.

1. Super-canonical sector. In super-canonical sector S^2 generators are frozen to constant and fermionic generators vanish so that infinite number of generators otherwise giving rise to degeneracy of massless states is eliminated. Color generators appear as pairs of Hamiltonian and its super-partner with an "anomalous" conformal weight determined by the color representation, and due to the breaking of conformal symmetry induced by CP_2 geometry reflecting itself as a massivation of spinor harmonics. Poisson bracket action does not conserve color conformal weights. This can be understood in terms of the breaking of conformal invariance. The ground states with negative conformal weight would be generated by color Hamiltonians and their spartners having same conformal weights. Color confinement suggests that the massless particles generated from these ground states cannot give rise to macroscopic long range forces.

2. SKM generators in NS representation.
N-S sector gives rise to super generators with conformal weight $n+1/2$,$n \geq 0$ since $h = -1/2$

generators are not allowed by the representation used. Therefore the dangerous $n = 0$ operators are absent.

3. Ramond sector of SKM algebra corresponding to $SO(3) \times SU(2)_L \times U(1)$ holonomies.
   $n = 0$ generators are absent in holonomy degrees of freedom. That the right handed neutrino is covariantly constant, is annihilated by charge matrices, and is orthogonal with $\lambda \neq 0$ modes of the modified Dirac operator D, implies that $n = 0$ fermionic generators vanish. Also the covariant constancy of em charge matrix and the anomalous conformal weight $h_c = 2$ of the left-handed electro-weak charge matrices is of importance. Hence no spartners are predicted in $SO(3) \times SU(2)_L \times U(1)$ degrees of freedom.

4. Ramond sector of SKM algebra corresponding to $SO3) \times SU(3)$ isometries.
   i) $n = 0$ bosonic $SO(3) \times SU(3)$ SKM generators act directly as operators $j^{Ar}D_r$ on the Hamiltonians of X^7 appearing in the definitions of configuration space Hamiltonians. In the same manner $j^{Ar}D_r$ transforms $j^{Bk}\Gamma_k$ to $j^{[A,B]k}\Gamma_k$ and does not affect the representation of H_B. Hence the KM algebra corresponding to isometries does not increase the "particle" number defined as the number of X^2 non-local operators in the state nor change the representation of $SO(3) \times SU(3)$.
   ii) Fermionic $SO(3)$ generators have $h_c = 0$ but for $n = 0$ they vanish by the orthogonality of ν_R and $\lambda > 0$ eigen modes of D. Fermionic $SU(3)$ SKM generators have an anomalous conformal weight $h_c = 1$.

The cautious conclusion would be that massless exotics are all created by color Hamiltonians and their spartners subject to the condition that tachyonic ground state is annihilated by SCV and SKMV generators $G_n,\ L_n,\ n < 0$. This might be enough to achieve consistency with the experimental facts since color confinement would restrict the new long range interactions to a finite range.

7.3.2 Improved vision

An objection against the effective absence of rotational degrees of freedom came from the realization that super-canonical degrees of freedom are absolutely essential for the understanding of the hadron mass spectrum [F4, F5].

1. Hadronic space-time sheet labelled $k = 107$ would be a carrier of many-particle state of super-canonical bosons carrying both spin and color quantum numbers. The additivity of the conformal weight implies string mass formula and gives a connection with the hadronic string model. String tension is predicted correctly and the states of the Regge trajectories correspond to many particle states for super-canonical bosons. Hadron masses are predicted with an accuracy better than one per cent.

2. The super-canonical part of the hadron is dark matter in a strict sense of the word and highly analogous to a black hole. This leads a model explaining RHIC events, where black-hole like states would be created in the collisions of heavy Gold nuclei by the fusion of the hadronic space-time sheets involving also the materialization of collision energy to super-canonical matter [34, 35]. The model also explains the re-incarnated Pomeron [33]. The strange cosmic ray events as well as the observation of cosmic rays with energy larger than the limiting energy 5×10^{10} GeV could be understood as resulting when extremely energetic proton has lost its valence quarks (Pomeron) and propagates as a mini black-hole without interactions with microwave background. LHC gives a possibility to test this picture.

3. The realization that neutron star can be regarded as a gigantic hadron leads to a microscopic description of black-holes as super-canonical black-holes and the requirement that horizon

radius equals to Compton length fixes the Planck constant to $\hbar_{gr} = 2GM^2$. This form is a generalization of the gravitational Planck constant appearing in the Bohr quantization of planetary orbits [D6].

To sum up, it seems that all basic ingredients of TGD Universe are present already at the level of the standard physics.

8 Particle massivation

In TGD framework p-adic thermodynamics provides a microscopic theory of particle massivation. The idea is very simple. The mass of the particle results from a thermal mixing of the massless states with CP_2 mass excitations of super-conformal algebra. In p-adic thermodynamics the Boltzmann weight $exp(-E/T)$ does not exist in general and must be replaced with p^{L_0/T_p} which exists for Virasoro generator L_0 if the inverse of the p-adic temperature is integer valued $T_p = 1/n$. The expansion in powers of p converges extremely rapidly for physical values of p, which are rather large. Therefore the three lowest terms in expansion give practically exact results. Thermal massivation does not not necessarily lead to light states and this drops a large number of exotic states from the spectrum of light particles. The partition functions of N-S and Ramond type representations are not changed in TGD framework despite the fact that fermionic super generators carry fermion numbers and are not Hermitian. Thus the practical calculations are relatively straightforward.

In free fermion picture the p-adic thermodynamics in the boson sector is for fermion-antifermion states associated with the two throats of the bosonic wormhole. The question is whether the thermodynamical mass squared is just the sum of the two independent fermionic contributions for Ramond representations or should one use N-S type representation resulting as a tensor product of Ramond representations. The latter option looks more plausible.

The overall conclusion about p-adic mass calculations is that fermionic mass spectrum is predicted in an excellent accuracy but that the thermal masses of the intermediate gauge bosons come 20-30 per cent to large for $T_p = 1$ and are completely negligible for $T_p = 1/2$. This forces to consider very seriously the possibility that thermal contribution to the bosonic mass is negligible and that TGD can, contrary to the original expectations, provide dynamical Higgs field as a fundamental field. The identification of Higgs as wormhole contact would provide this field. The bound state character of the boson states could be responsible for $T_p < 1$.

An alternative but not very plausible manner to understand the massivation of electro-weak gauge bosons is as reflecting the breaking of conformal invariance due to the lacking covariant constancy of the left handed parts of the charge matrices of electro-weak gauge bosons. In the following some aspects of the calculations are discussed: the rather extensive calculations can be found from the three chapters "p-Adic Particle Massivation:...." of [6].

8.1 Partition functions are not changed

One must write Super Virasoro conditions for L_n and *both* G_n and $G_n^\dagger$ rather than for L_n and G_n as in the case of the ordinary Super Virasoro algebra, and it is a priori not at all clear whether the partition functions for the Super Virasoro representations remain unchanged. This requirement is however crucial for the construction to work at all in the fermionic sector, since even the slightest changes for the degeneracies of the excited states can change light state to a state with mass of order m_0 in the p-adic thermodynamics.

8.1.1 Super conformal algebra

Super Virasoro algebra is generated by the bosonic the generators L_n (n is an integer valued index) and by the fermionic generators G_r, where r can be either integer (Ramond) or half odd integer

(NS). G_r creates quark/lepton for $r > 0$ and antiquark/antilepton for $r < 0$. For $r = 0$, G_0 creates lepton and its Hermitian conjugate anti-lepton. The defining commutation and anti-commutation relations are the following:

$$\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n} + \frac{c}{2}m(m^2-1)\delta_{m,-n} \ , \\
[L_m, G_r] &= (\frac{m}{2} - r)G_{m+r} \ , \\
[L_m, G_r^\dagger] &= (\frac{m}{2} - r)G_{m+r}^\dagger \ , \\
\{G_r, G_s^\dagger\} &= 2L_{r+s} + \frac{c}{3}(r^2 - \frac{1}{4})\delta_{m,-n} \ , \\
\{G_r, G_s\} &= 0 \ , \\
\{G_r^\dagger, G_s^\dagger\} &= 0 \ . \qquad (87)
\end{aligned}$$

By the inspection of these relations one finds some results of a great practical importance.

1. For the Ramond algebra G_0, G_1 and their Hermitian conjugates generate the $r \geq 0, n \geq 0$ part of the algebra via anti-commutations and commutations. Therefore all what is needed is to assume that Super Virasoro conditions are satisfied for these generators in case that G_0 and $G_0^\dagger$ annihilate the ground state. Situation changes if the states are *not* annihilated by G_0 and $G_0^\dagger$ since then one must assume the gauge conditions for both L_1, G_1 and $G_1^\dagger$ besides the mass shell conditions associated with G_0 and $G_0^\dagger$, which however do not affect the number of the Super Virasoro excitations but give mass shell condition and constraints on the state in the cm spin degrees of freedom. This will be assumed in the following. Note that for the ordinary Super Virasoro only the gauge conditions for L_1 and G_1 are needed.

2. NS algebra is generated by $G_{1/2}$ and $G_{3/2}$ and their Hermitian conjugates (note that $G_{3/2}$ cannot be expressed as the commutator of L_1 and $G_{1/2}$) so that only the gauge conditions associated with these generators are needed. For the ordinary Super Virasoro only the conditions for $G_{1/2}$ and $G_{3/2}$ are needed.

8.1.2 Conditions guaranteing that partition functions are not changed

The conditions guaranteing the invariance of the partition functions in the transition to the modified algebra must be such that they reduce the number of the excitations and gauge conditions for a given conformal weight to the same number as in the case of the ordinary Super Virasoro.

1. The requirement that physical states are invariant under $G \leftrightarrow G^\dagger$ corresponds to the charge conjugation symmetry and is very natural. As a consequence, the gauge conditions for G and $G^\dagger$ are not independent and their number reduces by a factor of one half and is the same as in the case of the ordinary Super Virasoro.

2. As far as the number of the thermal excitations for a given conformal weight is considered, the only remaining problem are the operators $G_n G_n^\dagger$, which for the ordinary Super Virasoro reduce to $G_n G_n = L_{2n}$ and do not therefore correspond to independent degrees of freedom. In present case this situation is achieved only if one requires

$$(G_n G_n^\dagger - G_n^\dagger G_n)|phys\rangle = 0 \ . \qquad (88)$$

   It is not clear whether this condition must be posed separately or whether it actually follows from the representation of the Super Virasoro algebra automatically.

8.1.3 Partition function for Ramond algebra

Under the assumptions just stated, the partition function for the Ramond states not satisfying any gauge conditions

$$Z(t) \quad = \quad 1 + 2t + 4t^2 + 8t^3 + 14t^4 + ... \ , \tag{89}$$

which is identical to that associated with the ordinary Ramond type Super Virasoro.

For a Super Virasoro representation with $N = 5$ sectors, of main interest in TGD, one has

$$\begin{aligned} Z_N(t) &= Z^{N=5}(t) = \sum D(n)t^n \\ &= 1 + 10t + 60t^2 + 280t^3 + ... \ . \end{aligned} \tag{90}$$

The degeneracies for the states satisfying gauge conditions are given by

$$d(n) \quad = \quad D(n) - 2D(n-1) \ . \tag{91}$$

corresponding to the gauge conditions for L_1 and G_1. Applying this formula one obtains for $N = 5$ sectors

$$d(0) = 1 \ , \quad d(1) = 8 \ , \quad d(2) = 40 \ , \quad d(3) = 160 \ . \tag{92}$$

The lowest order contribution to the p-adic mass squared is determined by the ratio

$$r(n) = \frac{D(n+1)}{D(n)} \ ,$$

where the value of n depends on the effective vacuum weight of the ground state fermion. Light state is obtained only provided the ratio is integer. The remarkable result is that for lowest lying states the ratio is integer and given by

$$r(1) = 8 \ , \quad r(2) = 5 \ , \quad r(3) = 4 \ . \tag{93}$$

It turns out that $r(2) = 5$ gives the best possible lowest order prediction for the charged lepton masses and in this manner one ends up with the condition $h_{vac} = -3$ for the tachyonic vacuum weight of Super Virasoro.

8.1.4 Partition function for NS algebra

For NS representations the calculation of the degeneracies of the physical states reduces to the calculation of the partition function for a single particle Super Virasoro

$$Z_{NS}(t) \quad = \quad \sum_n z(n/2)t^{n/2} \ . \tag{94}$$

Here $z(n/2)$ gives the number of Super Virasoro generators having conformal weight $n/2$. For a state with N active sectors (the sectors with a non-vanishing weight for a given ground state) the degeneracies can be read from the N-particle partition function expressible as

$$Z_N(t) \quad = \quad Z^N(t) \ . \tag{95}$$

Single particle partition function is given by the expression

$$Z(t) \quad = \quad 1 + t^{1/2} + t + 2t^{3/2} + 3t^2 + 4t^{5/2} + 5t^3 + ... \quad . \tag{96}$$

Using this representation it is an easy task to calculate the degeneracies for the operators of conformal weight Δ acting on a state having N active sectors.

One can also derive explicit formulas for the degeneracies and calculation gives

$$\begin{array}{ll} D(0,N) = 1 \ , & D(1/2,N) = N \ , \\ D(1,N) = \frac{N(N+1)}{2} \ , & D(3/2,N) = \frac{N}{6}(N^2+3N+8) \ , \\ D(2,N) = \frac{N}{2}(N^2+2N+3) \ , & D(5/2,N) = 9N(N-1) \ , \\ D(3,N) = 12N(N-1) + 2N(N-1) & . \end{array} \tag{97}$$

as a function of the conformal weight $\Delta = 0, 1/2, ..., 3$.

The number of states satisfying Super Virasoro gauge conditions created by the operators of a conformal weight Δ, when the number of the active sectors is N, is given by

$$d(\Delta,N) \quad = \quad D(\Delta,N) - D(\Delta - 1/2,N) - D(\Delta - 3/2,N) \ . \tag{98}$$

The expression derives from the observation that the physical states satisfying gauge conditions for $G^{1/2}$, $G^{3/2}$ satisfy the conditions for all Super Virasoro generators. For $T_p = 1$ light bosons correspond to the integer values of $d(\Delta+1,N)/d(\Delta,N)$ in case that massless states correspond to thermal excitations of conformal weight Δ: they are obtained for $\Delta = 0$ only (massless ground state). This is what is required since the thermal degeneracy of the light boson ground state would imply a corresponding factor in the energy density of the black body radiation at very high temperatures. For the physically most interesting nontrivial case with $N = 2$ two active sectors the degeneracies are

$$d(0,2) = 1 \ , \quad d(1,2) = 1 \ , \quad d(2,2) = 3 \ , \quad d(3,2) = 4 \ . \tag{99}$$

8.2 Fundamental length and mass scales

The basic difference between quantum TGD and super-string models is that the size of CP_2 is not of order Planck length but much larger: of order 10^4 Planck lengths. This conclusion is forced by several consistency arguments, the mass scale of electron, and by the cosmological data allowing to fix the string tension of the cosmic strings which are basic structures in TGD inspired cosmology.

8.2.1 The relationship between CP_2 radius and fundamental p-adic length scale

One can relate CP_2 'cosmological constant' to the p-adic mass scale: for $k_L = 1$ one has

$$m_0^2 \quad = \quad \frac{m_1^2}{k_L} = m_1^2 = 2\Lambda \ . \tag{100}$$

$k_L = 1$ results also by requiring that p-adic thermodynamics leaves charged leptons light and leads to optimal lowest order prediction for the charged lepton masses. Λ denotes the 'cosmological constant' of CP_2 (CP_2 satisfies Einstein equations $G^{\alpha\beta} = \Lambda g^{\alpha\beta}$ with cosmological term).

The real counterpart of the p-adic thermal expectation for the mass squared is sensitive to the choice of the unit of p-adic mass squared which is by definition mapped as such to the real unit in canonical identification. Thus an important factor in the p-adic mass calculations is the correct identification of the p-adic mass squared scale, which corresponds to the mass squared unit and hence to the unit of the p-adic numbers. This choice does not affect the spectrum of massless states but can affect the spectrum of light states in case of intermediate gauge bosons.

1. For the choice

$$M^2 = m_0^2 \leftrightarrow 1 \qquad (101)$$

the spectrum of L_0 is integer valued.

2. The requirement that all sufficiently small mass squared values for the color partial waves are mapped to real integers, would fix the value of p-adic mass squared unit to

$$M^2 = \frac{m_0^2}{3} \leftrightarrow 1 \ . \qquad (102)$$

For this choice the spectrum of L_0 comes in multiples of 3 and it is possible to have a first order contribution to the mass which cannot be of thermal origin (say $m^2 = p$). This indeed seems to happen for electro-weak gauge bosons.

p-Adic mass calculations [F3] allow to relate m_0 to electron mass and to Planck mass by the formula

$$\begin{aligned} \frac{m_0}{m_{Pl}} &= \frac{1}{\sqrt{5+Y_e}} \times 2^{127/2} \times \frac{m_e}{m_{Pl}} \ , \\ m_{Pl} &= \frac{1}{\sqrt{G}} \ . \end{aligned} \qquad (103)$$

For $Y_e = 0$ this gives $m_0 = .2437 \times 10^{-3} m_{Pl}$.

This means that CP_2 radius R defined by the length $L = 2\pi R$ of CP_2 geodesic is roughly 10^4 times the Planck length. More precisely, using the relationship

$$\Lambda = \frac{3}{2R^2} = M^2 = m_0^2 \ ,$$

one obtains for

$$L = 2\pi R = 2\pi \sqrt{\frac{3}{2}} \frac{1}{m_0} \simeq 3.1167 \times 10^4 \sqrt{G} \ \text{for} \ Y_e = 0 \ . \qquad (104)$$

The result came as a surprise: the first belief was that CP_2 radius is of order Planck length. It has however turned out that the new identification solved elegantly some long standing problems of TGD.

Y_e	0	.5	.7798
$(m_0/m_{Pl})10^3$	.2437	.2323	.2266
$K_R \times 10^{-7}$	2.5262	2.7788	2.9202
$(L_R/\sqrt{G}) \times 10^{-4}$	3.1580	3.3122	3.3954
$K \times 10^{-7}$	2.4606	2.4606	2.4606
$(L/\sqrt{G}) \times 10^{-4}$	3.1167	3.1167	3.1167
K_R/K	1.0267	1.1293	1.1868

Table 1. Table gives the values of the ratio $K_R = R^2/G$ and CP_2 geodesic length $L = 2\pi R$ for $Y_e \in \{0, 0.5, 0.7798\}$. Also the ratio of K_R/K, where $K = 2\times 3\times 5\times 7\times 11\times 13\times 17\times 19\times 23\times 2^{-3} * (15/17)$ is rational number producing R^2/G approximately is given.

The value of top quark mass favors $Y_e = 0$ and $Y_e = .5$ is largest value of Y_e marginally consistent with the limits on the value of top quark mass.

8.2.2 CP_2 radius as the fundamental p-adic length scale

The identification of CP_2 radius as the fundamental p-adic length scale is forced by the Super Virasoro invariance. The pleasant surprise was that the identification of the CP_2 size as the fundamental p-adic length scale rather than Planck length solved many long standing problems of older TGD.

1. The earliest formulation predicted cosmic strings with a string tension larger than the critical value giving the angle deficit 2π in Einstein's equations and thus excluded by General Relativity. The corrected value of CP_2 radius predicts the value k/G for the cosmic string tension with k in the range 10^{-7} $-$ 10^{-6} as required by the TGD inspired model for the galaxy formation solving the galactic dark matter problem.

2. In the earlier formulation there was no idea as how to derive the p-adic length scale $L \sim 10^4\sqrt{G}$ from the basic theory. Now this problem becomes trivial and one has to predict gravitational constant in terms of the p-adic length scale. This follows in principle as a prediction of quantum TGD. In fact, one can deduce G in terms of the p-adic length scale and the action exponential associated with the CP_2 extremal and gets a correct value if α_K approaches fine structure constant at electron length scale (due to the fact that electromagnetic field equals to the Kähler field if Z^0 field vanishes).

   Besides this, one obtains a precise prediction for the dependence of the Kähler coupling strength on the p-adic length scale by requiring that the gravitational coupling does not depend on the p-adic length scale. p-Adic prime p in turn has a nice physical interpretation: the critical value of α_K is same for the zero modes with given p. As already found, the construction of graviton state allows to understand the small value of the gravitational constant in terms of a de-coherence caused by multi-p fractality reducing the value of the gravitational constant from L_p^2 to G.

3. p-Adic length scale is also the length scale at which super-symmetry should be restored in standard super-symmetric theories. In TGD this scale corresponds to the transition to Euclidian field theory for CP_2 type extremals. There are strong reasons to believe that sparticles are however absent and that super-symmetry is present only in the sense that super-generators have complex conformal weights with $Re(h) = \pm 1/2$ rather than $h = 0$. The action of this super-symmetry changes the mass of the state by an amount of order CP_2 mass.

8.3 Spectrum of elementary particles

The assumption that $k = 1$ holds true for all particles forces to modify the earlier construction of quark states. This turns out to be possible without affecting the p-adic mass calculations whose outcome depend in an essential manner on the ground state conformal weights h_{gr} of the fermions (which can be negative).

8.3.1 Leptonic spectrum

For $k = 1$ the leptonic mass squared is integer valued in units of m_0^2 only for the states satisfying

$$p \; mod \; 3 \neq 2 \quad .$$

Only these representations can give rise to massless states. Neutrinos correspond to (p,p) representations with $p \geq 1$ whereas charged leptons correspond to $(p,p+3)$ representations. The earlier mass calculations demonstrate that leptonic masses can be understood if the ground state conformal weight is $h_{gr} = -1$ for charged leptons and $h_{gr} = -2$ for neutrinos.

The contribution of color partial wave to conformal weight is $h_c = (p^2+2p)/3$, $p \geq 1$, for neutrinos and $p = 1$ gives $h_c = 1$ (octet). For charged leptons $h_c = (p^2+5p+6)/3$ gives $h_c = 2$ for $p = 0$ (decuplet). In both cases super-canonical operator O must have a net conformal weight $h_{sc} = -3$ to produce a correct conformal weight for the ground state. p-adic considerations suggests the use of operators O with super-canonical conformal weight $z = -1/2 - i\sum n_k y_k$, where $s_k = 1/2 + iy_k$ corresponds to zero of Riemann ζ. If the operators in question are color Hamiltonians in octet representation net super-canonical conformal weight $h_{sc} = -3$ results. The tensor product of two octets with conjugate super-canonical conformal weights contains both octet and decuplet so that singlets are obtained. What strengthens the hopes that the construction is not adhoc is that the same operator appears in the construction of quark states too.

Right handed neutrino remains essentially massless. $p = 0$ right handed neutrino does not however generate $N = 1$ space-time (or rather, imbedding space) super symmetry so that no sparticles are predicted. The breaking of the electro-weak symmetry at the level of the masses comes out basically from the anomalous color electro-weak correlation for the Kaluza-Klein partial waves implying that the weights for the ground states of the fermions depend on the electromagnetic charge of the fermion. Interestingly, TGD predicts leptohadron physics based on color excitations of leptons and color bound states of these excitations could correspond topologically condensed on string like objects but not fundamental string like objects.

8.3.2 Spectrum of quarks

Earlier arguments [F4] related to a model of CKM matrix as a rational unitary matrix suggested that the string tension parameter k is different for quarks, leptons, and bosons. The basic mass formula read as

$$M^2 = m^2_{CP_2} + kL_0 \quad .$$

The values of k were $k_q = 2/3$ and $k_L = k_B = 1$. The general theory however predicts that $k = 1$ for all particles.

1. By earlier mass calculations and construction of CKM matrix the ground state conformal weights of U and D type quarks must be $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$. The formulas for the eigenvalues of CP_2 spinor Laplacian imply that if m_0^2 is used as unit, color conformal weight $h_c \equiv m^2_{CP_2}$ is integer for $p \; mod \; = \pm 1$ for U type quark belonging to $(p+1,p)$ type representation and obeying $h_c(U) = (p^2+3p+2)/3$ and for $p \; mod \; 3 = 1$ for D type quark belonging $(p,p+2)$ type representation and obeying $h_c(D) = (p^2+4p+4)/3$. Only these states can be massless since color Hamiltonians have integer valued conformal weights.

2. In the recent case $p = 1$ states correspond to $h_c(U) = 2$ and $h_c(D) = 3$. $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$ reproduce the previous results for quark masses required by the construction of CKM matrix. This forces the super-canonical operator O to compensate the anomalous color to have a net conformal weight $h_{sc} = -3$ just as in the leptonic case. The facts that the values of p are minimal for spinor harmonics and the super-canonical operator is same for both quarks and leptons suggest that the construction is not had hoc. The real justification would come from the demonstration that $h_{sc} = -3$ defines null state for SCV: this would also explain why h_{sc} would be same for all fermions.

3. It would seem that the tensor product of the spinor harmonic of quarks (as also leptons) with Hamiltonians gives rise to a large number of exotic colored states which have same thermodynamical mass as ordinary quarks (and leptons). Why these states have smaller values of p-adic prime that ordinary quarks and leptons, remains a challenge for the theory. Note that the decay widths of intermediate gauge bosons pose strong restrictions on the possible color excitations of quarks. On the other hand, the large number of fermionic color exotics can spoil the asymptotic freedom, and it is possible to have and entire p-adic length scale hierarchy of QCDs existing only in a finite length scale range without affecting the decay widths of gauge bosons.

The following table summarizes the color conformal weights and super-canonical vacuum conformal weights for the elementary particles.

	L	ν_L	U	D	W	γ, G, g
h_{vac}	-3	-3	-3	-3	-2	0
h_c	2	1	2	3	2	0

Table 2. The values of the parameters h_{vac} and h_c assuming that $k = 1$. The value of $h_{vac} \leq -h_c$ is determined from the requirement that p-adic mass calculations give best possible fit to the mass spectrum.

8.3.3 Photon, graviton and gluon

For photon, gluon and graviton the conformal weight of the $p = 0$ ground state is $h_{gr} = h_{vac} = 0$. The crucial condition is that $h = 0$ ground state is non-degenerate: otherwise one would obtain several physically more or less identical photons and this would be seen in the spectrum of black-body radiation. This occurs if one can construct several ground states not expressible in terms of the action of the Super Virasoro generators.

The only possibility to get exactly massless states in thermal sense is to have $\Delta = 0$ state with one active sector so that NS thermodynamics becomes trivial due to the absence of the thermodynamical excitations satisfying the gauge conditions. For neutral gauge bosons this is indeed achieved. For $T_p = 1/2$, which is required by the mass spectrum of intermediate gauge bosons, the thermal contribution to the mass squared is however extremely small even for W boson.

8.4 p-Adic thermodynamics does not explain the masses of intermediate gauge bosons

The requirement that the electron-intermediate gauge boson mass ratios are sensible, serves as a stringent test for the hypothesis that intermediate gauge boson masses result from the p-adic thermodynamics. It seems that the only possible option is that the parameter k has same value for both bosons, leptons, and quarks:

$$k_B = k_L = k_q = 1 \quad .$$

In this case all gauge bosons have $D(0) = 1$ and there are good changes to obtain boson masses correctly. $k = 1$ together with $T_p = 1$ implies that the thermal masses of very many boson states are extremely heavy so that the spectrum of the boson exotics is reduced drastically. For $T_p = 1/2$ the thermal contribution to the mass squared is completely negligible.

Contrary to the original optimistic beliefs based on calculational error, it seems however impossible to predict W/e and Z/e mass ratios correctly in p-adic thermodynamics scenario. Although the errors are of order 20-30 percent, they are enough to exclude p-adic thermodynamics explanation for the massivation of gauge bosons.

1. The thermal mass squared for a boson state with N active sectors (non-vanishing vacuum weight) is determined by the partition function for the tensor product of N NS type Super Virasoro algebras. The degeneracies of the excited states as a function of N and the weight Δ of the operator creating the massless state are given in the table below.

2. Both W and Z must correspond to $N = 2$ active Super Virasoro sectors for which $D(1) = 1$ and $D(2) = 3$ so that (using the formulas of p-adic thermodynamics [6, F3]) the thermal mass squared is $m^2 = k_B(p + 5p^2)$ for $T_p = 1$. The second order contribution to the thermal mass squared is extremely small so that Weinberg angle vanishes in the thermal approximation. $k_B = 1$ gives Z/e mass-ratio which is about 22 per cent too high. The thermal prediction for W-boson mass is the same as for Z^0 mass and thus even worse since the two masses are related $M_W^2 = M_Z^2 cos^2(\theta_W)$. For $T_p = 1/2$ thermal masses are completely negligible.

3. It seems that the Achilles's heel of the p-adic thermodynamics is bosonic sector whereas the weak point of the standard model is fermionic sector. This suggests that it might be possible to combine these two approaches. $T_p = 1/2$ is certainly the only possible p-adic temperature for intermediate gauge bosons so that gauge boson masses should result by a TGD variant of the Higgs mechanism. Contrary to the long-held belief, it is indeed possible to identify a candidate for Higgs boson with correct quantum numbers also in TGD framework. The point is that in quaternion-conformal Kac Moody algebra $su(3) = u(2) + t$ Kac-Moody charges decompose to two separately conserved parts Q_g and Q_J corresponding to variations with respect to induced metric and induced Kähler form. Q_J charges in $u(2)$ sub-algebra of $su(3)$ are identifiable as electro-weak charges whereas the charges in the complement t of $u(2)$ have interpretation as Higgs field possessing correct couplings to electro-weak gauge bosons. If t generates coherent state, standard electro-weak Higgs mechanism follows as a consequence. Sigma model type description in which the coupling to Higgs bosons induces only small shifts of fermion masses, suggests itself. In fact, this kind of mechanism has been also applied in the TGD inspired model of CP breaking in case of ordinary hadrons [F5].

4. An alternative option is based on the observation that the charge matrices of W and of left handed part of Z^0 are not covariantly constant and have the correct group theoretical properties to yield breaking of conformal invariance and thus mass squared as a thermodynamical vacuum expectation value.

5. The minimum p-adic mass squared is the p-adic mass squared unit $m_0^2/3$. This corresponds in a reasonable approximation to the mass of W boson so that the mass scale would be predicted correctly. The calculation of leptonic masses however requires the use of m_0^2 as a mass squared unit for which intermediate gauge boson masses are smaller than one unit. The way out of the difficulty is based on the use of a variant of the canonical identification I acting as $I_1(r/s) = I(r)/I(s)$. This map respects under certain additional conditions various symmetries and is the only sensible possibility at the level of scattering amplitudes. This variant predicts that the real counterpart of $m^2 = (m/n)p$ is $(m/n)/p$ rather than of order CP_2 squared so that intermediate gauge boson masses can be smaller than one unit even if $O(p)$ p-adically, and allows an elegant group theoretic description of m_W/m_Z mass ratio in terms of Weinberg angle. This point is discussed in [F4, F5].

N, Δ	0	1/2	1	3/2	2	5/2	3
2	1	1	1	3	3	4	4
3	1	2	3	9	11		
4	1	3	5	19	26		
5	1	4	10	24	150		

Table 3. Degeneracies $d(\Delta, N)$ of the operators satisfying NS type gauge conditions as a function of the number N of the active sectors and of the conformal weight Δ of the operator. Only those degeneracies, which are needed in the mass calculation for bosons are listed.

8.5 Some probabilistic considerations

There are uniqueness problems related to the mapping of p-adic probabilities to real ones. These problems find a nice resolution from the requirement that the map respects probability conservation. The implied modification of the original mapping does not change measurably the predictions for the masses of light particles.

8.5.1 How unique the map of p-adic probabilities and mass squared values are mapped to real numbers is?

The mapping of p-adic thermodynamical probabilities and mass squared values to real numbers is not completely unique.

1. Canonical identification $I : \sum x_n p^n \rightarrow \sum x_n p^{-n}$ takes care of this mapping but does not respect the sum of probabilities so that the real images $I(p_n)$ of the probabilities must be normalized. This is a somewhat alarming feature.

2. The modification of the canonical identification mapping rationals by the formula $I(r/s) = I(r)/I(s)$ has appeared naturally in various applications, in particular because it respects unitarity of unitary matrices with rational elements with $r < p, s < p$. In the case of p-adic thermodynamic the formula $I(g(n)p^n/Z) \rightarrow I(g(n)p^n)/I(Z)$ would be very natural although Z need not be rational anymore. For $g(n) < p$ the real counterparts of the p-adic probabilities would sum up to one automatically for this option. One cannot deny that this option is more convincing than the original one. The generalization of this formula to map p-adic mass squared to a real one is obvious.

3. Options 1) and 2) differ dramatically when the $n = 0$ massless ground state has ground state degeneracy $D > 1$. For option 1) the real mass is predicted to be of order CP_2 mass whereas for option 2) it would be by a factor $1/D$ smaller than the minimum mass predicted by the option a). Thus option 2) would predict a large number of additional exotic states. For those states which are light for option 1), the two options make identical predictions as far as the significant two lowest order terms are considered. Hence this interpretation would not change the predictions of the p-adic mass calculations in this respect. Option 2) is definitely more in accord with the real physics based intuitions and the main role of p-adic thermodynamics would be to guarantee the quantization of the temperature and fix practically uniquely the spectrum of the "Hamiltonian".

8.5.2 Under what conditions the mapping of p-adic ensemble probabilities to real probabilities respects probability conservation?

One can consider also a more general situation. Assume that one has an ensemble consisting of independent elementary events such that the number of events of type i is N_i. The probabilities are given by $p_i = N_i/N$ and $N = \sum N_i$ is the total number of elementary events. Even in the case that N is infinite as a real number it is natural to map the p-adic probabilities to their real counterparts using the rational canonical identification $I(p_i) = I(N_i)/I(N)$. Of course, N_i and N exist as well defined p-adic numbers under very stringent conditions only.

The question is under what conditions this map respects probability conservation. The answer becomes obvious by looking at the pinary expansions of N_i and N. If the integers N_i (possibly infinite as real integers) have pinary expansions having no common pinary digits, the sum of probabilities is conserved in the map. Note that this condition can assign also to a finite ensemble with finite number of a unique value of p.

This means that the selection of a basis for independent events corresponds to a decomposition of the set of integers labelling pinary digits to disjoint sets and brings in mind the selection of orthonormalized basis of quantum states in quantum theory. What is physically highly non-trivial that this "orthogonalization" alone puts strong constraints on probabilities of the allowed elementary events. One can say that the probabilities define distributions of pinary digits analogous to non-negative probability amplitudes in the space of integers labelling pinary digits, and the probabilities of independent events must be orthogonal with respect to the inner product defined by point-wise multiplication in the space of pinary digits.

p-Adic thermodynamics for which Boltzman weights $g(E)exp(-E/T)$ are replaced by $g(E)p^{E/T}$ such that one has $g(E) < p$ and E/T is integer valued, satisfies this constraint. The quantization of E/T to integer values implies quantization of both T and "energy" spectrum and forces so called super conformal invariance in TGD applications, which is indeed a basic symmetry of the theory.

There are infinitely many ways to choose the elementary events and each choice corresponds to a decomposition of the infinite set of integers n labelling the powers of p to disjoint subsets. These subsets can be also infinite. One can assign to this kind of decomposition a resolution which is the poorer the larger the subsets involved are. p-Adic thermodynamics would represent the situation in which the resolution is maximal since each set contains only single pinary digit. Note the analogy with the basis of completely localized wave functions in a lattice.

9 Modular contribution to the mass squared

The success of the p-adic mass calculations gives convincing support for the generation-genus correspondence. The basic physical picture is following.

1. Fermionic mass squared is dominated by partonic contribution, which is sum of cm and modular contributions: $M^2 = M^2(cm) + M^2(mod)$. Here 'cm' refers to the thermal contribution. Modular contribution can be assumed to depend on the genus of the boundary component only.

2. If Higgs contribution for diagonal (g, g) bosons (singlets with respect to "topological" $SU(3)$) dominates, the genus dependent contribution can be assumed to be negligible. This should be due to the bound state character of the wormhole contacts reducing thermal motion and thus the p-adic temperature.

3. Modular contribution to the mass squared can be estimated apart from an overall proportionality constant. The mass scale of the contribution is fixed by the p-adic length scale hypothesis. Elementary particle vacuum functionals are proportional to a product of all even theta functions and their conjugates, the number of even theta functions and their conjugates being $2N(g) = 2^g(2^g + 1)$. Also the thermal partition function must also be proportional to $2N(g)$:th power of some elementary partition function. This implies that thermal/ quantum

expectation $M^2(mod)$ must be proportional to $2N(g)$. Since single handle behaves effectively as particle, the contribution must be proportional to genus g also. The success of the resulting mass formula encourages the belief that the argument is essentially correct.

The challenge is to construct theoretical framework reproducing the modular contribution to mass squared. There are two alternative manners to understand the origin modular contribution.

1. The realization that super-canonical algebra is relevant for elementary particle physics leads to the idea that two thermodynamics are involved with the calculation of the vacuum conformal weight as a thermal expectation. The first thermodynamics corresponds to Super Kac-Moody algebra and second thermodynamics to super-canonical algebra. This approach allows a first principle understanding of the origin and general form of the modular contribution without any need to introduce additional structures in modular degrees of freedom. The very fact that super-canonical algebra does not commute with the modular degrees of freedom explains the dependence of the super-canonical contribution on moduli.

2. The earlier approach was based on the idea that he modular contribution could be regarded as a quantum mechanical expectation value of the Virasoro generator L_0 for the elementary particle vacuum functional. Quantum treatment would require generalization the concepts of the moduli space and theta function to the p-adic context and finding an acceptable definition of the Virasoro generator L_0 in modular degrees of freedom. The problem with this interpretation is that it forces to introduce, not only Virasoro generator L_0, but the entire super Virasoro algebra in modular degrees of freedom. One could also consider of interpreting the contribution of modular degrees of freedom to vacuum conformal weight as being analogous to that of CP_2 Laplacian but also this would raise the challenge of constructing corresponding Dirac operator. Obviously this approach has become obsolete.

The thermodynamical treatment taking into account the constraints from that p-adicization is possible might go along following lines.

1. In the real case the basic quantity is the thermal expectation value $h(M)$ of the conformal weight as a function of moduli. The average value of the deviation $\Delta h(M) = h(M) - h(M_0)$ over moduli space $\mathcal{M}$ must be calculated using elementary particle vacuum functional as a modular invariant partition function. Modular invariance is achieved if this function is proportional to the logarithm of elementary particle vacuum functional: this reproduces the qualitative features basic formula for the modular contribution to the conformal weight. p-Adicization leads to a slight modification of this formula.

2. The challenge of algebraically continuing this calculation to the p-adic context involves several sub-tasks. The notions of moduli space $\mathcal{M}_p$ and theta function must be defined in the p-adic context. An appropriately defined logarithm of the p-adic elementary particle vacuum functional should determine $\Delta h(M)$. The average of $\Delta h(M)$ requires an integration over $\mathcal{M}_p$. The problems related to the definition of this integral could be circumvented if the integral in the real case could be reduced to an algebraic expression, or if the moduli space is discrete in which case integral could be replaced by a sum.

3. The number theoretic existence of the p-adic Θ function leads to the quantization of the moduli so that the p-adic moduli space is discretized. Accepting the sharpened form of Riemann hypothesis [E8], the quantization means that the imaginary *resp.* real parts of the moduli are proportional to integers *resp.* combinations of imaginary parts of zeros of Riemann Zeta. This quantization could occur also for the real moduli for the maxima of Kähler function. This reduces the problematic p-adic integration to a sum and the resulting sum defining $\langle \Delta h \rangle$ converges extremely rapidly for physically interesting primes so that only the few lowest terms are needed.

9.1 Conformal symmetries and modular invariance

The full SKM invariance means that the super-conformal fields depend only on the conformal moduli of 2-surface characterizing the conformal equivalence class of the 2-surface. This means that all induced metrics differing by a mere Weyl scaling have same moduli. This symmetry is extremely powerful since the space of moduli is finite-dimensional and means that the entire infinite-dimensional space of deformations of parton 2-surface X^2 degenerates to a finite-dimensional moduli spaces under conformal equivalence. Obviously, the configurations of given parton correspond to a fiber space having moduli space as a base space. Super-canonical degrees of freedom could break conformal invariance in some appropriate sense.

9.1.1 Conformal and SKM symmetries leave moduli invariant

Conformal transformations and super Kac Moody symmetries must leave the moduli invariant. This means that they induce a mere Weyl scaling of the induced metric of X^2 and thus preserve its non-diagonal character $ds^2 = g_{z\overline{z}}dzd\overline{z}$. This is indeed true if

1. the Super Kac Moody symmetries are holomorphic isometries of $X^7 = \delta M^4_\pm \times CP_2$ made local with respect to the complex coordinate z of X^2, and
2. the complex coordinates of X^7 are holomorphic functions of z.

Using complex coordinates for X^7 the infinitesimal generators can be written in the form

$$J^{An} = z^n j^{Ak} D_k + \overline{z}^n j^{A\overline{k}} D_{\overline{k}} \ . \quad (105)$$

The intuitive picture is that it should be possible to choose X^2 freely. It is however not always possible to choose the coordinate z of X^2 in such a manner that X^7 coordinates are holomorphic functions of z since a consistency of inherent complex structure of X^2 with that induced from X^7 is required. Geometrically this is like meeting of two points in the space of moduli.

Lorentz boosts produce new inequivalent choices of S^2 with their own complex coordinate: this set of complex structures is parameterized by the hyperboloid of future light cone (Lobatchevski space or mass shell), but even this is not enough. The most plausible manner to circumvent the problem is that only the maxima of Kähler function correspond to the holomorphic situation so that super-canonical algebra representing quantum fluctuations would induce conformal anomaly.

9.1.2 The isometries of δM^4_+ are in one-one correspondence with conformal transformations

For CP_2 factor the isometries reduce to $SU(3)$ group acting also as canonical transformations. For $\delta M^4_+ = S^2 \times R_+$ one might expect that isometries reduce to Lorentz group containing rotation group of $SO(3)$ as conformal isometries. If r_M corresponds to a macroscopic length scale, then X^2 has a finite sized S^2 projection which spans a rather small solid angle so that group $SO(3)$ reduces in a good approximation to the group $E^2 \times SO(2)$ of translations and rotations of plane.

This expectation is however wrong! The light-likeness of δM^4_+ allows a dramatic generalization of the notion of isometry. The point is that the conformal transformations of S^2 induce a conformal factor $|df/dw|^2$ to the metric of δM^4_+ and the local radial scaling $r_M \rightarrow r_M/|df/dw|$ compensates it. Hence the group of conformal isometries consists of conformal transformations of S^2 with compensating radial scalings. This compensation of two kinds of conformal transformations is the deep geometric phenomenon which translates to the condition $L_{SC} - L_{SKM} = 0$ in the sub-space of physical states. Note that an analogous phenomenon occurs also for the light-like CDs X^3_l with respect to the metrically 2-dimensional induced metric.

The X^2-local radial scalings $r_M \rightarrow r_M(z, \overline{z})$ respect the conditions $g_{zz} = g_{\overline{zz}} = 0$ so that a mere Weyl scaling leaving moduli invariant results. By multiplying the conformal isometries of δM^4_+ by z^n (z is used as a complex coordinate for X^2 and w as a complex coordinate for S^2) a conformal localization of conformal isometries would result. Kind of double conformal transformations would be in question. Note however that this requires that X^7 coordinates are holomorphic functions of X^2 coordinate. These transformations deform X^2 unlike the conformal transformations of X^2. For X^3_l similar local scalings of the light like coordinate leave the moduli invariant but lead out of X^7.

9.1.3 Canonical transformations break the conformal invariance

In general, infinitesimal canonical transformations induce non-vanishing components $g_{zz}, g_{\overline{zz}}$ of the induced metric and can thus change the moduli of X^2. Thus the quantum fluctuations represented by super-canonical algebra and contributing to the configuration space metric are in general moduli changing. It would be interesting to know explicitly the conditions (the number of which is the dimension of moduli space for a given genus), which guarantee that the infinitesimal canonical transformation is moduli preserving.

9.2 The physical origin of the genus dependent contribution to the mass squared

Different p-adic length scales are not enough to explain the charged lepton mass ratios and an additional genus dependent contribution in the fermionic mass formula is required. The general form of this contribution can be guessed by regarding elementary particle vacuum functionals in the modular degrees of freedom as an analog of partition function and the modular contribution to the conformal weight as an analog of thermal energy obtained by averaging over moduli. p-Adic length scale hypothesis determines the overall scale of the contribution.

The exact physical origin of this contribution has remained mysterious but super-canonical degrees of freedom represent a good candidate for the physical origin of this contribution. This would mean a sigh of relief since there would be no need to assign conformal weights, super-algebra, Dirac operators, Laplacians, etc.. with these degrees of freedom.

9.2.1 Thermodynamics in super-canonical degrees of freedom as the origin of the modular contribution to the mass squared

The following general picture is the simplest found hitherto.

1. Elementary particle vacuum functionals are defined in the space of moduli of surfaces X^2 corresponding to the maxima of Kähler function. There some restrictions on X^2. In particular, p-adic length scale poses restrictions on the size of X^2. There is an infinite hierarchy of elementary particle vacuum functionals satisfying the general constraints but only the lowest elementary particle vacuum functionals are assumed to contribute significantly to the vacuum expectation value of conformal weight determining the mass squared value.

2. The contribution of Super-Kac Moody thermodynamics to the vacuum conformal weight h coming from Virasoro excitations of the $h = 0$ massless state is estimated in the previous calculations and does not depend on moduli. The new element is that for a partonic 2-surface X^2 with given moduli, Virasoro thermodynamics is present also in super-canonical degrees of freedom.

   Super-canonical thermodynamics means that, besides the ground state with $h_{gr} = -h_{SC}$ with minimal value of super-canonical conformal weight h_{SC}, also thermal excitations of this state by super-canonical Virasoro algebra having $h_{gr} = -h_{SC} - n$ are possible. For these ground states the SKM Virasoro generators creating states with net conformal weight $h = h_{SKM} - h_{SC} - n \geq 0$ have larger conformal weight so that the SKM thermal average h depends on n. It depends also on the moduli M of X^2 since the Beltrami differentials representing a tangent space basis for the moduli space $\mathcal{M}$ do not commute with the super-canonical algebra. Hence the thermally averaged SKM conformal weight h_{SKM} for given values of moduli satisfies

$$h_{SKM} = h(n, M) \ . \tag{106}$$

3. The average conformal weight induced by this double thermodynamics can be expressed as a super-canonical thermal average $\langle \cdot \rangle_{SC}$ of the SKM thermal average $h(n, M)$:

$$h(M) = \langle h(n, M) \rangle_{SC} = \sum p_n(M) h(n) \ , \tag{107}$$

where the moduli dependent probability $p_n(M)$ of the super-canonical Virasoro excitation with conformal weight n should be consistent with the p-adic thermodynamics. It is convenient to write $h(M)$ as

$$h(M) = h_0 + \Delta h(M) \ , \tag{108}$$

where h_0 is the minimum value of $h(M)$ in the space of moduli. The form of the elementary particle vacuum functionals suggest that h_0 corresponds to moduli with $Im(\Omega_{ij}) = 0$ and thus to singular configurations for which handles degenerate to one-dimensional lines attached to a sphere.

4. There is a further averaging of $\Delta h(M)$ over the moduli space $\mathcal{M}$ by using the modulus squared of elementary particle vacuum functional so that one has

$$h = h_0 + \langle \Delta h(M) \rangle_{\mathcal{M}} \ . \tag{109}$$

Modular invariance allows to pose very strong conditions on the functional form of $\Delta h(M)$. The simplest assumption guaranteing this and thermodynamical interpretation is that $\Delta h(M)$ is proportional to the logarithm of the vacuum functional Ω:

$$\Delta h(M) \propto -log(\frac{\Omega(M)}{\Omega_{max}}) \ . \tag{110}$$

Here Ω_{max} corresponds to the maximum of Ω for which $\Delta h(M)$ vanishes.

9.2.2 Justification for the general form of the mass formula

The proposed general ansatz for $\Delta h(M)$ provides a justification for the general form of the mass formula deduced by intuitive arguments.

1. The factorization of the elementary particle vacuum functional Ω into a product of $2N(g) = 2^g(2^g+1)$ terms and the logarithmic expression for $\Delta h(M)$ imply that the thermal expectation values is a sum over thermal expectation values over $2N(g)$ terms associated with various even characteristics (a,b), where a and b are g-dimensional vectors with components equal to $1/2$ or 0 and the inner product $4a\cdot b$ is an even integer. If each term gives the same result in the averaging using Ω_{vac} as a partition function, the proportionality to $2N_g$ follows.

2. For genus $g\geq 2$ the partition function defines an average in $3g-3$ complex-dimensional space of moduli. The analogy of $\langle \Delta h\rangle$ and thermal energy suggests that the contribution is proportional to the complex dimension $3g-3$ of this space. For $g\leq 1$ the contribution the complex dimension of moduli space is g and the contribution would be proportional to g.

$$\begin{aligned} \langle \Delta h\rangle &\propto g\times X(g) \ \text{ for } \ g\leq 1 \ , \\ \langle \Delta h\rangle &\propto (3g-3)\times X(g) \ \text{ for } \ g\geq 2 \ , \\ X(g) &= 2^g(2^g+1) \ . \end{aligned} \tag{111}$$

   If X^2 is hyper-elliptic for the maxima of Kähler function, this expression makes sense only for $g\leq 2$ since vacuum functionals vanish for hyper-elliptic surfaces.

3. The earlier argument, inspired by the interpretation of elementary particle vacuum functional as a partition function, was that each factor of the elementary particle vacuum functional gives the same contribution to $\langle \Delta h\rangle$, and that this contribution is proportional to g since each handle behaves like a particle:

$$\langle \Delta h\rangle \propto g\times X(g) \ . \tag{112}$$

   The prediction following from the previous differs by a factor $(3g-3)/g$ for $g\geq 2$. This would scale up the dominant modular contribution to the masses of the third $g=2$ fermionic generation by a factor $\sqrt{3/2}\simeq 1.22$. One must of course remember, that these rough arguments allow $g-$ dependent numerical factors of order one so that it is not possible to exclude either argument.

9.3 Generalization of Θ functions and quantization of p-adic moduli

The task is to find p-adic counterparts for theta functions and elementary particle vacuum functionals. The constraints come from the p-adic existence of the exponentials appearing as the summands of the theta functions and from the convergence of the sum. The exponentials must be proportional to powers of p just as the Boltzmann weights defining the p-adic partition function. The outcome is a quantization of moduli so that integration can be replaced with a summation and the average of $\Delta h(M)$ over moduli is well defined.

It is instructive to study the problem for torus in parallel with the general case. The ordinary moduli space of torus is parameterized by single complex number τ. The points related by $SL(2,Z)$ are equivalent, which means that the transformation $\tau\rightarrow (A\tau+B)/(C\tau+D)$ produces a point equivalent with τ. These transformations are generated by the shift $\tau\rightarrow\tau+1$ and $\tau\rightarrow -1/\tau$. One can choose the fundamental domain of moduli space to be the intersection of the slice $Re(\tau)\in[-1/2,1/2]$ with the exterior of unit circle $|\tau|=1$. The idea is to start directly from physics and to look whether one might some define p-adic version of elementary particle vacuum functionals in the p-adic counter part of this set or in some modular invariant subset of this set.

Elementary particle vacuum functionals are expressible in terms of theta functions using the functions $\Theta^4[a,b]\overline{\Theta}^4[a,b]$ as a building block. The general expression for the theta function reads as

$$\Theta[a,b](\Omega) \ = \ \sum_n exp(i\pi(n+a)\cdot\Omega\cdot(n+a))exp(2i\pi(n+a)\cdot b) \ . \qquad (113)$$

The latter exponential phase gives only a factor $\pm i$ or ± 1 since $4a\cdot b$ is integer. For $p \ mod \ 4 = 3$ imaginary unit exists in an algebraic extension of p-adic numbers. In the case of torus (a,b) has the values $(0,0)$, $(1/2,0)$ and $(0,1/2)$ for torus since only even characteristics are allowed.

Concerning the p-adicization of the first exponential appearing in the summands in Eq. 113, the obvious problem is that π does not exists p-adically. The introduction of the scaled variable $\hat{\tau} = \pi\tau$ resolves this problem. The second modification is the replacement of the factors $exp(X)$ with $p^{X/log(p)}$ in order to achieve a rapid p-adic convergence of the sum defining the theta function. This requires a further scaling so that one has $\Omega_p = \pi\Omega/log(p)$ is the appropriate variable and the terms in the sum are apart from the phase factor of form $p^{i(n+a)\cdot\Omega_p\cdot(n+a)}$.

If the exponents

$$p^{i(n+a)\cdot Im(\Omega_{ij,p})\cdot(n+a)} = p^{-a\cdot Im(\Omega_{ij,p})\cdot a} \times p^{-2a\cdot Im(\Omega_{ij,p})n} \times p^{-n\cdot Im(\Omega_{ij,p})\cdot n}$$

are integer powers of p, $\Theta_{[a,b]}$ exist in R_p. A milder condition is that only the building blocks $\Theta^4[a,b]\overline{\Theta}^4[a,b]$ exist in R_p. The problematic factor is the first exponent since the components of the vector a can have values $1/2$ and 0 and its existence implies a quantization of $Im(\Omega_{ij,p})$ as

$$Im(\Omega_{ij,p}) \ = \ -Kn_{ij} \ , \ n_{ij}\in Z \ , \ n_{ij}\geq 1 \ , \qquad (114)$$

$K = 4$ guarantees the existence of Θ functions and $K = 1$ the existence of elementary particle vacuum functionals. Obviously the sum defining Θ converges rapidly with respect to the p-adic norm.

The problem is that the condition $Im(\Omega_{ij,p}) > 0$ is not satisfied. There is however no reason why the p-adic theta function could not be defined by changing the sign of the exponents so that one would have

$$\begin{aligned} \Theta[a,b](\Omega)_p \ &= \ \sum_n p^{-i(n+a)\cdot\Omega_p\cdot(n+a)} \times exp\left[2i\pi(n+a)\cdot b\right] \ , \\ Im(\Omega_{ij,p}) \ &= \ Kn_{ij} \ , \ n_{ij}\geq 1 \ . \end{aligned} \qquad (115)$$

$K = 4$ guarantees the existence of Θ functions in R_p and $K = 1$ the existence of elementary vacuum functional in R_p: in this case $\Theta_{[a,b]}$ exists in appropriate algebraic extension of R_p. Note that a similar change of sign must be performed in p-adic thermodynamics for powers of p to map p-adic probabilities to real ones.

A further requirement is that the phases $p^{-iRe(\Omega_{ij,p})/4}$ exist p-adically. A weaker condition that only the phases $p^{-iRe(\Omega_{ij,p})}$ exist p-adically guarantees that elementary particle vacuum functionals exist p-adically. The condition that p^{iy} exists for certain preferred values of y for all values of prime p is encountered repeatedly in the algebraic continuation of quantum TGD to p-adic context. The sharpening of the Riemann Hypothesis [E8] stating that the partition functions $1/(1-p^z)$ appearing in the product expansion of Rieman Zeta in various p-adic number fields exist for the zeros $z = 1/2 + iy$ of Riemann Zeta, is number theoretically highly attractive.

This conjecture implies that p^{iy} is in general a product of a phase factor $exp(i2\pi m/n)$ in some algebraic extension of p-adic numbers and of a Pythagorean phase $(k+il)/\sqrt{k^2+l^2}$, $k^2+l^2=n^2$. A potential problem is that this phase factor does not possess unit p-adic norm in the general case.

The explicit form for the allowed (k,l) and (l,k) pairs is given by

$$\begin{aligned} k &= 2rs \ , \\ l &= r^2-s^2 \ , \\ n &= r^2+s^2 \ . \end{aligned} \qquad (116)$$

where r and s are relatively prime integers, not both odd. Note that (l,k) is also an allowed solution. An important point to be noticed is that the p-adic norm of Pythagorean phase is not larger than one for physically most interesting primes satisfying $p \ mod \ 4 = 3$ since $n \ mod \ 4 = 1$ holds true as a simple calculation shows. This guarantees that the phase factors of the Θ function cannot spoil the p-adic convergence of the sum defining the p-adic theta function.

The sharpening of the Riemann hypothesis, when combined with the requirement that the logarithmic radial waves $(r_M/r_0)^{iz}$ exists in some finite-dimensional extension of any p-adic number fields when r_M/r_0 is rational valued, implies that the radial conformal weights z of the super-canonical algebra correspond to the zeros of Zeta and their appropriate combinations. The quantization condition is

$$Re(\Omega_{ij,p}) = K\sum n_k y_k \ , \qquad (117)$$

where y_k correspond to zeros of Zeta. $K=4$ guarantees that Θ functions exist p-adically. $K=1$ is enough to guarantee the existence of elementary particle vacuum functionals.

In the real context the quantization of moduli of torus would correspond to

$$\begin{aligned} \tau &= K(\sum n_k y_k + in) \times \frac{log(p)}{\pi} \ , \\ |\tau| &= K\sqrt{n^2+(\sum_k n_k y_k)^2} \ , \\ \Phi &= atan(\frac{n}{\sum_k n_k y_k}) \ . \end{aligned} \qquad (118)$$

$K=1$ guarantees the existence of elementary particle vacuum functionals and $K=4$ the existence of Theta functions. The ratio for the complex vectors defining the sides of the plane parallelogram defining torus via the identification of the parallel sides is quantized. In other words, the angles Φ between the sides and the ratios of the sides given by $|\tau|$ have quantized values.

The quantization rules for the moduli of the higher genera read as

$$\Omega_{ij} = K\left[\sum n_k(i,j)y_k + in(i,j)\right] \times \frac{log(p)}{\pi} \ , \qquad (119)$$

If the quantization rules hold true also for the maxima of Kähler function in the real context, there are good hopes that the p-adicized expression for Δh is obtained by a simple algebraic continuation of the real formula. Thus p-adic length scale characterizes partonic surface X^2 rather than the light like causal determinant X^3_l containing X^2. Therefore the idea that various p-adic primes label various X^3_l connecting fixed partonic surfaces X^2_i would not be correct.

The set of the moduli allowed by the quantization rules is not invariant under modular transformations. For instance, in the case of torus the $SL(2, Z)$ Möbius transformations $\Omega \rightarrow \Omega + n$ and $\Omega \rightarrow 1/\Omega$ lead out of the allowed moduli space. This is not however a problem if there ar no modular transformations relating quantized moduli so that they can be thought of as forming single fundamental domain containing possibly non-equivalent moduli from several fundamental domains in the conventional sense of the word.

Quite generally, the quantization of moduli means that the allowed 2-dimensional shapes form a lattice and are thus additive. It also means that the maxima of Kähler function obey a linear superposition in an extreme abstract sense. The proposed number theoretical quantization is expected to apply for any complex space allowing some preferred complex coordinates. In particular, configuration space of 2-surfaces could allow this kind of quantization in the complex coordinates naturally associated with isometries and this could allow to define configuration space integration, at least the counterpart of integration in zero mode degrees of freedom, as a summation.

9.4 The calculation of the modular contribution $\langle \Delta h \rangle$ to the conformal weight

The quantization of the moduli implies that the integral over moduli can be defined as a sum over moduli. The theta function $\Theta[a, b](\Omega)_p(\tau_p)$ is proportional to $p^{a \cdot a Im(\Omega_{ij,p})} = p^{K n_{ij} m(a)/4}$ for $a \cdot a = m(a)/4$, where $K = 1$ *resp.* $K = 4$ corresponds to the existence existence of elementary particle vacuum functionals *resp.* theta functions in R_p. These powers of p can be extracted from the thetas defining the vacuum functional. The numerator of the vacuum functional gives $(p^n)^{2K \sum_{a,b} m(a)}$. The numerator gives $(p^n)^{2K \sum_{a,b} m(a_0)}$, where a_0 corresponds to the minimum value of $m(a)$. $a_0 = (0, 0, .., 0)$ is allowed and gives $m(a_0) = 0$ so that the p-adic norm of the denominator equals to one. Hence one has

$$|\Omega_{vac}(\Omega_p)|_p \quad = \quad p^{-2nK \sum_{a,b} m(a)} \tag{120}$$

The sum converges extremely rapidly for large values of p as function of n so that in practice only few moduli contribute.

The definition of $log(\Omega_{vac})$ poses however problems since in $log(p)$ does not exist as a p-adic number in any p-adic number field. The argument of the logarithm should have a unit p-adic norm. The simplest manner to circumvent the difficulty is to use the fact that the p-adic norm $|\Omega_p|_p$ is also a modular invariant, and assume that the contribution to conformal weight depends on moduli as

$$\Delta h_p(\Omega_p) \quad \propto \quad log(\frac{\Omega_{vac}}{|\Omega_{vac}|_p}) \ . \tag{121}$$

The sum defining $\langle \Delta h_p \rangle$ converges extremely rapidly and gives a result of order $O(p)$ p-adically as required.

The p-adic expression for $\langle \Delta h_p \rangle$ should result from the corresponding real expression by an algebraic continuation. This encourages the conjecture that the allowed moduli are quantized for the maxima of Kähler function, so that the integral over the moduli space is replaced with a sum also in the real case, and that Δh given by the double thermodynamics as a function of moduli can be defined as in the p-adic case. The positive power of p multiplying the numerator could be interpreted as a degeneracy factor. In fact, the moduli are not primary dynamical variables in the case of the induced metric, and there must be a modular invariant weight factor telling how many 2-surfaces correspond to given values of moduli. The power of p could correspond to this factor.

10 Appendix: Gauge bosons in the original scenario

The construction of gauge boson states is more or less trivial if bosons correspond to wormhole contacts and has been already discussed. The construction of gauge boson states is is more intricate process than in the original scenario which did not involve super-canonical algebra and the effective 2-dimensionality. Since the construction gives some insights about the construction in the case that the only partonic fermion states are free many-fermion states plus the states obtained from them by applying super-canonical and Super-Kac Moody algebra, I decided to keep this section.

10.1 Bi-locality of boson states

The gauge boson could correspond to either a local fermion current contracted with j^{Ak} or to a bi-local operator.

1. For a local operator no normal ordering of the current would be needed since at a given space-time sheet and for TGD quantization avoiding infinite vacuum energy, the current would contain only anti-commuting creation operators of a positive (negative) energy fermion and negative (positive) energy anti-fermion. The vacuum expectation value would involve only the contractions of fermion and anti-fermion with the legs of the current and a well-defined and finite integral over X^2 would result. This is however not enough: also the norms of the boson states must be finite and locality implies an infinite norm as an integral of $\delta(0)$ over X^2. Hence local operators seem to be excluded.

2. If photon is created by a bi-local operator, it would involve a kind of structure function in $X^2 \times X^2$ allowing visualization as a line connecting two points x and y having fermion and anti-fermion at its ends. The bi-local current would be sum of two terms

$$
\begin{aligned}
B &= \int_{X^2\times X^2} dV_x dV_y B(x,y)\left[\overline{\Psi}(x)E(y)\Psi(y) + \overline{\Psi}(x)E(x)\Psi(y)\right] \ , \\
E &= j^{Ak}\gamma_k \ .
\end{aligned}
\tag{122}
$$

   The vacuum expectation value determining the vertex would boil down to a correlation function defined as integral over $X^2 \times X^2$ for this Hamiltonian and bilinear of functions formed from positive energy fermion and anti-fermion. $B(x,y)$ could be determined by the super conformal invariance as a correlation function.

Conformal invariance suggests that correlation functions obeying simple power scaling laws as a function of the distance r between fermion and anti-fermion are associated with the boson states. The power law holds true with respect to the distance r measured in the induced metric.

The distance r between fermion and antifermion in the induced metric of X^2 is expressed to behave fractally as function of Δr_M, where r_M is the light like radial coordinate of δM^4_+. For large values of r_M $r(\Delta r_M)$ is expected to grow very slowly since X^2 becomes almost light like in this direction. For small distance the growth is expected to be very rapid by the p-adic fractality of X^2 meaning that X^2 becomes 2-D version for the coast line of Britain. The scaling behavior

$$
\begin{aligned}
\frac{r}{r_0} &= x^{\Delta} \ , \ \Delta < 1 \ , \\
x &= \frac{\Delta r_M}{r_{M,0}}
\end{aligned}
\tag{123}
$$

is expected. A good guess for $r_{M,0}$ is as a p-adic length scale: $r_{M,0} = L_p$.

10.2 Bosonic charge matrices, conformal invariance, and coupling constants

Bosons are represented by fermion-antifermion bilinears. The requirement that boson state has a finite norm implies that bosons are bi-local operators creating fermion antifermion states in X^2. The bilinear representing the boson can be regarded as a second quantized version of the charge matrix of the charge represented by the boson in question. Also a polarization vector contracted with M^4 gamma matrices is involved. In the case of graviton/gluon charge matrix involve momentum operators of M^4/color rotation generators of CP_2 acting in center of mass degrees of freedom acting of the second quantized spinor fields. The normalization factors of the bosonic states determine their couplings to fermion pairs, which appear as fundamental couplings: the proportionality $1/\sqrt{N} \propto g$ between the coupling g and proportionality factor is predicted.

10.3 The ground states associated with gauge bosons

The experience about p-adic mass calculations gives some hints about the ground state conformal weights of intermediate gauge bosons.

1. If p-adic temperature is $T = 1/2$ for bosons instead of $T = 1$ for fermions, p-adic thermodynamics does not significantly contribute to boson masses except if ground states have vanishing conformal weight so that ground state degeneracy is absent and $T = 1/2$ gives completely negligible thermal contribution to the conformal weight.

2. If the mass in the case of gauge bosons is dominantly due to the coupling to Higgs, all electro-weak bosons could have vanishing conformal weights in the ground state. This would be in conflict with the assumption $h_{vac}(W) = -2$ of the earlier model following from the following argument. W boson is not a color singlet although it does not of course belong to an irreducible representation of $SU(3)$. One could argue that W charge matrix and the left handed part of Z charge matrix correspond to a $j = 1$ triplet of $SU_L(2) \subset SU(3)$ and thus has $h_c = j(j + 1) = 2$ for $j = 1$. Also, because W behaves like $e\overline{\nu}$ pair the W charge matrix must have the color conformal weight $h_c = 2$ of $e\overline{\nu}$. Also the requirement that the ground state conformal weight is conserved in electro-weak vertices supports this picture. $h_c = 2$ would be compensated by the negative conformal weight of the super-canonical operator from super-canonical generators.

   Z^0 would be superposition of states with different super-canonical ground state conformal weights. The left handed part of charge matrix proportional to I_L^3 would have ground state super-canonical conformal weight $h_{vac} = -2$ and the vectorial part of Z^0 charge matrix proportional to Q_{em} would have $h_c = 0$ and there would be no compensating super-canonical factor. Photon of course has a vanishing ground state conformal weight.

3. In the case of gluons the isometry generator $J^A = j^{Ak} D_k$ does not change the representation associated with a color Hamiltonian. The assumption that this operator carries a conformal weight $Re[h_c] = 0$ conforms with the masslessness of gluon and with the fact that also translation generators possess a vanishing conformal weight in the stringy mass formula. If h_c has imaginary part, one could distinguish between gluons and their phase conjugates. If quarks have a net complex conformal weight as previous considerations suggest and if hadrons have real net conformal weight (by no means necessary), gluons should have net conformal weights of form $h = iy$ compensating the conformal weights of quarks. Operators giving rise to purely imaginary conformal weight might serve as counterparts for the color electric flux tubes connecting the 2-surfaces associated with quarks.

10.4 Bosonic charge matrices

In the following more detailed forms of bosonic charge matrices are listed.

10.4.1 Photon and intermediate gauge bosons

Photonic charge matrix is of the form

$$q_{em} \;\; = \;\; a \times Id + b \times J_{kl}\Sigma^{kl} \;\; , \tag{124}$$

and is covariantly constant. As a consequence photon the charge matrix does not develop any contribution to the mass squared and for $T = 1/2$ photon remains in an excellent approximation massless in p-adic thermodynamics.

The charge matrix of W^- is simply the left handed isospin matric I_L^3 and if the previous arguments are correct it carries a conformal weight $h_c = 2$. In standard model Z^0 charge matrix is a linear combination of the left handed electro-weak isospin I_L^3 and electromagnetic charge Q_{em}.

$$Q_Z \;\; = \;\; I_L^3 - sin^2(\theta_W)Q_{em} \;\; . \tag{125}$$

In standard model the mixing of I_L^3 and Q_{em} corresponds to the mixing of $U(1)$ and $U(1)_L$ bosons by Weinberg angle θ_W. I_L^3 has ground state conformal weight -2 whereas for Q_{em} the weight vanishes.

10.4.2 Graviton and gluon

Graviton corresponds to a charge matrix, or rather charge operator acting on fermion bi-linear, defined as

$$O \;\; = \;\; E^{kl}\gamma_k(\partial_{1,l} - \partial_{2,l}) \tag{126}$$

with gamma matrix contracted between fermion and antifermion: here the flatness of M^4 is essential. E_{kl} is the polarization tensor of graviton satisfying obvious constraints. The derivative operators with respect to Minkowski coordinates act on fermion and anti-fermion.

For the gluon the charge matrix is given by

$$\begin{aligned} O &= E_k\gamma^k(J_1^A - J_2^A) \;\; , \\ J^A &= j^{Ak}D_k \;\; . \end{aligned} \tag{127}$$

E_k is the polarization vector of gluon.

In the case of graviton and gluon the question about the action of the isometry generator arises since the second quantized induced spinor field Ψ and the correlation function $B(x,y)$ depend on X^2 coordinates rather than imbedding space coordinates. The problem is analogous to that of interpreting the coordinate z of X^2 in the anti-commutators and commutators of Super Kac-Moody and super-canonical generators as an imbedding space coordinate. As found, the problem can be circumvented if z is identifiable in terms of a unique imbedding coordinate w for a representative 2-surface $Y^2(X^2)$ assignable to a maximum of the Kähler function whose perturbations by super-canonical algebra appear in the configuration space functional integral.

10.5 $BF\overline{F}$ couplings and the general form of bosonic configuration space spinor fields

Conformal theory alone gives no hint about how the coupling constant appears, and configuration space-integral is necessary to understand the emergence of the gauge coupling.

1. A strong hint comes from the facts that all $BF\overline{F}$ coupling constants, except possibly gravitational constant, must be proportional to the Kähler coupling g_K. The most natural manner to achieve this is to require that the bosonic configuration space spinor fields vanish at the maximum of the Kähler function where the perturbation series is developed. That bosons should correspond to small perturbations around the maximum of the Kähler function is in accordance with the assumption that quantum fields correspond to the perturbations around the extrema of the action functional. This means that one can write $B(x,y)$ in the form

$$\begin{aligned} B(x,y) &= \partial_I K B^I(x,y) \ , \\ \partial_I K(X^3) &= 0 \ \text{at the maximum of } K \ . \end{aligned} \tag{128}$$

   Gere $\partial_I K$ denotes partial derivatives of Kähler function with respect to the configuration space coordinates X^I vanishing at the maximum of K.

2. The functional integral in the lowest order approximation is obtained by expanding $B(x,y)$ in lowest order to functional Taylor series in using the coordinates X^I

$$B(x,y) = \partial_R K \times B^I(x,y) \times X^R \ , \tag{129}$$

   It is understood that also $B^I(x,y)$ allows functional power series expansion as a functional of X^3. In the lowest order approximation the norm N of the boson state is given by the functional integral

$$\begin{aligned} N &= \langle \int_{X^2\times Y^2} \overline{B}(x,y)B(x,y)dV_x dV_y\rangle = A_{IJ}\times B^{IJ} \ , \\ A_{IJ} &= \partial_I\partial_R K \times \partial_J\partial_S K \times \langle X^R X^S\rangle \ , \\ B^{IJ} &= \int_{X^2\times Y^2} \overline{B}^I(x,y)B^J(x,y)dV_x dV_y \ . \end{aligned} \tag{130}$$

   Here $\langle X^R X^S\rangle$ is a two point function defined by the functional integral over small perturbations around the maximum of Kähler function Specifying the coordinates to complex coordinates and using the covariant Kahler metric $G_{K\overline{L}} = \partial_K\partial_{\overline{L}}K$ as the kinetic term. Since the contravariant Kähler metric defines the propagator, the lowest order approximation gives

$$N = G_{K\overline{L}} \times B^{K\overline{L}} \ . \tag{131}$$

   What is nice that the symmetry considerations allow to determine the covariant metric highly uniquely and the propagator disappears from the final formula. The normalization factor $1/\sqrt{N}$ of the boson state is obviously proportional to g_K since the Kähler function K is proportional to $1/\alpha_K$.

3. Fermion boson vertex is indeed proportional to g_K. $B(x,y)$ must be expanded in a functional Taylor series up to a second order term

$$B(x,y) = \partial_R K B^I(x,y) X^R + \partial_R K \times \partial_S B^I(x,y) \times X^R X^S + \cdots \quad . \tag{132}$$

The general expression of the $BF\overline{F}$ vertex is

$$\begin{aligned} V_{BF\overline{F}} &= \frac{1}{\sqrt{N}} \langle \int_{X^2 \times Y^2} \overline{B}(x,y) \Gamma dV_x dV_y \rangle = \frac{1}{\sqrt{N}} A \quad , \\ A &= \int_{X^2 \times Y^2} \partial_I \overline{B}^I(x,y) \Gamma dV_x dV_y \quad . \end{aligned} \tag{133}$$

The propagator compensates the second order derivatives of Kähler function in the functional integral average, and the vertex is indeed proportional to g_K.

References

Online books about TGD

[1] M. Pitkänen (2006), *Topological Geometrodynamics: Overview.* http://www.helsinki.fi/~matpitka/tgdview/tgdview.html.

[2] M. Pitkänen (2006), *Quantum Physics as Infinite-Dimensional Geometry.* http://www.helsinki.fi/~matpitka/tgdgeom/tgdgeom.html.

[3] M. Pitkänen (2006), *Physics in Many-Sheeted Space-Time.* http://www.helsinki.fi/~matpitka/tgdclass/tgdclass.html.

[4] M. Pitkänen (2006), *Quantum TGD.* http://www.helsinki.fi/~matpitka/tgdquant/tgdquant.html.

[5] M. Pitkänen (2006), *TGD as a Generalized Number Theory.* http://www.helsinki.fi/~matpitka/tgdnumber/tgdnumber.html.

[6] M. Pitkänen (2006), *p-Adic length Scale Hypothesis and Dark Matter Hierarchy.* http://www.helsinki.fi/~matpitka/paddark/paddark.html.

[7] M. Pitkänen (2006), *TGD and Fringe Physics.* http://www.helsinki.fi/~matpitka/freenergy/freenergy.html.

Online books about TGD inspired theory of consciousness and quantum biology

[8] M. Pitkänen (2006), *Bio-Systems as Self-Organizing Quantum Systems.* http://www.helsinki.fi/~matpitka/bioselforg/bioselforg.html.

[9] M. Pitkänen (2006), *Quantum Hardware of Living Matter.* http://www.helsinki.fi/~matpitka/bioware/bioware.html.

[10] M. Pitkänen (2006), *TGD Inspired Theory of Consciousness*.
http://www.helsinki.fi/∼matpitka/tgdconsc/tgdconsc.html.

[11] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*.
http://www.helsinki.fi/∼matpitka/genememe/genememe.html.

[12] M. Pitkänen (2006), *TGD and EEG*.
http://www.helsinki.fi/∼matpitka/tgdeeg/tgdeeg/tgdeeg.html.

[13] M. Pitkänen (2006), *Bio-Systems as Conscious Holograms*.
http://www.helsinki.fi/∼matpitka/hologram/hologram.html.

[14] M. Pitkänen (2006), *Magnetospheric Consciousness*.
http://www.helsinki.fi/∼matpitka/magnconsc/magnconsc.html.

[15] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*.
http://www.helsinki.fi/∼matpitka/magnconsc/mathconsc.html.

References to the chapters of books

[A8] The chapter *Was von Neumann Right After All* of [4].
http://www.helsinki.fi/∼matpitka/tgdview/tgdview.html#vNeumann.

[A9] The chapter *Does TGD Predict the Spectrum of Planck Constants?* of [1].
http://www.helsinki.fi/∼matpitka/tgdview/tgdview.html#Planck.

[B1] The chapter *Identification of the Configuration Space Kähler Function* of [2].
http://www.helsinki.fi/∼matpitka/tgdgeom/tgdgeom.html#kahler.

[B2] The chapter *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part I* of [2].
http://www.helsinki.fi/∼matpitka/tgdgeom/tgdgeom.html#compl1.

[B3] The chapter *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part II* of [2].
http://www.helsinki.fi/∼matpitka/tgdgeom/tgdgeom.html#compl2.

[B4] The chapter *Configuration Space Spinor Structure* of [2].
http://www.helsinki.fi/∼matpitka/tgdgeom/tgdgeom.html#cspin.

[C1] The chapter *Construction of Quantum Theory: Symmetries* of [4].
http://www.helsinki.fi/∼matpitka/tgdquant/tgdquant.html#quthe.

[C2] The chapter *Construction of Quantum Theory: S-matrix* of [4].
http://www.helsinki.fi/∼matpitka/tgdquant/tgdquant.html#towards.

[C3] The chapter *Hyper-Finite Factors and Construction of S-matrix* of [4].
http://www.helsinki.fi/∼matpitka/tgdquant/tgdquant.html#HFSmatrix.

[C4] The chapter *Construction of S-matrix* of [4].
http://www.helsinki.fi/∼matpitka/tgdquant/tgdquant.html#smatrix.

[C5] The chapter *Is it Possible to Understand Coupling Constant Evolution at Space-Time Level?* of [4].
http://www.helsinki.fi/∼matpitka/tgdquant/tgdquant.html#rgflow.

[C7] The chapter *Equivalence of Loop Diagrams with Tree Diagrams and Cancellation of Infinities in Quantum TGD* of [4].
http://www.helsinki.fi/∼matpitka/tgdquant/tgdquant.html#bialgebra.

[D1] The chapter *Basic Extremals of Kähler Action* of [3].
http://www.helsinki.fi/∼matpitka/tgdclass/tgdclass.html#class.

[D3] The chapter *The Relationship Between TGD and GRT* of [3].
http://www.helsinki.fi/∼matpitka/tgdclass/tgdclass.html#tgdgrt.

[D4] The chapter *Cosmic Strings* of [3].
http://www.helsinki.fi/∼matpitka/tgdclass/tgdclass.html#cstrings.

[D5] The chapter *TGD and Cosmology* of [3].
http://www.helsinki.fi/∼matpitka/tgdclass/tgdclass.html#cosmo.

[D6] The chapter *TGD and Astrophysics* of [3].
http://www.helsinki.fi/∼matpitka/tgdclass/tgdclass.html#astro.

[E1] The chapter *TGD as a Generalized Number Theory: p-Adicization Program* of [5].
http://www.helsinki.fi/∼matpitka/tgdnumber/tgdnumber.html#visiona.

[E2] The chapter *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [5].
http://www.helsinki.fi/∼matpitka/tgdnumber/tgdnumber.html#visionb.

[E3] The chapter *TGD as a Generalized Number Theory: Infinite Primes* of [5].
http://www.helsinki.fi/∼matpitka/tgdnumber/tgdnumber.html#visionc.

[E8] The chapter *Riemann Hypothesis and Physics* of [5].
http://www.helsinki.fi/∼matpitka/tgdnumber/tgdnumber.html#riema.

[E9] The chapter *Topological Quantum Computation in TGD Universe* of [5].
http://www.helsinki.fi/∼matpitka/tgdnumber/tgdnumber.html#tqc.

[F1] The chapter *Elementary Particle Vacuum Functionals* of [6].
http://www.helsinki.fi/∼matpitka/paddark/paddark.html#elvafu.

[F3] The chapter *p-Adic Particle Massivation: Hadron Masses* of [6].
http://www.helsinki.fi/∼matpitka/paddark/paddark.html#padmass2.

[F4] The chapter *p-Adic Particle Massivation: Hadron Masses* of [6].
http://www.helsinki.fi/∼matpitka/paddark/paddark.html#padmass3.

[F5] The chapter *p-Adic Particle Massivation: New Physics* of [6].
http://www.helsinki.fi/∼matpitka/paddark/paddark.html#padmass4.

[F7] The chapter *The Recent Status of Leptohadron Hypothesis* of [6].
http://www.helsinki.fi/∼matpitka/paddark/paddark.html#leptc.

[F8] The chapter *TGD and Nuclear Physics* of [6].
http://www.helsinki.fi/∼matpitka/paddark/paddark.html#padnucl.

[F9] The chapter *Dark Nuclear Physics and Condensed Matter* of [6].
http://www.helsinki.fi/∼matpitka/paddark/paddark.html#exonuclear.

[J1] The chapter *Bio-Systems as Super-Conductors: part I* of [9].
http://www.helsinki.fi/∼matpitka/bioware/bioware.html#superc1.

[J2] The chapter *Bio-Systems as Super-Conductors: part II* of [9].
http://www.helsinki.fi/∼matpitka/bioware/bioware.html#superc2.

[J3] The chapter *Bio-Systems as Super-Conductors: part III* of [9].
http://www.helsinki.fi/∼matpitka/bioware/bioware.html#superc3.

[J6] The chapter *Coherent Dark Matter and Bio-Systems as Macroscopic Quantum Systems* of [9].
http://www.helsinki.fi/∼matpitka/bioware/bioware.html#darkbio.

[K1] The chapter *Time, Spacetime and Consciousness* of [13].
http://www.helsinki.fi/∼matpitka/hologram/hologram.html#time.

[K3] The chapter *General Theory of Qualia* of [13].
http://www.helsinki.fi/∼matpitka/hologram/hologram.html#qualia.

[M3] The chapter *Dark Matter Hierarchy and Hierarchy of EEGs* of [12].
http://www.helsinki.fi/∼matpitka/tgdeeg/tgdeeg/tgdeeg.html#eegdark.

[M6] The chapter *Quantum Model for Hearing* of [12].
http://www.helsinki.fi/∼matpitka/tgdeeg/tgdeeg/tgdeeg.html#hearing.

Mathematics related references

[16] M. Pitkänen (2002), *A Strategy for Proving Riemann Hypothesis*, matharXiv.org/0111262.

[17] M. Pitkänen (2003), *A Strategy for Proving Riemann Hypothesis*, Acta Math. Univ. Comeniae, vol. 72.

[18] Pope, C., N. (1980): *Eigenfunctions and Spin^c Structures on CP_2* D.A.M.T.P. preprint.

Theoretical Physics

[19] P. Goddard, A. Kent, D. Olive (1986), Commun. Math. Phys 103, 105-119.

[20] J. Rasmussen (2000), *Comments on $N = 4$ super-conformal algebras*,
hep-th/0003035. This version talks about $SL(2)\times U(1)$ Kac-Moody algebra. A slightly different variant of the article with same name http://www.pims.math.ca talks about $SU(2) \times U(1)$ Kac-Moody algebra so that it both versions seem to be possible.

[21] S. Gukov *et al*(2004), *An Index for 2D field theories with large N=4 super-conformal symmetry.* hep-th/0404023.

[22] Schwartz, J., H. (ed) (1985): *Super strings. The first 15 years of Superstring Theory.* World Scientific.

[23] Green, M., B., Schwartz, J., H. and Witten,E. (1987): *Superstring Theory.* Cambridge University Press.

[24] C. Kassel (1995), *Quantum Groups*, Springer Verlag.

[25] C. Gomez, M. Ruiz-Altaba, G. Sierra (1996), *Quantum Groups and Two-Dimensional Physics*, Cambridge University Press.

[26] P.Dorey (1998). *Exact S-matrices*, arXiv.org:hep-th/9810026.

[27] C. N. Yang, M. L. Ge (1989), *Braid Group, Knot Theory, and Statistical Mechanics*, World Scientific.

[28] D. Zagier (1994), *Values of Zeta Functions and Their Applications*, First European Congress of Mathematics (Paris, 1992), Vol. II, Progress in Mathematics 120, Birkhauser, 497-512.

[29] U. Mueller and C. Schubert (2002), *A Quantum Field Theoretical Representation of Euler-Zagier Sums*, arXiv:math.QA/9908067. Int. J. Math. Sc. Vol 31, issue 3 (2002), 127-148.

[30] M. Kontsevich (1999), *Operads and Motives in Deformation Quantization*, arXiv: math.QA/9904055.

[31] D. J. Boradhurst and D. Kreimer (1996), *Association of multiple zeta values with Feynman diagrams up to 9 loops*, arXiv: hep-th/96069128.

[32] P. Cartier (2001), *A Mad Day's Work: From Grothendienck to Connes and Kontsevich: the Evolution of Concepts of Space and Symmetry*, Bulletin of the American Mathematical Society, Vol 38, No 4, pp. 389-408.

Physics related references

[33] M. Derrick *et al*(1993), Phys. Lett B 315, p. 481.

[34] E. S. Reich (2005), *Black hole like phenomenon created by collider*, New Scientist 19, issue 2491.

[35] T. Ludham and L. McLerran (2003), *What Have We Learned From the Relativistic Heavy Ion Collider?*, Physics Today, October issue.
http://www.physicstoday.org/vol-56/iss-10/p48.html.

[36] H. Muir (2000),*Trailblazer*, New Scientist, 23, September, No 2257.

[37] D. B. Kaplan, A. E. Nelson and N. Weiner (2004), *Neutrino Oscillations as a Probe of Dark Energy*,hep-ph/0401099.

[38] A. Cohen and S. Glashow (2006), *Very special relativity*, Phys. Rev. Lett. 97, 021601.
S. Glashow (2006), *Simplicity, Symmetry and Small Matrices.*
http://www.df.unibo.it/honorem2006/Glashow-laurea-eng.pdf.
See also the New Scientist article *Sending Einstein into Spin* at http://www.newscientist.com/contents/issue/2587.html.