Elementary Particle Vacuum Functionals

M. Pitkänen[1], February 1, 2006

[1] Department of Physical Sciences, High Energy Physics Division,
PL 64, FIN-00014, University of Helsinki, Finland.
matpitka@rock.helsinki.fi, http://www.physics.helsinki.fi/∼matpitka/.
Recent address: Puutarhurinkatu 10,10960, Hanko, Finland.

Contents

Abstract

Genus-generation correspondence is one of the basic ideas of TGD approach. In order to answer various questions concerning the plausibility of the idea, one should know something about the dependence of the elementary particle vacuum functionals on the vibrational degrees of freedom for the boundary component. The construction of the elementary particle vacuum functionals based on Diff invariance, 2-dimensional conformal symmetry, modular invariance plus natural stability requirements indeed leads to an essentially unique form of the vacuum functionals and one can understand why $g > 2$ bosonic families are experimentally absent and why lepton numbers are conserved separately.

An argument suggesting that the number of the light fermion families is three, is developed. The argument goes as follows. Elementary particle vacuum functionals represent bound states of g handles and vanish identically for hyper-elliptic surfaces having $g > 2$. Since all $g \leq 2$ surfaces are hyper-elliptic, $g \leq 2$ and $g > 2$ elementary particles cannot appear in same non-vanishing vertex and therefore decouple. The $g > 2$ vacuum functionals not vanishing for hyper-elliptic surfaces represent many particle states of $g \leq 2$ elementary particle states being thus unstable against the decay to $g \leq 2$ states. The failure of Z_2 conformal symmetry for $g > 2$ elementary particle vacuum functionals would in turn explain why they are heavy: this however not absolutely necessary since these particles would behave like dark matter in any case.

1 Introduction

One of the basic ideas of TGD approach is genus-generation correspondence: boundary components of the 3-surface should be carriers of elementary particle numbers and the observed particle families should correspond to various boundary topologies. A more general hypothesis is that the 2-surfaces in question sections of 3-D lightlike causal determinants, say those associated with wormhole contacts carrying parton quantum numbers

1.1 First series of questions

The most attractive feature of this idea is universality: if the generalized string model vertices are identified as particle vertices, different particle families are predicted to behave identically with respect to the known interactions in accordance with observational facts.

Before one can accept this identification, one should however answer several questions:

a) Also elementary bosons are predicted to possess family degeneracy: why the higher boson families have not been observed? Why only $g = 0$, "spherical", bosons seem to be the bosons produced in particle accelerators? Are $g > 0$ bosons very massive or are their couplings to fermions very small?

b) Topological reactions changing the genus of boundary component are possible (some of the handles of 2-surfaces suffers pinch or new handle is created): why however different lepton numbers are conserved in such a good approximation?

c) Why the number of the observed elementary particle families seems to be three [27]?

1.2 Second series of questions

The questions above are obvious if one accepts string model picture about particle vertices. 25 years with TGD however leads to question the string model based interpretation of particle vertices and stimulates a slightly different series of questions.

a) What really happens in particle vertices? Is the generalization of string model diagrams the proper description of particle reactions in TGD framework? Or should one assume that vertices are direct generalizations of ordinary Feynmann diagrams so that the Feynmann diagrams correspond to singular 4-manifolds and vertices to non-singular 3-manifolds at which the ends of space-time sheets representing particles meet? The elegant treatment of fermion number and other conserved quantum numbers in the vertices and construction of the vertices themselves [C7] provides a considerable support for this view. In this framework string model type vertices would be interpreted in terms of a propagation of the particle through several paths simultaneously as in double-slit experiment.

b) The new picture about vertices predicts a profound difference between fermions and bosons: the lowest bosonic vacuum wave functionals must be completely delocalized with respect to the genus to guarantee that the gauge couplings to the fermions are universal. Why this delocalization does not occur for fermions as the successful calculation of elementary particle masses strongly suggests [6]? Why would bosonic families correspond to a hierarchy of delocalized states having $g < 3$ with a phase phase factor $expi2\pi ng/3$, $n = 0, 1, 2$ characterizing the particle family. Why would fermions correspond to states localized to $g \leq 2$? What makes bells ringing is that for topologically delocalized bosons the finiteness of the vertices would require an effective reduction of the number of particle families to a finite number N. For instance, one can consider a decomposition of the lattice $\{g \geq 0\}$ to disjoint sublattices with a complete bosonic delocalization inside each lattice.

c) Why the number of genera is just three? $g \leq 2$ Riemann surfaces are always hyper-elliptic (have global Z_2 conformal symmetry) unlike $g > 2$ surfaces. Why the complete bosonic de-localization of the light families should be restricted inside the hyper-elliptic sector? Could the reason be that $g > 2$ elementary particle vacuum functionals vanish for hyper-elliptic surfaces so that states localized to $g \leq 2$ surfaces are not transformed to $g > 2$ surfaces? Does the Z_2 symmetry make these states light?

d) There is also a second intriguing observation. Configuration space Clifford algebra is a direct integral over von Neumann algebras known as hyperfinite factors of type II_1 [21, C8]. The hierarchy of Jones inclusions for von Neumann algebras is characterized by a quantum phase $q = exp(i\pi/N)$, $N \geq 3$. $N = 3$ corresponds to the simplest algebraic extension of rationals and is TGD framework physically completely unique as compared to $N > 3$ since the value of the inverse of $\hbar$ vanishes for $N = 3$ apart from small gravitational corrections [C8]. The huge value of Planck constant means maximal quantum coherence time natural for elementary particles.

Is the number of light particle families three because elementary particles correspond to the lowest level in the hierarchy of Jones inclusions and to the maximally quantal situation perhaps correlating with the hyper-elliptic symmetry? Could the lattice $\{g \geq 0\}$ decompose into a union of disjoint de-localization sub-lattices with $n = 3, 4, 5...$ elements corresponding to $q = exp(i\pi/n)$?

1.3 The notion of elementary particle vacuum functional

In order to provide answers to either series of questions one must know something about the dependence of the elementary particle state functionals on the geometric properties of the boundary component and in the sequel an attempt to construct what might be called elementary particle vacuum functionals, is made. Irrespective of what identification of interaction vertices is adopted, the arguments involved with the construction involve only the string model type vertices so that the previous discussion seems to apply more or less as such.

The basic assumptions underlying the construction are the following ones:

a) Elementary particle vacuum functionals depend on the geometric properties of the two-surface X^2 representing elementary particle.

b) Vacuum functionals possess extended Diff invariance: all 2-surfaces on the orbit of the 2-surface X^2 correspond to the same value of the vacuum functional. This condition is satisfied if vacuum functionals have as their argument, not X^2 as such, but some 2- surface Y^2 belonging to the unique orbit of X^2 (determined by the principle selecting preferred extremal of the Kähler action as a generalized Bohr orbit [B1]) and determined in $Diff^3$ invariant manner.

c) Vacuum functionals possess conformal invariance and therefore for a given genus depend on a finite number of variables specifying the conformal equivalence class of Y^2.

d) Vacuum functionals satisfy the cluster decomposition property: when the surface Y^2 degenerates to a union of two disjoint surfaces (particle decay in string model inspired picture), vacuum functional decomposes into a product of the vacuum functionals associated with disjoint surfaces.

e) Elementary particle vacuum functionals are stable against the two-particle decay $g \rightarrow g_1 + g_2$ and one particle decay $g \rightarrow g - 1$.

In the following the construction will be described in more detail.

i) Some basic concepts related to the description of the space of the conformal equivalence classes of Riemann surfaces are introduced and the concept of hyper-ellipticity is introduced. Since theta functions will play a central role in the construction of the vacuum functionals, also their basic properties are discussed.

ii) After these preliminaries the construction of elementary particle vacuum functionals is carried out.

iii) Possible explanations for the experimental absence of the higher fermion families are considered.

2 Basic facts about Riemann surfaces

In the following some basic aspects about Riemann surfaces will be summarized. The basic topological concepts, in particular the concept of the mapping class group, are introduced, and the Teichmueller parameters are defined as conformal invariants of the Riemann surface, which in fact specify the conformal equivalence class of the Riemann surface completely.

2.1 Mapping class group

The first homology group $H_1(X^2)$ of a Riemann surface of genus g contains $2g$ generators [17, 19, 18]: this is easy to understand geometrically since

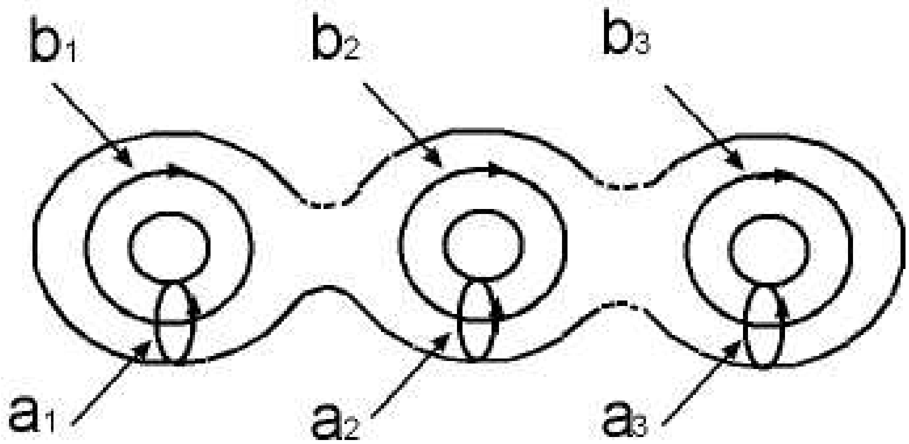

Figure 1: Definition of the canonical homology basis

each handle contributes two homology generators. The so called canonical homology basis can be identified as in Fig. 2.1.

One can define the so called intersection number $J(a,b)$ for two elements a and b of the homology group as the number of intersection points for the curves a and b counting the orientation. Since $J(a,b)$ depends on the homology classes of a and b only, it defines an antisymmetric quadratic form in $H_1(X^2)$. In the canonical homology basis the non-vanishing elements of the intersection matrix are:

$$J(a_i,b_j) \quad = \quad -J(b_j,a_i) = \delta_{i,j} \ . \tag{1}$$

J clearly defines symplectic structure in the homology group.

The dual to the canonical homology basis consists of the harmonic one-forms $\alpha_i, \beta_i, i = 1,..,g$ on X^2. These 1-forms satisfy the defining conditions

$$\begin{array}{ll} \int_{a_i} \alpha_j = \delta_{i,j} & \int_{b_i} \alpha_j = 0 \ , \\ \int_{a_i} \beta_j = 0 & \int_{b_i} \beta_j = \delta_{i,j} \ . \end{array} \tag{2}$$

The following identity helps to understand the basic properties of the Teichmueller parameters

$$\int_{X^2} \theta \wedge \eta \;\; = \;\; \sum_{i=1,..,g} [\int_{a_i} \theta \int_{b_i} \eta - \int_{b_i} \theta \int_{a_i} \eta] \ . \tag{3}$$

The existence of topologically nontrivial diffeomorphisms, when X^2 has genus $g > 0$, plays an important role in the sequel. Denoting by $Diff$ the group of the diffeomorphisms of X^2 and by $Diff_0$ the normal subgroup of the diffeomorphisms homotopic to identity, one can define the mapping class group M as the coset group

$$M \;\; = \;\; Diff/Diff_0 \ . \tag{4}$$

The generators of M are so called Dehn twists along closed curves a of X^2. Dehn twist is defined by excising a small tubular neighborhood of a, twisting one boundary of the resulting tube by 2π and gluing the tube back into the surface: see Fig. 2.1.

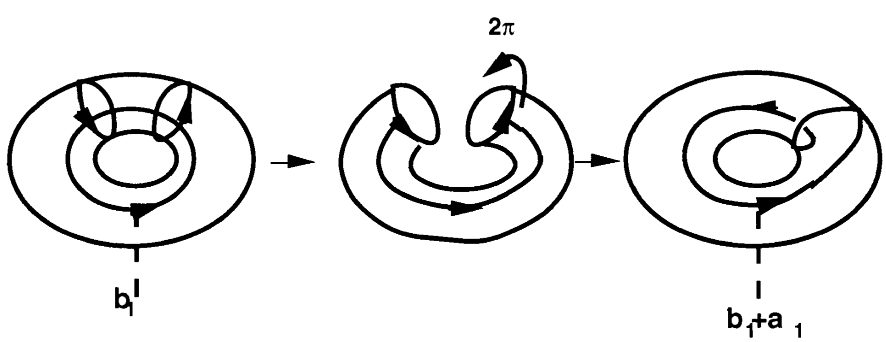

Figure 2: Definition of the Dehn twist

It can be shown that a minimal set of generators is defined by the following curves

$$a_1, b_1, a_1^{-1}a_2^{-1}, a_2, b_2, a_2^{-1}a_3^{-11}, ..., a_g, b_g \quad . \tag{5}$$

The action of these transformations in the homology group can be regarded as a symplectic linear transformation preserving the symplectic form defined by the intersection matrix. Therefore the matrix representing the action of $Diff$ on $H_1(X^2)$ is $2g \times 2g$ matrix M with integer entries leaving J invariant: $MJM^T = J$. Mapping class group is often referred also as a symplectic modular group and denoted by $Sp(2g, Z)$. The matrix representing the action of M in the canonical homology basis decomposes into four $g \times g$ blocks A, B, C and D

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad , \tag{6}$$

where A and D operate in the subspaces spanned by the homology generators a_i and b_i respectively and C and D map these spaces to each other. The notation $D = [A, B; C, D]$ will be used in the sequel: in this notation the representation of the symplectic form J is $J = [0, 1; -1, 0]$.

2.2 Teichmueller parameters

The induced metric on the two-surface X^2 defines a unique complex structure. Locally the metric can always be written in the form

$$ds^2 \;\; = \;\; e^{2\phi} dz d\bar{z} \quad . \tag{7}$$

where z is local complex coordinate. When one covers X^2 by coordinate patches, where the line element has the above described form, the transition functions between coordinate patches are holomorphic and therefore define a complex structure.

The conformal transformations ξ of X^2 are defined as the transformations leaving invariant the angles between the vectors of X^2 tangent space invariant: the angle between the vectors X and Y at point x is same as the angle between the images of the vectors under Jacobian map at the image point $\xi(x)$. These transformations need not be globally defined and in each coordinate patch they correspond to holomorphic (anti-holomorphic) mappings as is clear from the diagonal form of the metric in the local complex coordinates. A distinction should be made between local conformal transformations and globally defined conformal transformations, which will be referred to as conformal symmetries: for instance, for hyper-elliptic surfaces the group of the conformal symmetries contains two-element group Z_2.

Using the complex structure one can decompose one-forms to linear combinations of one-forms of type $(1,0)$ $(f(z,\bar{z})dz)$ and $(0,1)$ $(f(z,\bar{z})d\bar{z})$. $(1,0)$ form ω is holomorphic if the function f is holomorphic: $\omega = f(z)dz$ on each coordinate patch.

There are g independent holomorphic one forms ω_i known also as Abelian differentials of the first kind [17, 19, 18] and one can fix their normalization by the condition

$$\int_{a_i} \omega_j = \delta_{ij} \ . \tag{8}$$

This condition completely specifies ω_i.

Teichmueller parameters Ω_{ij} are defined as the values of the forms ω_i for the homology generators b_j

$$\Omega_{ij} = \int_{b_j} \omega_i \ . \tag{9}$$

The basic properties of Teichmueller parameters are the following:
i) The $g \times g$ matrix Ω is symmetric: this is seen by applying the formula (3) for $\theta = \omega_i$ and $\eta = \omega_j$.
ii) The imaginary part of Ω is positive: $Im(\Omega) > 0$. This is seen by the application of the same formula for $\theta = \eta$. The space of the matrices satisfying these conditions is known as Siegel upper half plane.
iii) The space of Teichmueller parameters can be regarded as a coset space $Sp(2g,R)/U(g)$ [19]: the action of $Sp(2g,R)$ is of the same form as the action of $Sp(2g,Z)$ and $U(g) \subset Sp(2g,R)$ is the isotropy group of a given point of Teichmueller space.
iv) Teichmueller parameters are conformal invariants as is clear from the holomorphy of the defining one-forms.
v) Teichmueller parameters specify completely the conformal structure of Riemann surface [18].

Although Teichmueller parameters fix the conformal structure of the 2-surface completely, they are not in one-to-one correspondence with the conformal equivalence classes of the two-surfaces:

i) The dimension for the space of the conformal equivalence classes is $D = 3g-3$, when $g>1$ and smaller than the dimension of Teichmueller space given by $d = (g\times g+g)/2$ for $g>3$: all Teichmueller matrices do not correspond to a Riemann surface. In TGD approach this does not produce any problems as will be found later.
ii) The action of the topologically nontrivial diffeomorphisms on Teichmueller parameters is nontrivial and can be deduced from the action of the diffeomorphisms on the homology ($Sp(2g,Z)$ transformation) and from the defining condition $\int_{a_i}\omega_j = \delta_{i,j}$: diffeomorphisms correspond to elements $[A,B;C,D]$ of $Sp(2g,Z)$ and act as generalized Möbius transformations

$$\Omega \rightarrow (A\Omega+B)(C\Omega+D)^{-1} \ . \tag{10}$$

All Teichmueller parameters related by $Sp(2g,Z)$ transformations correspond to the same Riemann surface.
iii) The definition of the Teichmueller parameters is not unique since the definition of the canonical homology basis involves an arbitrary numbering of the homology basis. The permutation S of the handles is represented by same $g\times g$ orthogonal matrix both in the basis $\{a_i\}$ and $\{b_i\}$ and induces a similarity transformation in the space of the Teichmueller parameters

$$\Omega \rightarrow S\Omega S^{-1} \ . \tag{11}$$

Clearly, the Teichmueller matrices related by a similarity transformations correspond to the same conformal equivalence class. It is easy to show that handle permutations in fact correspond to $Sp(2g,Z)$ transformations.

2.3 Hyper-ellipticity

The motivation for considering hyper-elliptic surfaces comes from the fact, that $g>2$ elementary particle vacuum functionals turn out to be vanishing for hyper-elliptic surfaces and this in turn will be later used to provide a possible explanation the non-observability of $g>2$ particles.

Hyper-elliptic surface X can be defined abstractly as two-fold branched cover of the sphere having the group Z_2 as the group of conformal symmetries (see [19, 16, 18]. Thus there exists a map $\pi: X\rightarrow S^2$ so that the inverse image $\pi^{-1}(z)$ for a given point z of S^2 contains two points except at a finite number (say p) of points z_i (branch points) for which the inverse image contains only one point. Z_2 acts as conformal symmetries permuting the two points in $\pi^{-1}(z)$ and branch points are fixed points of the involution.

The concept can be generalized [16]: g-hyper-elliptic surface can be defined as a 2-fold covering of genus g surface with a finite number of branch points. One can consider also p-fold coverings instead of 2-fold coverings: a common feature of these Riemann surfaces is the existence of a discrete group of conformal symmetries.

A concrete representation for the hyper-elliptic surfaces [19] is obtained by studying the surface of C^2 determined by the algebraic equation

$$w^2 - P_n(z) \quad = \quad 0 \ , \tag{12}$$

where w and z are complex variables and $P_n(z)$ is a complex polynomial. One can solve w from the above equation

$$w_{\pm} \quad = \quad \pm\sqrt{P_n(z)} \ , \tag{13}$$

where the square root is determined so that it has a cut along the positive real axis. What happens that w has in general two roots (two-fold covering property), which coincide at the roots z_i of $P_n(z)$ and if n is odd, also at $z = \infty$: these points correspond to branch points of the hyper-elliptic surface and their number r is always even: $r = 2k$. w is discontinuous at the cuts associated with the square root in general joining two roots of $P_n(z)$ or if n is odd, also some root of P_n and the point $z = \infty$. The representation of the hyper-elliptic surface is obtained by identifying the two branches of w along the cuts. From the construction it is clear that the surface obtained in this manner has genus $k-1$. Also it is clear that Z_2 permutes the different roots $w_{\pm}$ with each other and that $r = 2k$ branch points correspond to fixed points of the involution.

The following facts about the hyper-elliptic surfaces [19, 18] turn out to be important in the sequel:
i) All $g < 3$ surfaces are hyper-elliptic.
ii) $g \geq 3$ hyper-elliptic surfaces are not in general hyper-elliptic and form a set of codimension 2 in the space of the conformal equivalence classes [19].

2.4 Theta functions

An extensive and detailed account of the theta functions and their applications can be found in the book of Mumford [19]. Theta functions appear also in the loop calculations of string model [17]. In the following the so called Riemann theta function and theta functions with half integer characteristics will be defined as sections (not strictly speaking functions) of the so called Jacobian variety.

For a given Teichmueller matrix Ω, Jacobian variety is defined as the $2g$-dimensional torus obtained by identifying the points z of C^g (vectors with g complex components) under the equivalence

$$z \sim z + \Omega m + n \ , \tag{14}$$

where m and n are points of Z^g (vectors with g integer valued components) and Ω acts in Z^g by matrix multiplication.

The definition of Riemann theta function reads as

$$\Theta(z|\Omega) = \sum_n exp(i\pi n \cdot \Omega \cdot n + i2\pi n \cdot z) \ . \tag{15}$$

Here $\cdot$ denotes standard inner product in C^g. Theta functions with half integer characteristics are defined in the following manner. Let a and b denote vectors of C^g with half integer components (component either vanishes or equals to 1/2). Theta function with characteristics $[a,b]$ is defined through the following formula

$$\Theta[a,b](z|\Omega) = \sum_n exp\left[i\pi(n+a)\cdot\Omega\cdot(n+a) + i2\pi(n+a)\cdot(z+b)\right] \ . \tag{16}$$

A brief calculation shows that the following identity is satisfied

$$\Theta[a,b](z|\Omega) = exp(i\pi a\cdot\Omega\cdot a + i2\pi a\cdot b)\times\Theta(z+\Omega a+b|\Omega) \tag{17}$$

Theta functions are not strictly speaking functions in the Jacobian variety but rather sections in an appropriate bundle as can be seen from the identities

$$\begin{aligned}
\Theta[a,b](z+m|\Omega) &= exp(i2\pi a\cdot m)\Theta[a,b](z\Omega) \ , \\
\Theta[a,b](z+\Omega m|\Omega) &= exp(\alpha)\Theta[a,b](z|\Omega) \ , \\
exp(\alpha) &= exp(-i2\pi b\cdot m)exp(-i\pi m\cdot\Omega\cdot m - 2\pi m\cdot z) \ .
\end{aligned} \tag{18}$$

The number of theta functions is 2^{2g} and same as the number of nonequivalent spinor structures defined on two-surfaces. This is not an accident [17]: theta functions with given characteristics turn out to be in a close relation to the functional determinants associated with the Dirac operators defined on the two-surface. It is useful to divide the theta functions to even and odd theta functions according to whether the inner product $4a \cdot b$ is even or odd integer. The numbers of even and odd theta functions are $2^{g-1}(2^g+1)$ and $2^{g-1}(2^g-1)$ respectively.

The values of the theta functions at the origin of the Jacobian variety understood as functions of Teichmueller parameters turn out to be of special interest in the following and the following notation will be used:

$$\Theta[a,b](\Omega) \quad \equiv \quad \Theta[a,b](0|\Omega) \ , \tag{19}$$

$\Theta[a,b](\Omega)$ will be referred to as theta functions in the sequel. From the defining properties of odd theta functions it can be found that they are odd functions of z and therefore vanish at the origin of the Jacobian variety so that only even theta functions will be of interest in the sequel.

An important result is that also some *even* theta functions vanish for $g>2$ hyper-elliptic surfaces : in fact one can characterize $g>2$ hyper-elliptic surfaces by the vanishing properties of the theta functions [19, 18]. The vanishing property derives from conformal symmetry (Z_2 in the case of hyper-elliptic surfaces) and the vanishing phenomenon is rather general [16]: theta functions tend to vanish for Riemann surfaces possessing discrete conformal symmetries. It is not clear (to the author) whether the presence of a conformal symmetry is in fact equivalent with the vanishing of some theta functions. As already noticed, spinor structures and the theta functions with half integer characteristics are in one-to-one correspondence and the vanishing of theta function with given half integer characteristics is equivalent with the vanishing of the Dirac determinant associated with the corresponding spinor structure or equivalently: with the existence of a zero mode for the Dirac operator [17]. For odd characteristics zero mode exists always: for even characteristics zero modes exist, when the surface is hyper-elliptic or possesses more general conformal symmetries.

3 Elementary particle vacuum functionals

The basic assumption is that elementary particle families correspond to various elementary particle vacuum functionals associated with the 2-dimensional boundary components of the 3-surface. These functionals need not be localized to a single boundary topology. Neither need their dependence on the boundary component be local. An important role in the following considerations is played by the fact that the minimization requirement of the Kähler action associates a unique 3-surface to each boundary component, the "Bohr orbit" of the boundary and this surface provides a considerable (and necessarily needed) flexibility in the definition of the elementary particle vacuum functionals. There are several natural constraints to be satisfied by elementary particle vacuum functionals.

3.1 Extended Diff invariance and Lorentz invariance

Extended Diff invariance is completely analogous to the extension of 3-dimensional Diff invariance to four-dimensional Diff invariance in the interior of the 3-surface. Vacuum functional must be invariant not only under diffeomorphisms of the boundary component but also under the diffeomorphisms of the 3- dimensional "orbit" Y^3 of the boundary component. In other words: the value of the vacuum functional must be same for any time slice on the orbit the boundary component. This is guaranteed if vacuum functional is functional of some two-surface Y^2 belonging to the orbit and defined in $Diff^3$ invariant manner.

An additional natural requirement is Poincare invariance. In the original formulation of the theory only Lorentz transformations of the light cone were exact symmetries of the theory. In this framework the definition of Y^2 as the intersection of the orbit with the hyperboloid $\sqrt{m_{kl}m^k m^l} = a$ is $Diff^3$ and Lorentz invariant.

3.1.1 Interaction vertices as generalization of stringy vertices

For stringy diagrams Poincare invariance of conformal equivalence class and general coordinate invariance are far from being a trivial issues. Vertices are now not completely unique since there is an infinite number of singular 3-manifolds which can be identified as vertices even if one assumes space-likeness. One should be able to select a unique singular 3-manifold to fix the conformal equivalence class.

One might hope that Lorentz invariant invariant and general coordinate invariant definition of Y^2 results by introducing light cone proper time a as a height function specifying uniquely the point at which 3-surface is singular (stringy diagrams help to visualize what is involved), and by restricting the singular 3-surface to be the intersection of $a = constant$ hyperboloid of M^4

containing the singular point with the space-time surface. There would be non-uniqueness of the conformal equivalence class due to the choice of the origin of the light cone but the decomposition of the configuration space of 3-surfaces to a union of configuration spaces characterized by unions of future and past light cones could resolve this difficulty.

3.1.2 Interaction vertices as generalization of ordinary ones

If the interaction vertices are identified as intersections for the ends of space-time sheets representing particles, the conformal equivalence class is naturally identified as the one associated with the intersection of the boundary component or light like causal determinant with the vertex. Poincare invariance of the conformal equivalence class and generalized general coordinate invariance follow trivially in this case.

3.2 Conformal invariance

Conformal invariance implies that vacuum functionals depend on the conformal equivalence class of the surface Y^2 only. What makes this idea so attractive is that for a given genus g configuration space becomes effectively finite-dimensional. A second nice feature is that instead of trying to find coordinates for the space of the conformal equivalence classes one can construct vacuum functionals as functions of the Teichmueller parameters.

That one can construct this kind of functions as suitable functions of the Teichmueller parameters is not trivial. The essential point is that the boundary components can be regarded as submanifolds of $M^4_+ \times CP_2$: as a consequence vacuum functional can be regarded as a composite function:

2-surface $\rightarrow$ Teichmueller matrix Ω determined by the induced metric $\rightarrow \Omega_{vac}(\Omega)$

Therefore the fact that there are Teichmueller parameters which do not correspond to any Riemann surface, doesn't produce any trouble. It should be noticed that the situation differs from that in the Polyakov formulation of string models, where one doesn't assume that the metric of the two-surface is induced metric (although classical equations of motion imply this).

3.3 Diff invariance

Since several values of the Teichmueller parameters correspond to the same conformal equivalence class, one must pose additional conditions on the functions of the Teichmueller parameters in order to obtain single valued functions of the conformal equivalence class.

The first requirement of this kind is the invariance under topologically nontrivial Diff transformations inducing $Sp(2g, Z)$ transformation $(A, B; C, D)$ in the homology basis. The action of these transformations on Teichmueller parameters is deduced by requiring that holomorphic one-forms satisfy the defining conditions in the transformed homology basis. It turns out that the action of the topologically nontrivial diffeomorphism on Teichmueller parameters can be regarded as a generalized Möbius transformation:

$$\Omega \rightarrow (A\Omega + B)(C\Omega + D)^{-1} \ . \tag{20}$$

Vacuum functional must be invariant under these transformations. It should be noticed that the situation differs from that encountered in the string models. In TGD the integration measure over the configuration space is Diff invariant: in string models the integration measure is the integration measure of the Teichmueller space and this is not invariant under $Sp(2g, Z)$ but transforms like a density: as a consequence the counterpart of the vacuum functional must be also modular covariant since it is the product of vacuum functional and integration measure, which must be modular invariant.

It is possible to show that the quantities

$$(\Theta[a, b]/\Theta[c, d])^4 \ . \tag{21}$$

and their complex conjugates are $Sp(2g, Z)$ invariants [19] and therefore can be regarded as basic building blocks of the vacuum functionals.

Teichmueller parameters are not uniquely determined since one can always perform a permutation of the g handles of the Riemann surface inducing a redefinition of the canonical homology basis (permutation of g generators). These transformations act as similarities of the Teichmueller matrix:

$$\Omega \rightarrow S\Omega S^{-1} \ , \tag{22}$$

where S is the $g \times g$ matrix representing the permutation of the homology generators understood as orthonormal vectors in the g- dimensional vector space. Therefore the Teichmueller parameters related by these similarity transformations correspond to the same conformal equivalence class of the

Riemann surfaces and vacuum functionals must be invariant under these similarities.

It is easy to find out that these similarities permute the components of the theta characteristics: $[a,b] \rightarrow [S(a), S(b)]$. Therefore the invariance requirement states that the handles of the Riemann surface behave like bosons: the vacuum functional constructed from the theta functions is invariant under the permutations of the theta characteristics. In fact, this requirement brings in nothing new. Handle permutations can be regarded as $Sp(2g, Z)$ transformations so that the modular invariance alone guarantees invariance under handle permutations.

3.4 Cluster decomposition property

Consider next the behavior of the vacuum functional in the limit, when boundary component with genus g splits to two separate boundary components of genera g_1 and g_2 respectively. The splitting into two separate boundary components corresponds to the reduction of the Teichmueller matrix Ω^g to a direct sum of $g_1 \times g_1$ and $g_2 \times g_2$ matrices ($g_1 + g_2 = g$):

$$\Omega^g = \Omega^{g_1} \oplus \Omega^{g_2} \ , \tag{23}$$

when a suitable definition of the Teichmueller parameters is adopted. The splitting can also take place without a reduction to a direct sum: the Teichmueller parameters obtained via $Sp(2g, Z)$ transformation from $\Omega^g = \Omega^{g_1} \oplus \Omega^{g_2}$ do not possess direct sum property in general.

The physical interpretation is obvious: the non-diagonal elements of the Teichmueller matrix describe the geometric interaction between handles and at this limit the interaction between the handles belonging to the separate surfaces vanishes. On the physical grounds it is natural to require that vacuum functionals satisfy cluster decomposition property at this limit: that is they reduce to the product of appropriate vacuum functionals associated with the composite surfaces.

Theta functions satisfy cluster decomposition property [19, 17]. Theta characteristics reduce to the direct sums of the theta characteristics associated with g_1 and g_2 ($a = a_1 \oplus a_2, \ b = b_1 \oplus b_2$) and the dependence on the Teichmueller parameters is essentially exponential so that the cluster decomposition property indeed results:

$$\Theta[a,b](\Omega^g) \ = \ \Theta[a_1,b_1](\Omega^{g_1})\Theta[a_2,b_2](\Omega^{g_2}) \ . \tag{24}$$

Cluster decomposition property holds also true for the products of theta functions. This property is also satisfied by suitable homogenous polynomials of thetas. In particular, the following quantity playing central role in the construction of the vacuum functional obeys this property

$$Q_0 \quad = \quad \sum_{[a,b]} \Theta[a,b]^4 \bar{\Theta}[a,b]^4 \ , \tag{25}$$

where the summation is over all even theta characteristics (recall that odd theta functions vanish at the origin of C^g).

Together with the $Sp(2g,Z)$ invariance the requirement of cluster decomposition property implies that the vacuum functional must be representable in the form

$$\Omega_{vac} \quad = \quad P_{M,N}(\Theta^4, \bar{\Theta}^4)/Q_{MN}(\Theta^4, \bar{\Theta}^4) \tag{26}$$

where the homogenous polynomials $P_{M,N}$ and $Q_{M,N}$ have same degrees (M and N as polynomials of $\Theta[a,b]^4$ and $\bar{\Theta}[a,b]^4$.

3.5 Finiteness requirement

Vacuum functional should be finite. Finiteness requirement is satisfied provided the numerator $Q_{M,N}$ of the vacuum functional is real and positive definite. The simplest quantity of this type is the quantity Q_0 defined previously and its various powers. $Sp(2g,Z)$ invariance and finiteness requirement are satisfied provided vacuum functionals are of the following general form

$$\Omega_{vac} \quad = \quad \frac{P_{N,N}(\Theta^4, \bar{\Theta}^4)}{Q_0^N} \ , \tag{27}$$

where $P_{N,N}$ is homogenous polynomial of degree N with respect to $\Theta[a,b]^4$ and $\bar{\Theta}[a,b]^4$. In addition $P_{N,N}$ is invariant under the permutations of the theta characteristics and satisfies cluster decomposition property.

3.6 Stability against the decay $g \rightarrow g_1 + g_2$

Elementary particle vacuum functionals must be stable against the genus conserving decays $g \rightarrow g_1 + g_2$. This decay corresponds to the limit at which Teichmueller matrix reduces to a direct sum of the matrices associated with g_1 and g_2 (note however the presence of $Sp(2g,Z)$ degeneracy). In accordance with the topological description of the particle reactions one expects that this decay doesn't occur if the vacuum functional in question vanishes at this limit.

In general the theta functions are non-vanishing at this limit and vanish provided the theta characteristics reduce to a direct sum of the odd theta characteristics. For $g<2$ surfaces this condition is trivial and gives no constraints on the form of the vacuum functional. For $g=2$ surfaces the theta function $\Theta(a,b)$, with $a=b=(1/2,1/2)$ satisfies the stability criterion identically (odd theta functions vanish identically), when Teichmueller parameters separate into a direct sum. One can however perform $Sp(2g,Z)$ transformations giving new points of Teichmueller space describing the decay. Since these transformations transform theta characteristics in a nontrivial manner to each other and since all even theta characteristics belong to same $Sp(2g,Z)$ orbit [19, 17], the conclusion is that stability condition is satisfied provided $g=2$ vacuum functional is proportional to the product of fourth powers of all even theta functions multiplied by its complex conjugate.

If $g>2$ there always exists some theta functions, which vanish at this limit and the minimal vacuum functional satisfying this stability condition is of the same form as in $g=2$ case, that is proportional to the product of the fourth powers of all even Theta functions multiplied by its complex conjugate:

$$\Omega_{vac} = \prod_{[a,b]} \Theta[a,b]^4 \bar{\Theta}[a,b]^4 / Q_0^N \ , \tag{28}$$

where N is the number of even theta functions. The results obtained imply that genus-generation correspondence is one to one for $g>1$ for the minimal vacuum functionals. Of course, the multiplication of the minimal vacuum functionals with functionals satisfying all criteria except stability criterion gives new elementary particle vacuum functionals: a possible physical identification of these vacuum functionals is most naturally as some kind of excited states.

One of the questions posed in the beginning was related to the experimental absence of $g>0$, possibly massless, elementary bosons. The proposed stability criterion suggests a nice explanation. The point is that elementary particles are stable against decays $g \rightarrow g_1 + g_2$ but not with respect to the decay $g \rightarrow g +$ *sphere*. As a consequence the direct emission of $g>0$

gauge bosons is impossible unlike the emission of $g = 0$ bosons: for instance the decay muon $\rightarrow$ electron $+(g = 1)$ photon is forbidden.

3.7 Stability against the decay $g \rightarrow g - 1$

This stability criterion states that the vacuum functional is stable against single particle decay $g \rightarrow g - 1$ and, if satisfied, implies that vacuum functional vanishes, when the genus of the surface is smaller than g. In stringy framework this criterion is equivalent to a separate conservation of various lepton numbers: for instance, the spontaneous transformation of muon to electron is forbidden. Notice that this condition doesn't imply that that the vacuum functional is localized to a single genus: rather the vacuum functional of genus g vanishes for all surfaces with genus smaller than g. This hierarchical structure should have a close relationship to Cabibbo-Kobayashi-Maskawa mixing of the quarks.

The stability criterion implies that the vacuum functional must vanish at the limit, when one of the handles of the Riemann surface suffers a pinch. To deduce the behavior of the theta functions at this limit, one must find the behavior of Teichmueller parameters, when i:th handle suffers a pinch. Pinch implies that a suitable representative of the homology generator a_i or b_i contracts to a point.

Consider first the case, when a_i contracts to a point. The normalization of the holomorphic one-form ω_i must be preserved so that that ω_i must behaves as $1/z$, where z is the complex coordinate vanishing at pinch. Since the homology generator b_i goes through the pinch it seems obvious that the imaginary part of the Teichmueller parameter $\Omega_{ii} = \int_{b_i} \omega_i$ diverges at this limit (this conclusion is made also in [19]): $Im(\Omega_{ii}) \rightarrow \infty$.

Of course, this criterion doesn't cover all possible manners the pinch can occur: pinch might take place also, when the components of the Teichmueller matrix remain finite. In the case of torus topology one finds that $Sp(2g, Z)$ element $(A, B; C, D)$ takes $Im(\Omega) = \infty$ to the point C/D of real axis. This suggests that pinch occurs always at the boundary of the Teichmueller space: the imaginary part of Ω_{ij} either vanishes or some matrix element of $Im(\Omega)$ diverges.

Consider next the situation, when b_i contracts to a point. From the definition of the Teichmueller parameters it is clear that the matrix elements Ω_{kl}, with $k, l \neq i$ suffer no change. The matrix element Ω_{ki} obviously vanishes at this limit. The conclusion is that i:th row of Teichmueller matrix vanishes at this limit. This result is obtained also by deriving the $Sp(2g, Z)$ transformation permuting a_i and b_i with each other: in case of torus this transformation reads $\Omega \rightarrow -1/\Omega$.

Consider now the behavior of the theta functions, when pinch occurs. Consider first the limit, when $Im(\Omega_{ii})$ diverges. Using the general definition of $\Theta[a,b]$ it is easy to find out that all theta functions for which the i:th component a_i of the theta characteristic is non-vanishing (that is $a_i = 1/2$) are proportional to the exponent $exp(-\pi\Omega_{ii}/4)$ and therefore vanish at the limit. The theta functions with $a_i = 0$ reduce to $g-1$ dimensional theta functions with theta characteristic obtained by dropping i:th components of a_i and b_i and replacing Teichmueller matrix with Teichmueller matrix obtained by dropping i:th row and column. The conclusion is that all theta functions of type $\Theta(a,b)$ with $a = (1/2, 1/2, ..., 1/2)$ satisfy the stability criterion in this case.

What happens for the $Sp(2g,Z)$ transformed points on the real axis? The transformation formula for theta function is given by [19, 17]

$$\Theta[a,b]((A\Omega+B)(C\Omega+D)^{-1}) = exp(i\phi)det(C\Omega+D)^{1/2}\Theta[c,d](\Omega) \ , \tag{29}$$

where

$$\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \left(\begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} (CD^T)_d/2 \\ (AB^T)_d/2 \end{pmatrix} \right) \ . \tag{30}$$

Here ϕ is a phase factor irrelevant for the recent purposes and the index d refers to the diagonal part of the matrix in question.

The first thing to notice is the appearance of the diverging square root factor, which however disappears from the vacuum functionals (P and Q have same degree with respect to thetas). The essential point is that theta characteristics transform to each other: as already noticed all even theta characteristics belong to the same $Sp(2g,Z)$ orbit. Therefore the theta functions vanishing at $Im(\Omega_{ii}) = \infty$ do not vanish at the transformed points. It is however clear that for a given Teichmueller parametrization of pinch some theta functions vanish always.

Similar considerations in the case $\Omega_{ik} = 0$, i fixed, show that all theta functions with $b = (1/2, ..., 1/2)$ vanish identically at the pinch. Also it is clear that for $Sp(2g,Z)$ transformed points one can always find some vanishing theta functions. The overall conclusion is that the elementary particle vacuum functionals obtained by using $g \to g_1 + g_2$ stability criterion satisfy also $g \to g-1$ stability criterion since they are proportional to the product of all even theta functions. Therefore the only nontrivial consequence of $g \to g-1$ criterion is that also $g=1$ vacuum functionals are of the same general form as $g>1$ vacuum functionals.

A second manner to deduce the same result is by restricting the consideration to the hyper-elliptic surfaces and using the representation of the theta functions in terms of the roots of the polynomial appearing in the definition of the hyper-elliptic surface [19]. When the genus of the surface is smaller than three (the interesting case), this representation is all what is needed since all surfaces of genus $g<3$ are hyper-elliptic.

Since hyper-elliptic surfaces can be regarded as surfaces obtained by gluing two compactified complex planes along the cuts connecting various roots of the defining polynomial it is obvious that the process $g \to g-1$ corresponds to the limit, when two roots of the defining polynomial coincide. This limit corresponds either to disappearance of a cut or the fusion of two cuts to a single cut. Theta functions are expressible as the products of differences of various roots (Thomae's formula [19])

$$\Theta[a,b]^4 \propto \prod_{i<j\in T}(z_i - z_j) \prod_{k<l\in CT}(z_k - z_l) \ , \tag{31}$$

where T denotes some subset of $\{1,2,...,2g\}$ containing $g+1$ elements and CT its complement. Hence the product of all even theta functions vanishes, when two roots coincide. Furthermore, stability criterion is satisfied only by the product of the theta functions.

Lowest dimensional vacuum functionals are worth of more detailed consideration.

i) $g=0$ particle family corresponds to a constant vacuum functional: by continuity this vacuum functional is constant for all topologies.

ii) For $g=1$ the degree of P and Q as polynomials of the theta functions is 24: the critical number of transversal degrees of freedom in bosonic string model! Probably this result is not an accident.

ii) For $g=2$ the corresponding degree is 80 since there are 10 even genus 2 theta functions.

There are large numbers of vacuum functionals satisfying the relevant criteria, which do not satisfy the proposed stability criteria. These vacuum functionals correspond either to many particle states or to unstable single particle states.

3.8 Continuation of the vacuum functionals to higher genus topologies

From continuity it follows that vacuum functionals cannot be localized to single boundary topology. Besides continuity and the requirements listed above, a natural requirement is that the continuation of the vacuum functional from the sector g to the sector $g+k$ reduces to the product of the original vacuum functional associated with genus g and $g=0$ vacuum functional at the limit when the surface with genus $g+k$ decays to surfaces with genus g and k: this requirement should guarantee the conservation of separate lepton numbers although different boundary topologies suffer mixing in the vacuum functional. These requirements are satisfied provided the continuation is constructed using the following rule:

Perform the replacement

$$\Theta[a,b]^4 \quad \rightarrow \quad \sum_{c,d} \Theta[a \oplus c, b \oplus d]^4 \tag{32}$$

for each fourth power of the theta function. Here c and d are Theta characteristics associated with a surface with genus k. The same replacement is performed for the complex conjugates of the theta function. It is straightforward to check that the continuations of elementary particle vacuum functionals indeed satisfy the cluster decomposition property and are continuous.

To summarize, the construction has provided hoped for answers to some questions stated in the beginning: stability requirements explain the separate conservation of lepton numbers and the experimental absence of $g>0$ elementary bosons. What has not not been explained is the experimental absence of $g>2$ fermion families. The vanishing of the $g>2$ elementary particle vacuum functionals for the hyper-elliptic surfaces however suggest a possible explanation: under some conditions on the surface X^2 the surfaces Y^2 are hyper-elliptic or possess some conformal symmetry so that elementary particle vacuum functionals vanish for them. This conjecture indeed might make sense since the surfaces Y^2 are determined by the asymptotic dynamics and one might hope that the surfaces Y^2 are analogous to the final states of a dissipative system.

4 Explanations for the absence of the $g>2$ elementary particles from spectrum

The decay properties of the intermediate gauge bosons [27] are consistent with the assumption that the number of the light neutrinos is $N=3$. Also cosmological considerations pose upper bounds on the number of the light neutrino families and $N = 3$ seems to be favored [28]. It must be however emphasized that p-adic considerations [F5] encourage the consideration the existence of higher genera with neutrino masses such that they are not produced in the laboratory at present energies. In any case, for TGD approach the finite number of light fermion families is a potential difficulty since genus-generation correspondence suggests that the number of the fermion (and possibly also boson) families is infinite. Therefore one had better to find a good argument showing that the number of the observed neutrino families, or more generally, of the observed elementary particle families, is small also in the world described by TGD.

It will be later found that also TGD inspired cosmology requires that the number of the effectively massless fermion families must be small after Planck time. This suggests that boundary topologies with handle number $g > 2$ are unstable and/or very massive so that they, if present in the spectrum, disappear from it after Planck time, which correspond to the value of the light cone proper time $a \simeq 10^{-11}$ seconds.

In accordance with the spirit of TGD approach it is natural to wonder whether some geometric property differentiating between $g > 2$ and $g < 3$ boundary topologies might explain why only $g < 3$ boundary components are observable. One can indeed find a good candidate for this kind of property: namely hyper-ellipticity, which states that Riemann surface is a two-fold branched covering of sphere possessing two-element group Z_2 as conformal automorphisms. All $g < 3$ Riemann surfaces are hyper-elliptic unlike $g > 2$ Riemann surfaces, which in general do not posses this property. Thus it is natural to consider the possibility that hyper-ellipticity or more general conformal symmetries might explain why only $g < 2$ topologies correspond to the observed elementary particles.

As regards to the present problem the crucial observation is that some even theta functions vanish for the hyper-elliptic surfaces with genus $g > 2$ [19]. What is essential is that these surfaces have the group Z_2 as conformal symmetries. Indeed, the vanishing phenomenon is more general. Theta functions tend to vanish for $g > 2$ two-surfaces possessing discrete group of conformal symmetries [16]: for instance, instead of sphere one can consider branched coverings of higher genus surfaces.

From the general expression of the elementary particle vacuum functional it is clear that elementary particle vacuum functionals vanish, when Y^2 is hyper-elliptic surface with genus $g > 2$ and one might hope that this is enough to explain why the number of elementary particle families is three.

4.1 Hyper-ellipticity implies the separation of $g \leq 2$ and $g > 2$ sectors to separate worlds

If the vertices are defined as intersections of space-time sheets of elementary particles and if elementary particle vacuum functionals are required to have Z_2 symmetry, the localization of elementary particle vacuum functionals to $g \leq 2$ topologies occurs automatically. Even if one allows as limiting case vertices for which 2-manifolds are pinched to topologies intermediate between $g > 2$ and $g \leq 2$ topologies, Z_2 symmetry present for both topological interpretations implies the vanishing of this kind of vertices. This applies also in the case of stringy vertices so that also particle propagation would respect the effective number of particle families. $g > 2$ and $g \leq 2$ topologies would behave much like their own worlds in this approach. This is enough to explain the experimental findings if one can understand why the $g > 2$ particle families are absent as incoming and outgoing states or are very heavy.

4.2 What about $g > 2$ vacuum functionals which do not vanish for hyper-elliptic surfaces?

The vanishing of all $g \geq 2$ vacuum functionals for hyper-elliptic surfaces cannot hold true generally. There must exist vacuum functionals which do satisfy this condition. This suggest that elementary particle vacuum functionals for $g > 2$ states have interpretation as bound states of g handles and that the more general states which do not vanish for hyper-elliptic surfaces correspond to many-particle states composed of bound states $g \leq 2$ handles and cannot thus appear as incoming and outgoing states. Thus $g > 2$ elementary particles would decouple from $g \leq 2$ states.

4.3 Should higher elementary particle families be heavy?

TGD predicts an entire hierarchy of scaled up variants of standard model physics for which particles do not appear in the vertices containing the known elementary particles and thus behave like dark matter [A1, C8]. Also $g > 2$ elementary particles would behave like dark matter and in principle there is no absolute need for them to be heavy.

The safest option would be that $g > 2$ elementary particles are heavy and the breaking of Z_2 symmetry for $g \geq 2$ states could guarantee this. p-Adic considerations lead to a general mass formula for elementary particles such that the mass of the particle is proportional to $\frac{1}{\sqrt{p}}$ [6]. Also the dependence of the mass on particle genus is completely fixed by this formula. What remains however open is what determines the p-adic prime associated with a particle with given quantum numbers. Of course, it could quite well occur that p is much smaller for $g > 2$ genera than for $g \leq 2$ genera.

5 Could also gauge bosons correspond to wormhole contacts?

The developments in the formulation of quantum TGD which have taken place during the period 2005-2007 [C1, C2, C3] suggest dramatic simplifications of the general picture discussed above. p-Adic mass calculations [F3, F4, F5] leave a lot of freedom concerning the detailed identification of elementary particles. The basic open question is whether the *theory is free at parton level* as suggested by the recent view about the construction of S-matrix and by the almost topological QFT property of quantum TGD at parton level [C2, C3]. Or more concretely: do partonic 2-surfaces carry only free many-fermion states or can they carry also bound states of fermions and anti-fermions identifiable as bosons?

What is known that Higgs boson corresponds naturally to a wormhole contact [C5]. The wormhole contact connects two space-time sheets with induced metric having Minkowski signature. Wormhole contact itself has an Euclidian metric signature so that there are two wormhole throats which are light-like 3-surfaces and would carry fermion and anti-fermion number in the case of Higgs. Irrespective of the identification of the remaining elementary particles MEs (massless extremals, topological light rays) would serve as space-time correlates for elementary bosons. Higgs type wormhole contacts would connect MEs to the larger space-time sheet and the coherent state of neutral Higgs would generate gauge boson mass and could contribute also to fermion mass.

The basic question is whether this identification applies also to gauge bosons (certainly not to graviton). This identification would imply quite a dramatic simplification since the theory would be free at single parton level and the only stable parton states would be fermions and anti-fermions. As will be found this identification allows to understand the dramatic difference between graviton and other gauge bosons and the weakness of gravitational coupling, gives a connection with the string picture of gravitons, and predicts that stringy states are directly relevant for nuclear and condensed matter physics as has been proposed already earlier [F8, J1, J2]. In order to avoid confusion it must be emphasized that this picture is not consistent with the older picture discussed in previous sections.

5.1 Option I: Only Higgs as a wormhole contact

The only possibility considered hitherto has been that elementary bosons correspond to partonic 2-surfaces carrying fermion-anti-fermion pair such that either fermion or anti-fermion has a non-physical polarization. For this option CP_2 type extremals condensed on MEs and travelling with light velocity would serve as a model for both fermions and bosons. MEs are not absolutely necessary for this option. The couplings of fermions and gauge bosons to Higgs would be very similar topologically. Consider now the counter arguments.

a) This option fails if the theory at partonic level is free field theory so that anti-fermions and elementary bosons cannot be identified as bound states of fermion and anti-fermion with either of them having non-physical polarization.

b) Mathematically oriented mind could also argue that the asymmetry between Higgs and elementary gauge bosons is not plausible whereas asymmetry between fermions and gauge bosons is. Mathematician could continue by arguing that if wormhole contacts with net quantum numbers of Higgs boson are possible, also those with gauge boson quantum numbers are unavoidable.

c) Physics oriented thinker could argue that since gauge bosons do not exhibit family replication phenomenon (having topological explanation in TGD framework) there must be a profound difference between fermions and bosons.

5.2 Option II: All elementary bosons as wormhole contacts

The hypothesis that quantum TGD reduces to a free field theory at parton level is consistent with the almost topological QFT character of the theory at this level. Hence there are good motivations for studying explicitly the consequences of this hypothesis.

5.2.1 Elementary bosons must correspond to wormhole contacts if the theory is free at parton level

Also gauge bosons could correspond to wormhole contacts connecting MEs [D1] to larger space-time sheet and propagating with light velocity. For this option there would be no need to assume the presence of non-physical fermion or anti-fermion polarization since fermion and anti-fermion would reside at different wormhole throats. Only the definition of what it is to be non-physical would be different on the light-like 3-surfaces defining the throats.

The difference would naturally relate to the different time orientations of wormhole throats and make itself manifest via the definition of light-like operator $o = x^k\gamma_k$ appearing in the generalized eigenvalue equation for the modified Dirac operator [B4, C1]. For the first throat o^k would correspond to a light-like tangent vector t^k of the partonic 3-surface and for the second throat to its M^4 dual $\hat{t}^k$ in a preferred rest system in M^4 (implied by the basic construction of quantum TGD). What is nice that this picture non-asks the question whether t^k or $\hat{t}^k$ should appear in the modified Dirac operator.

Rather satisfactorily, MEs (massless extremals, topological light rays) would be necessary for the propagation of wormhole contacts so that they would naturally emerge as classical correlates of bosons. The simplest model for fermions would be as CP_2 type extremals topologically condensed on MEs and for bosons as pieces of CP_2 type extremals connecting ME to the larger space-time sheet. For fermions topological condensation is possible to either space-time sheet.

5.2.2 What about light-like boundaries and macroscopic wormhole contacts?

Light-like boundaries of the space-time sheet can have macroscopic size and can carry free many-fermion states but not elementary bosons. Number theoretic braids and anyons might be assignable to these structures. Deformations of cosmic strings to magnetic flux tubes with a light-like outer boundary are especially interesting in this respect.

If the ends of a string like object move with light velocity as implied by the usual stringy boundary conditions they indeed define light-like 3-surfaces. Many-fermion states could be assigned at the ends of string. One could also connect in pairwise manner the ends of two time-like strings having opposite time orientation using two space-like strings so that the analog of boson state consisting of two wormhole contacts and analogous to graviton would result. "Wormhole throats" could have arbitrarily long distance in M^4.

Wormhole contacts can be regarded as slightly deformed CP_2 type extremals only if the size of M^4 projection is not larger than CP_2 size. The natural question is whether one can construct macroscopic wormhole contacts at all.

a) The throats of wormhole contacts cannot belong to vacuum extremals. One might however hope that small deformations of macrosopic vacuum extremals could yield non-vacuum wormhole contacts of macroscopic size.

b) A large class of macroscopic wormhole contacts which are vacuum extremals consists of surfaces of form $X_1^2 \times X_2^2 \subset (M^1 \times Y^2) \times E^3$, where Y^2 is Lagrangian manifold of CP_2 (induced Kähler form vanishes) and $M^4 = M^1 \times E^3$ represents decomposition of M^1 to time-like and space-like subspaces. X_2^2 is a stationary surface of E^3. Both $X_1^2 \subset M^1 \times CP_2$ and X_2^2 have an Euclidian signature of metric except at light-like boundaries $X_a^1 \times X_2^2$ and $X_b^1 \times X_2^2$ defined by ends of X_1^2 defining the throats of the wormhole contact.

c) This kind of vacuum extremals could define an extremely general class of macroscopic wormhole contacts as their deformations. These wormhole contacts describe an interaction of wormhole throats regarded as closed strings as is clear from the fact that X^2 can be visualized as an analog of closed string world sheet X_1^2 in $M^1 \times Y^2$ describing a reaction leading from a state with a given number of incoming closed strings to a state with a given number of outgoing closed strings which correspond to wormhole throats at the two space-time sheets involved.

5.2.3 Phase conjugate states and matter- antimatter asymmetry

By fermion number conservation fermion-boson and boson-boson couplings must involve the fusion of partonic 3-surfaces along their ends identified as wormhole throats. Bosonic couplings would differ from fermionic couplings only in that the process would be $2 \rightarrow 4$ rather than $1 \rightarrow 3$ at the level of throats.

The decay of boson to an ordinary fermion pair with fermion and anti-fermion at the same space-time sheet would take place via the basic vertex at which the 2-dimensional ends of light-like 3-surfaces are identified. The sign of the boson energy would tell whether boson is ordinary boson or its phase conjugate (say phase conjugate photon of laser light) and also dictate the sign of the time orientation of fermion and anti-fermion resulting in the decay.

The two space-time sheets of opposite time orientation associated with bosons would naturally serve as space-time correlates for the positive and negative energy parts of the zero energy state and the sign of boson energy would tell whether it is phase conjugate or not. In the case of fermions second space-time sheet is not absolutely necessary and one can imagine that fermions in initial/final states correspond to single space-time sheet of positive/negative time orientation.

Also a candidate for a new kind interaction vertex emerges. The splitting of bosonic wormhole contact would generate fermion and time-reversed anti-fermion having interpretation as a phase conjugate fermion. This process cannot correspond to a decay of boson to ordinary fermion pair. The splitting process could generate matter-antimatter asymmetry in the sense that fermionic antimatter would consist dominantly of negative energy anti-fermions at space-time sheets having negative time orientation [D5, D6].

This vertex would define the fundamental interaction between matter and phase conjugate matter. Phase conjugate photons are in a key role in TGD based quantum model of living matter. This involves model for memory as communications in time reversed direction, mechanism of intentional action involving signalling to geometric past, and mechanism of remote metabolism involving sending of negative energy photons to the energy reservoir [K1]. The splitting of wormhole contacts has been considered as a candidate for a mechanism realizing Boolean cognition in terms of "cognitive neutrino pairs" resulting in the splitting of wormhole contacts with net quantum numbers of Z^0 boson [J3, M6].

5.3 Graviton and other stringy states

Fermion and anti-fermion can give rise to only single unit of spin since it is impossible to assign angular momentum with the relative motion of wormhole throats. Hence the identification of graviton as single wormhole contact is not possible. The only conclusion is that graviton must be a superposition of fermion-anti-fermion pairs and boson-anti-boson pairs with coefficients determined by the coupling of the parton to graviton. Graviton-graviton pairs might emerge in higher orders. Fermion and anti-fermion would reside at the same space-time sheet and would have a non-vanishing relative angular momentum. Also bosons could have non-vanishing relative angular momentum and Higgs bosons must indeed possess it.

Gravitons are stable if the throats of wormhole contacts carry non-vanishing gauge fluxes so that the throats of wormhole contacts are connected by flux tubes carrying the gauge flux. The mechanism producing gravitons would the splitting of partonic 2-surfaces via the basic vertex. A connection with string picture emerges with the counterpart of string identified as the flux tube connecting the wormhole throats. Gravitational constant would relate directly to the value of the string tension.

The TGD view about coupling constant evolution [C5] predicts $G \propto L_p^2$, where L_p is p-adic length scale, and that physical graviton corresponds to $p = M_{127} = 2^{127} - 1$. Thus graviton would have geometric size of order Compton length of electron which is something totally new from the point of view of usual Planck length scale dogmatism. In principle an entire p-adic hierarchy of gravitational forces is possible with increasing value of G.

The explanation for the small value of the gravitational coupling strength serves as a test for the proposed picture. The exchange of ordinary gauge boson involves the exchange of single CP_2 type extremal giving the exponent of Kähler action compensated by state normalization. In the case of graviton exchange two wormhole contacts are exchanged and this gives second power for the exponent of Kähler action which is not compensated. It would be this additional exponent that would give rise to the huge reduction of gravitational coupling strength from the naive estimate $G \sim L_p^2$.

Gravitons are obviously not the only stringy states. For instance, one obtains spin 1 states when the ends of string correspond to gauge boson and Higgs. Also non-vanishing electro-weak and color quantum numbers are possible and stringy states couple to elementary partons via standard couplings in this case. TGD based model for nuclei as nuclear strings having length of order $L(127)$ [F8] suggests that the strings with light M_{127} quark and anti-quark at their ends identifiable as companions of the ordinary graviton are responsible for the strong nuclear force instead of exchanges of ordinary mesons or color van der Waals forces.

Also the TGD based model of high T_c super-conductivity involves stringy states connecting the space-time sheets associated with the electrons of the exotic Cooper pair [J1, J2]. Thus stringy states would play a key role in nuclear and condensed matter physics, which means a profound departure from stringy wisdom, and breakdown of the standard reductionistic picture.

5.4 Spectrum of non-stringy states

The 1-throat character of fermions is consistent with the generation-genus correspondence. The 2-throat character of bosons predicts that bosons are characterized by the genera (g_1, g_2) of the wormhole throats. Note that the interpretation of fundamental fermions as wormhole contacts with second throat identified as a Fock vacuum is excluded.

The general bosonic wave-function would be expressible as a matrix M_{g_1,g_2} and ordinary gauge bosons would correspond to a diagonal matrix $M_{g_1,g_2} = \delta_{g_1,g_2}$ as required by the absence of neutral flavor changing currents (say gluons transforming quark genera to each other). 8 new gauge bosons are predicted if one allows all 3×3 matrices with complex entries orthonormalized with respect to trace meaning additional dynamical $SU(3)$ symmetry. Ordinary gauge bosons would be $SU(3)$ singlets in this sense. The existing bounds on flavor changing neutral currents give bounds on the masses of the boson octet. The 2-throat character of bosons should relate to the low value $T = 1/n \ll 1$ for the p-adic temperature of gauge bosons as contrasted to $T = 1$ for fermions.

If one forgets the complications due to the stringy states (including graviton), the spectrum of elementary fermions and bosons is amazingly simple and almost reduces to the spectrum of standard model. In the fermionic sector one would have fermions of standard model. By simple counting leptonic wormhole throat could carry $2^3 = 8$ states corresponding to 2 polarization states, 2 charge states, and sign of lepton number giving 8+8=16 states altogether. Taking into account phase conjugates gives 16+16=32 states.

In the non-stringy boson sector one would have bound states of fermions and phase conjugate fermions. Since only two polarization states are allowed for massless states, one obtains $(2 + 1) \times (3 + 1) = 12$ states plus phase conjugates giving 12+12=24 states. The addition of color singlet states for quarks gives 48 gauge bosons with vanishing fermion number and color quantum numbers. Besides 12 electro-weak bosons and their 12 phase conjugates there are 12 exotic bosons and their 12 phase conjugates. For the exotic bosons the couplings to quarks and leptons are determined by the orthogonality of the coupling matrices of ordinary and boson states. For exotic counterparts of W bosons and Higgs the sign of the coupling to quarks is opposite. For photon and Z^0 also the relative magnitudes of the couplings to quarks must change. Altogether this makes 48+16+16=80 states. Gluons would result as color octet states. Family replication would extend each elementary boson state into $SU(3)$ octet and singlet and elementary fermion states into $SU(3)$ triplets.

5.5 Higgs mechanism

Consider next the generation of mass as a vacuum expectation value of Higgs when also gauge bosons correspond to wormhole contacts. The presence of Higgs condensate should make the simple rectilinear ME curved so that the average propagation of fields would occur with a velocity less than light velocity. Field equations allow MEs of this kind as solutions [D1].

The finite range of interaction characterized by the gauge boson mass should correlate with the finite range for the free propagation of wormhole contacts representing bosons along corresponding ME. The finite range would result from the emission of Higgs like wormhole contacts from gauge boson like wormhole contact leading to the generation of coherent states of neutral Higgs particles. The emission would also induce non-rectilinearity of ME as a correlate for the recoil in the emission of Higgs.

Higgs expectation should have space-time correlate appearing in the modified Dirac operator. A good candidate is p-adic thermal average for the generalized eigenvalue λ of the modified Dirac operator vanishing for the zero modes. Thermal mass squared as opposed to Higgs contribution would correspond to the average of integer valued conformal weight. For bosons (in particular Higgs boson!) it is simply the sum of expectations for the two wormhole throats.

Both contributions are basically thermal which raises the question whether the interpretation in terms of coherent state of Higgs field (and essentially quantal notion) is really appropriate unless also thermal states can be regarded as genuine quantum states. The matrix characterizing time-like entanglement for the zero energy quantum state can be also thermal S-matrix with respect to the incoming and outgoing partons (hyper-finite factors of type III allow the analog of thermal QFT at the level of quantum states. This allows also a first principle description of p-adic thermodynamics.

6 Elementary particle vacuum functionals for dark matter

One of the open questions is how dark matter hierarchy reflects itself in the properties of the elementary particles. The basic questions are how the quantum phase $q = ep(2i\pi/n)$ makes itself visible in the solution spectrum of the modified Dirac operator D and how elementary particle vacuum functionals depend on q. Considerable understanding of these questions emerged recently. One can generalize modular invariance to fractional modular invariance for Riemann surfaces possessing Z_n symmetry and perform a similar generalization for theta functions and elementary particle vacuum functionals. In particular, without any further assumptions $n = 2$ dark fermions have only three families. The existence of space-time correlate for fermionic 2-valuedness suggests that fermions indeed correspond to $n = 2$, or more generally to even values of n, so that this result would hold quite generally. Elementary bosons (actually exotic particles) would correspond to $n = 1$, and more generally odd values of n, and could have also higher families.

6.1 Connection between Hurwitz zetas, quantum groups, and hierarchy of Planck constants?

The action of modular group SL(2,Z) on Riemann zeta [23] is induced by its action on theta function [24]. The action of the generator $\tau \rightarrow -1/\tau$ on theta function is essential in providing the functional equation for Riemann Zeta.

Usually the action of the generator $\tau \to \tau + 1$ on Zeta is not considered explicitly. The surprise was that the action of the generator $\tau \to \tau + 1$ on Riemann Zeta does not give back Riemann zeta but a more general function known as Hurwitz zeta $\zeta(s,z)$ for $z = 1/2$. One finds that Hurwitz zetas for certain rational values of argument define in a well defined sense representations of fractional modular group to which quantum group can be assigned naturally. This could allow to code the value of the quantum phase $q = exp(i2\pi/n)$ to the solution spectrum of the modified Dirac operator D.

6.1.1 Hurwitz zetas

Hurwitz zeta is obtained by replacing integers m with $m + z$ in the defining sum formula for Riemann Zeta:

$$\zeta(s,z) = \sum_m (m+z)^{-s} \quad . \tag{33}$$

Riemann zeta results for $z = n$.
Hurwitz zeta obeys the following functional equation for rational $z = m/n$ of the second argument [22]:

$$\zeta(1-s, \frac{m}{n}) = \frac{2\Gamma(s)}{2\pi n}^s \sum_{k=1}^{n} cos(\frac{\pi s}{2} - \frac{2\pi k m}{n}) \zeta(s, \frac{k}{n}) \quad . \tag{34}$$

The representation of Hurwitz zeta in terms of θ [22] is given by the equation

$$\int_0^\infty [\theta(z,it) - 1]\, t^{s/2} \frac{dt}{t} = \pi^{(1-s)/2} \Gamma(\frac{1-s}{2}) \left[\zeta(1-s,z) + \zeta(1-s,1-z)\right] \quad .(35)$$

By the periodicity of theta function this gives for $z = n$ Riemann zeta.

6.1.2 The action of $\tau \to \tau + 1$ transforms $\zeta(s,0)$ to $\zeta(s,1/2)$

The action of the transformations $\tau \to \tau + 1$ on the integral representation of Riemann Zeta [23] in terms of θ function [24]

$$\theta(z;\tau) - 1 = 2 \sum_{n=1}^{\infty} [exp(i\pi\tau)]^{n^2} cos(2\pi n z) \tag{36}$$

is given by

$$\pi^{-s/2}\Gamma(\frac{s}{2})\zeta(s) = \int_0^\infty [\theta(0;it) - 1]t^{s/2}\frac{dt}{t} \quad . \tag{37}$$

Using the first formula one finds that the shift $\tau = it \to \tau+1$ in the argument θ induces the shift $\theta(0;\tau) \to \theta(1/2;\tau)$. Hence the result is Hurwitz zeta $\zeta(s,1/2)$. For $\tau \to \tau+2$ one obtains Riemann Zeta.

Thus $\zeta(s,0)$ and $\zeta(s,1/2)$ behave like a doublet under modular transformations. Under the subgroup of modular group obtained by replacing $\tau \to \tau+1$ with $\tau \to \tau+2$ Riemann Zeta forms a singlet. The functional equation for Hurwitz zeta relates $\zeta(1-s,1/2)$ to $\zeta(s,1/2)$ and $\zeta(s,1) = \zeta(s,0)$ so that also now one obtains a doublet, which is not surprising since the functional equations directly reflects the modular transformation properties of theta functions. This doublet might be the proper object to study instead of singlet if one considers full modular invariance.

6.1.3 Hurwitz zetas form n-plets closed under the action of fractional modular group

The inspection of the functional equation for Hurwitz zeta given above demonstrates that $\zeta(s,m/n)$, $m = 0,1,...,n$, form in a well-defined sense an n-plet under fractional modular transformations obtained by using generators $\tau \to -1/\tau$ and $\tau \to \tau + 2/n$. The latter corresponds to the unimodular matrix $(a,b;c,d) = (1,2/n;0,1)$. These matrices obviously form a group. Note that Riemann zeta is always one member of the multiplet containing n Hurwitz zetas.

These observations bring in mind fractionization of quantum numbers, quantum groups corresponding to the quantum phase $q = exp(i2\pi/n)$, and the inclusions for hyper-finite factors of type II_1 partially characterized by these quantum phases. Fractional modular group obtained using generator $\tau \to \tau+2/n$ and Hurwitz zetas $\zeta(s,k/n)$ could very naturally relate to these and related structures.

6.2 Hurwitz zetas and dark matter

These observations suggest a direct application to quantum TGD.

6.2.1 Basic vision bout dark matter

a) In TGD framework inclusions of HFFs of type II_1 are directly related to the hierarchy of Planck constants involving a generalization of the notion of imbedding space obtained by gluing together copies of 8-D $H = M^4 \times CP_2$ with a discrete bundle structure $H \rightarrow H/Z_{n_a} \times Z_{n_b}$ together along the 4-D intersections of the associated base spaces [C9]. A book like structure results and various levels of dark matter correspond to the pages of this book. One can say that elementary particles proper are maximally quantum critical and live in the 4-D intersection of these imbedding spaces whereas their "field bodies" reside at the pages of the Big Book. Note that analogous book like structures results when real and various p-adic variants of the imbedding space are glued together along common algebraic points.

b) The integers n_a and n_b give Planck constant as $\hbar/\hbar_0 = n_a/n_b$, whose most general value is a rational number. In Platonic spirit one can argue that number theoretically simple integers involving only powers of 2 and Fermat primes are favored physically. Phase transitions between different matters occur at the intersection.

c) The inclusions $\mathcal{N} \subset \mathcal{M}$ of HFFs relate also to quantum measurement theory with finite measurement resolution with $\mathcal{N}$ defining the measurement resolution so that N-rays replace complex rays in the projection postulate and quantum space $\mathcal{M}/\mathcal{N}$ having fractional dimension effectively replaces $\mathcal{M}$.

d) The basic hypothesis is that the inverses of zeta function or of more general variants of zeta coding information about the algebraic structure of the partonic 2-surface appear in the admittedly speculative fundamental formula for the generalized eigenvalues of modified Dirac operator D. This formula is consistent with the generalized eigenvalue equation for D but is not the only one that one can imagine.

e) The generalized eigen spectrum of D should code information both about the p-adic prime p characterizing particle and about quantum phases $q = exp(i2\pi/n)$ assignable to the particle in M^4 and CP_2 degrees of freedom. I understand how p-adic primes appear in the spectrum of D and therefore how coupling constant evolution emerges at the level of free field theory so that radiative corrections can vanish without the loss of coupling constant evolution [C5]. The problem has been to understand how the quantum phase characterizing the sector of the generalized imbedding space could make itself visible in these formulas and therefore in quantum dynamics at the level of free spinor fields. The replacement of Riemann zeta with an n-plet of Hurwitz zetas would resolve this problem.

f) Geometrically the fractional modular invariance would naturally relate to the fact that Riemann surface (partonic 2-surface) can be seen as an $n_a \times n_b$-fold covering of its projection to the base space of H: fractional modular transformations corresponding to n_a and n_b would relate points at different sheets of the covering of M^4 and CP_2. This means $Z_{n_a n_b} = Z_{n_a} \times Z_{n_b}$ conformal symmetry. This suggests that the fractionization could be a completely general phenomenon happening also for more general zeta functions.

6.2.2 What about exceptional cases $n = 1$ and $n = 2$?

Also $n = 1$ and $n = 2$ are present in the hierarchy of Hurwitz zetas (singlet and doublet). They do not correspond to allowed Jones inclusion since one has $n > 2$ for them. What could this mean?

a) It would seem that the fractionization of modular group relates to Jones inclusions ($n > 2$) giving rise to fractional statistics. $n = 2$ corresponding to the full modular group Sl(2,Z) could relate to the very special role of 2-valued logic, to the degeneracy of $n = 2$ polygon in plane, to the very special role played by 2-component spinors playing exceptional role in Riemann geometry with spinor structure, and to the canonical representation of HFFs of type II_1 as fermionic Fock space (spinors in the world of classical worlds). Note also that SU(2) defines the building block of compact non-commutative Lie groups and one can obtain Lie-algebra generators of Lie groups from n copies of SU(2) triplets and posing relations which distinguish the resulting algebra from a direct sum of SU(2) algebras.

b) Also $n = 2$-fold coverings $M^4 \to M^4/Z_2$ and $CP_2 \to CP_2/Z_2$ seem to make sense. One can argue that by quantum classical correspondence the spin half property of imbedding space spinors should have space-time correlate. Could $n = 2$ coverings allow to define the space-time correlates for particles having half odd integer spin or weak isospin? If so, bosons would correspond to $n = 1$ and fermions to $n = 2$. One could of course counter argue that induced spinor fields already represent fermions at space-time level and there is no need for the doubling of the representation.

The trivial group Z_1 and Z_2 are exceptional since Z_1 does not define any quantization axis and Z_2 allows any quantization axis orthogonal to the line connecting two points. For $n \geq 3$ Z_n fixes the direction of quantization axis uniquely. This obviously correlates with $n \geq 3$ for Jones inclusions.

6.2.3 Dark elementary particle functionals

One might wonder what might be the dark counterparts of elementary particle vacuum functionals. Theta functions $\theta_{[a,b]}(z, \Omega)$ with characteristic $[a, b]$ for Riemann surface of genus g as functions of z and Teichmueller parameters Ω are the basic building blocks of modular invariant vacuum functionals defined in the finite-dimensional moduli space whose points characterize the conformal equivalence class of the induced metric of the partonic 2-surface. Obviously, kind of spinorial variants of theta functions are in question with $g+g$ spinor indices for genus g.

The recent case corresponds to $g=1$ Riemann surface (torus) so that a and b are $g=1$-component vectors having values 0 or 1/2 and Hurwitz zeta corresponds to $\theta_{[0,1/2]}$. The four Jacobi theta functions listed in Wikipedia [24] correspond to these thetas for torus. The values for a and b are 0 and 1 for them but this is a mere convention.

The extensions of modular group to fractional modular groups obtained by replacing integers with integers shifted by multiples of $1/n$ suggest the existence of new kind of q-theta functions with characteristics $[a,b]$ with a and b being g-component vectors having fractional values k/n, $k=0,1...n-1$. There exists also a definition of q-theta functions working for $0 \leq |q| < 1$ but not for roots of unity [25]. The q-theta functions assigned to roots of unity would be associated with Riemann surfaces with additional Z_n conformal symmetry but not with generic Riemann surfaces and obtained by simply replacing the value range of characteristics $[a,b]$ with the new value range in the defining formula

$$\Theta[a,b](z|\Omega) = \sum_n exp\left[i\pi(n+a)\cdot\Omega\cdot(n+a)+i2\pi(n+a)\cdot(z+b)\right] . \quad (38)$$

for theta functions. If Z_n conformal symmetry is relevant for the definition of fractional thetas it is probably so because it would make the generalized theta functions sections in a bundle with a finite fiber having Z_n action.

This hierarchy would correspond to the hierarchy of quantum groups for roots of unity and Jones inclusions and one could probably define also corresponding zeta function multiplets. These theta functions would be building blocks of the elementary particle vacuum functionals for dark variants of elementary particles invariant under fractional modular group. They would also define a hierarchy of fractal variants of number theoretic functions: it would be interesting to see what this means from the point of view of Langlands program [26] discussed also in TGD framework [E11] involving ordinary modular invariance in an essential manner.

This hierarchy would correspond to the hierarchy of quantum groups for roots of unity and Jones inclusions and one could probably define also corresponding zeta function multiplets. These theta functions would be building

blocks of the elementary particle vacuum functionals for dark variants of elementary particles invariant under fractional modular group.

6.2.4 Hierarchy of Planck constants defines a hierarchy of quantum critical systems

Dark matter hierarchy corresponds to a hierarchy of conformal symmetries Z_n of partonic 2-surfaces with genus $g \geq 1$ such that factors of n define subgroups of conformal symmetries of Z_n. By the decomposition $Z_n = \prod_{p|n} Z_p$, where $p|n$ tells that p divides n, this hierarchy corresponds to an hierarchy of increasingly quantum critical systems in modular degrees of freedom. For a given prime p one has a sub-hierarchy Z_p, $Z_{p^2} = Z_p \times Z_p$, etc... such that the moduli at n+1:th level are contained by n:th level. In the similar manner the moduli of Z_n are sub-moduli for each prime factor of n. This mapping of integers to quantum critical systems conforms nicely with the general vision that biological evolution corresponds to the increase of quantum criticality as Planck constant increases.

The group of conformal symmetries could be also non-commutative discrete group having Z_n as a subgroup. This inspires a very short-lived conjecture that only the discrete subgroups of SU(2) allowed by Jones inclusions are possible as conformal symmetries of Riemann surfaces having $g \geq 1$. Besides Z_n one could have tedrahedral and icosahedral groups plus cyclic group Z_{2n} with reflection added but not Z_{2n+1} nor the symmetry group of cube. The conjecture is wrong. Consider the orbit of the subgroup of rotational group on standard sphere of E^3, put a handle at one of the orbits such that it is invariant under rotations around the axis going through the point, and apply the elements of subgroup. You obtain a Riemann surface having the subgroup as its isometries. Hence all discrete subgroups of SU(2) can act even as isometries for some value of g.

The number theoretically simple ruler-and-compass integers having as factors only first powers of Fermat primes and power of 2 would define a physically preferred sub-hierarchy of quantum criticality for which subsequent levels would correspond to powers of 2: a connection with p-adic length scale hypothesis suggests itself.

Spherical topology is exceptional since in this case the space of conformal moduli is trivial and conformal symmetries correspond to the entire $SL(2,C)$. This would suggest that only the fermions of lowest generation corresponding to the spherical topology are maximally quantum critical. This brings in mind Jones inclusions for which the defining subgroup equals to $SU(2)$ and Jones index equals to $\mathcal{M}/\mathcal{N} = 4$. In this case all discrete subgroups of $SU(2)$ label the inclusions. These inclusions would correspond to fiber space $CP_2 \rightarrow CP_2/U(2)$ consisting of geodesic spheres of CP_2. In this case the discrete subgroup might correspond to a selection of a subgroup of $SU(2) \subset SU(3)$ acting non-trivially on the geodesic sphere. Cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$ having geodesic spheres of CP_2 as their ends could correspond to this phase dominating the very early cosmology.

6.2.5 Fermions in TGD Universe allow only three families

What is nice that if fermions correspond to $n = 2$ dark matter with Z_2 conformal symmetry as strong quantum classical correspondence suggests, the number of ordinary fermion families is three without any further assumptions. To see this suppose that also the sectors corresponding to $M^4 \rightarrow M^4/Z_2$ and $CP_2 \rightarrow CP_2/Z_2$ coverings are possible. Z_2 conformal symmetry implies that partonic Riemann surfaces are hyper-elliptic. For genera $g > 2$ this means that some theta functions of $\theta_{[a,b]}$ appearing in the product of theta functions defining the vacuum functional vanish. Hence fermionic elementary particle vacuum functionals would vanish for $g > 2$ and only 3 fermion families would be possible for $n = 2$ dark matter.

This results can be strengthened. The existence of space-time correlate for the fermionic 2-valuedness suggests that fermions quite generally to even values of n, so that this result would hold for all fermions. Elementary bosons (actually exotic particles belonging to Kac-Moody type representations) would correspond to odd values of n, and could possess also higher families. There is a nice argument supporting this hypothesis. n-fold discretization provided by covering associated with H corresponds to discretization for angular momentum eigenstates. Minimal discretization for $2j + 1$ states corresponds to $n = 2j + 1$. $j = 1/2$ requires $n = 2$ at least, $j = 1$ requires $n = 3$ at least, and so on. $n = 2j + 1$ allows spins $j \leq n - 1/2$. This spin-quantum phase connection at the level of space-time correlates has counterpart for the representations of quantum SU(2).

These rules would hold only for genuinely elementary particles corresponding to single partonic component and all bosonic particles of this kind are exotics (excitations in only "vibrational" degrees of freedom of partonic 2-surface with modular invariance eliminating quite a number of them.

References

Online books about TGD

[1] M. Pitkänen (2006), *Topological Geometrodynamics: Overview.* http://www.helsinki.fi/~matpitka/tgdview/tgdview.html.

[2] M. Pitkänen (2006), *Quantum Physics as Infinite-Dimensional Geometry.* http://www.helsinki.fi/~matpitka/tgdgeom/tgdgeom.html.

[3] M. Pitkänen (2006), *Physics in Many-Sheeted Space-Time.* http://www.helsinki.fi/~matpitka/tgdclass/tgdclass.html.

[4] M. Pitkänen (2006), *Quantum TGD.* http://www.helsinki.fi/~matpitka/tgdquant/tgdquant.html.

[5] M. Pitkänen (2006), *TGD as a Generalized Number Theory.* http://www.helsinki.fi/~matpitka/tgdnumber/tgdnumber.html.

[6] M. Pitkänen (2006), *p-Adic length Scale Hypothesis and Dark Matter Hierarchy.* http://www.helsinki.fi/~matpitka/paddark/paddark.html.

[7] M. Pitkänen (2006), *TGD and Fringe Physics.* http://www.helsinki.fi/~matpitka/freenergy/freenergy.html.

Online books about TGD inspired theory of consciousness and quantum biology

[8] M. Pitkänen (2006), *Bio-Systems as Self-Organizing Quantum Systems.* http://www.helsinki.fi/~matpitka/bioselforg/bioselforg.html.

[9] M. Pitkänen (2006), *Quantum Hardware of Living Matter.* http://www.helsinki.fi/~matpitka/bioware/bioware.html.

[10] M. Pitkänen (2006), *TGD Inspired Theory of Consciousness.* http://www.helsinki.fi/~matpitka/tgdconsc/tgdconsc.html.

[11] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory.* http://www.helsinki.fi/~matpitka/genememe/genememe.html.

[12] M. Pitkänen (2006), *TGD and EEG*.
http://www.helsinki.fi/~matpitka/tgdeeg/tgdeeg/tgdeeg.html.

[13] M. Pitkänen (2006), *Bio-Systems as Conscious Holograms*.
http://www.helsinki.fi/~matpitka/hologram/hologram.html.

[14] M. Pitkänen (2006), *Magnetospheric Consciousness*.
http://www.helsinki.fi/~matpitka/magnconsc/magnconsc.html.

[15] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*.
http://www.helsinki.fi/~matpitka/magnconsc/mathconsc.html.

References to the chapters of books

[A1] The chapter *An Overview about the Evolution of Quantum TGD* of [1].
http://www.helsinki.fi/~matpitka/tgdview/tgdview.html#tgdevoI.

[B1] The chapter *Identification of the Configuration Space Kähler Function* of [2].
http://www.helsinki.fi/~matpitka/tgdgeom/tgdgeom.html#kahler.

[B4] The chapter *Configuration Space Spinor Structure* of [2].
http://www.helsinki.fi/~matpitka/tgdgeom/tgdgeom.html#cspin.

[C1] The chapter *Construction of Quantum Theory: Symmetries* of [4].
http://www.helsinki.fi/~matpitka/tgdquant/tgdquant.html#quthe.

[C2] The chapter *Construction of Quantum Theory: S-matrix* of [4].
http://www.helsinki.fi/~matpitka/tgdquant/tgdquant.html#towards.

[C3] The chapter *Hyper-Finite Factors and Construction of S-matrix* of [4].
http://www.helsinki.fi/~matpitka/tgdquant/tgdquant.html#HFSmatrix.

[C5] The chapter *Is it Possible to Understand Coupling Constant Evolution at Space-Time Level?* of [4].
http://www.helsinki.fi/~matpitka/tgdquant/tgdquant.html#rgflow.

[C7] The chapter *Equivalence of Loop Diagrams with Tree Diagrams and Cancellation of Infinities in Quantum TGD* of [4].
http://www.helsinki.fi/~matpitka/tgdquant/tgdquant.html#bialgebra.

[C8] The chapter *Was von Neumann Right After All* of [4].
http://www.helsinki.fi/~matpitka/tgdquant/tgdquant.html#vNeumann.

[C9] The chapter *Does TGD Predict the Spectrum of Planck Constants?* of [4].
http://www.helsinki.fi/~matpitka/tgdquant/tgdquant.html#Planck.

[D1] The chapter *Basic Extremals of Kähler Action* of [3].
http://www.helsinki.fi/~matpitka/tgdclass/tgdclass.html#class.

[D5] The chapter *TGD and Cosmology* of [3].
http://www.helsinki.fi/~matpitka/tgdclass/tgdclass.html#cosmo.

[D6] The chapter *TGD and Astrophysics* of [3].
http://www.helsinki.fi/~matpitka/tgdclass/tgdclass.html#astro.

[E8] The chapter *Riemann Hypothesis and Physics* of [5].
http://www.helsinki.fi/~matpitka/tgdnumber/tgdnumber.html#riema.

[E11] The chapter *Langlands Program and TGD* of [5].
http://www.helsinki.fi/~matpitka/tgdnumber/tgdeeg/tgdnumber.html#Langlandia.

[F3] The chapter *p-Adic Particle Massivation: Elementary particle Masses* of [6].
http://www.helsinki.fi/~matpitka/paddark/paddark.html#padmass2.

[F4] The chapter *p-Adic Particle Massivation: Hadron Masses* of [6].
http://www.helsinki.fi/~matpitka/paddark/paddark.html#padmass3.

[F5] The chapter *p-Adic Particle Massivation: New Physics* of [6].
http://www.helsinki.fi/~matpitka/paddark/paddark.html#padmass4.

[F8] The chapter *TGD and Nuclear Physics* of [6].
http://www.helsinki.fi/~matpitka/paddark/paddark.html#padnucl.

[J1] The chapter *Bio-Systems as Super-Conductors: part I* of [9].
http://www.helsinki.fi/~matpitka/bioware/bioware.html#superc1.

[J2] The chapter *Bio-Systems as Super-Conductors: part II* of [9].
http://www.helsinki.fi/~matpitka/bioware/bioware.html#superc2.

[J3] The chapter *Bio-Systems as Super-Conductors: part III* of [9].
http://www.helsinki.fi/~matpitka/bioware/bioware.html#superc3.

[K1] The chapter *Time, Spacetime and Consciousness* of [13].
http://www.helsinki.fi/~matpitka/hologram/hologram.html#time.

[M6] The chapter *Quantum Model for Hearing* of [12].
http://www.helsinki.fi/~matpitka/tgdeeg/tgdeeg/tgdeeg.html#hearing.

Mathematics related references

[16] Accola, R. (1975): *Riemann Surfaces, Theta functions and Abelian Automorphism Groups.* Lecture Notes in Mathematics 483. Springer Verlag.

[17] Alvarez-Gaume L.,Moore G. and Vafa C. (1986): *Theta functions, Modular invariance and Strings.* Commun. Math. Phys. 106. 40-87.

[18] Farkas, H., M. and Kra, I. (1980): *Riemann Surfaces.* Springer Verlag.

[19] Mumford, D.(1983,1984): *Tata Lectures on Theta I,II,III.* Birkhäuser.

[20] P. S. Aspinwall, B. R. Greene, and D. R. Morrison (1993), *Calabi-Yau Moduli Space, Mirror Manifolds, and Space-time Topology Change in String Theory*, hep-th/9309097.

[21] M. Redei and M. Stöltzner (eds) (2001), *John von Neumann and the Foundations of Quantum Physics*, Vol. 8, Dordrecht: Kluwer Academic Publishers. V. S. Sunder (1987), *An Invitation to von Neumann Algebras*, New York: Springer-Verlag.

[22] *Hurwitz zeta function*, http://en.wikipedia.org/wiki/Hurwitz_zeta_function.

[23] *Riemann Zeta function*, http://en.wikipedia.org/wiki/Riemann_zeta.

[24] *Theta function*, http://en.wikipedia.org/wiki/Theta_function.

[25] *Q-theta function*, http://en.wikipedia.org/wiki/Q-theta_function.

[26] *Langlands program*, http://en.wikipedia.org/wiki/Langlands_program.

Physics related references

[27] Decamp *et al* (Aleph Collaboration)(1989): CERN-EP/89-141, preprint.

[28] Denegri, D. *et* (1989): *The number of neutrino species.* CERN EP/89-72, preprint.