An Overview About Quantum TGD

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Abstract

This chapter provides a summary about quantum TGD. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the "world of the classical worlds" identified as the infinite-dimensional configuration space of 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis this vision.

1. Geometric ideas

TGD relies heavily on geometric ideas, which have gradually generalized during the years.

a) The basic vision is that it is possible to reduce quantum theory to configuration space geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes configuration space Kähler geometry uniquely. Accordingly, configuration space can be regarded as a union of infinite-dimensional symmetric spaces labelled by zero modes labelling classical non-quantum fluctuating degrees of freedom. The huge symmetries of the configuration space geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

b) Configuration space spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the configuration space. Configuration space spinors define a von Neumann algebra known as hyper-finite factor of type II_1 (HFFs). This has led to a profound understanding of quantum TGD. The outcome of this approach is that the exponents of Kähler function and Chern-Simons action are not fundamental objects but reduce to the Dirac determinant associated with the modified Dirac operator assigned to the light-like 3-surfaces.

c) p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both configuration space geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no adhoc elements and is inherent to the physics of TGD.

d) The work with HFFs combined with experimental input led to the notion of hierarchy of Planck constants interpreted in terms of dark

matter. The hierarchy is realized via a generalization of the notion of imbedding space obtained by gluing infinite number of its variants along common lower-dimensional quantum critical sub-manifolds. This leads to the identification of number theoretical braids as points of partonic 2-surface which correspond to the minima of generalized eigenvalue of Dirac operator, a scalar field to which Higgs vacuum expectation is proportional to. Higgs vacuum expectation has thus a purely geometric interpretation. This leads to an explicit formula for the Dirac determinant. What is remarkable is that the construction gives also the 4-D space-time sheets associated with the light-like orbits of partonic 2-surfaces: they should correspond to preferred extremals of Kähler action. Thus hierarchy of Planck constants is now an essential part of construction of quantum TGD and of mathematical realization of the notion of quantum criticality.

e) HFFs lead also to an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, CP_2 could be interpreted as a structure related to octonions. This would mean that TGD could be seen also as a generalized number theory.

2. Ideas related to the construction of S-matrix

The construction of S-matrix involves several ideas that have emerged during last years.

a) Zero energy ontology motivated originally by TGD inspired cosmology means that physical states have vanishing net quantum numbers and are decomposable to positive and negative energy parts separated by a temporal distance characterizing the system as space-time sheet of finite size in time direction. The particle physics interpretation is as initial and final states of a particle reaction. S-matrix and density matrix are unified to the notion of M-matrix expressible as a product of square root of density matrix and of unitary S-matrix. Thermodynamics becomes therefore a part of quantum theory. One must distinguish M-matrix from U-matrix defined between zero energy states and analogous to S-matrix and characterizing the unitary process associated with quantum jump. U-matrix is most naturally related to the description of intentional action since in a well-defined sense it has elements between physical systems corresponding to different number fields.

b) The notion of measurement resolution represented in terms of inclusions of HFFs is an essential element of the picture. Measurement resolution corresponds to the action of the included sub-algebra creating zero energy states in time scales shorter than the cutoff scale. This algebra effectively replaces complex numbers as coefficient fields and the condition that its action commutes with the M-matrix implies that M-matrix corresponds to Connes tensor product. Together with super-conformal symmetries this fixes possible M-matrices to a very high degree.

c) Light-likeness of the basic fundamental objects implies that TGD is almost topological QFT so that the formulation in terms of category theoretical notions is expected to work. M-matrices form in a natural manner a functor from the category of cobordisms to the category of pairs of Hilbert spaces and this gives additional strong constraints on the theory.

1 Introduction

This chapter provides a summary about quantum TGD. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the "world of the classical worlds" identified as the infinite-dimensional configuration space of 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis this vision.

1.1 Geometric ideas

TGD relies heavily on geometric ideas, which have gradually generalized during the years.

1. The basic dynamical objects of TGD are 3-surfaces of 8-D imbedding space fixed uniquely by the symmetries of particle physics and the structure of standard model. 4-D general coordinate invariance allows to assume that these surfaces are light-like and the interpretation is as random light-like orbits of 2-dimensional partons. This picture leads immediately to an understanding of the fundamental super-conformal symmetries of the theory and realization that TGD can be seen as an almost topological quantum field theory.

2. The basic vision is that it is possible to reduce quantum theory to configuration space geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes configuration space Kähler geometry uniquely. Accordingly, configuration space can be regarded as a union of infinite-dimensional symmetric spaces labelled by zero modes labelling classical non-quantum fluctuating degrees of freedom. The huge symmetries of the configuration space geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

3. Configuration space spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the configuration space. Configuration space spinors define a von Neumann algebra known as hyper-finite factor of type II_1 (HFFs). This has led to a profound understanding of quantum TGD. The outcome of this approach is that the exponents of Kähler function and Chern-Simons action are not fundamental objects but reduce to the Dirac determinant associated with the modified Dirac operator assigned to the light-like 3-surfaces.

4. p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both configuration space geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no ad hoc elements and is inherent to the physics of TGD.

5. The work with HFFs combined with experimental input led to the notion of hierarchy of Planck constants interpreted in terms of dark matter. The hierarchy is realized via a generalization of the notion of imbedding space obtained by gluing infinite number of its variants along common lower-dimensional quantum critical sub-manifolds. This leads to the identification of number theoretical braids as points of partonic 2-surface which correspond to the minima of generalized eigenvalue of Dirac operator, a scalar field to which Higgs vacuum expectation is proportional to. Higgs vacuum expectation has thus a purely geometric interpretation. This leads to an explicit formula for the Dirac determinant. The zeta function associated with the eigenvalues in turn defines the super-canonical conformal weights as its zeros so that very beautiful picture results. What is remarkable is that the construction gives also the 4-D space-time sheets associated with the light-like orbits of partonic 2-surfaces: they should correspond to preferred extremals of Kähler action. Thus hierarchy of Planck constants is now an essential part of construction of quantum TGD and of mathematical realization of the notion of quantum criticality.

6. HFFs lead also to an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, CP_2 could be interpreted as a structure related to octonions. This would mean that TGD could be seen also as a generalized number theory. The vision about TGD as a generalized number theory involves also the notion of infinite primes. This notion leads to a further generalization of the ideas about geometry: this time the notion of space-time point generalizes so that it has an infinitely complex number theoretical anatomy not visible in real topology.

1.2 Ideas related to the construction of S-matrix

The construction of S-matrix has been the most difficult challenge of TGD and involves several ideas that have emerged during last years. It is not possible to represent explicit formulas yet but the general principles behind S-matrix, or rather its generalization to M-matrix, are reasonably well understood now.

1. Zero energy ontology motivated originally by TGD inspired cosmology means that physical states have vanishing net quantum numbers and are decomposable to positive and negative energy parts separated by a temporal distance characterizing the system as space-time sheet of finite size in time direction. The particle physics interpretation is as initial and final states of a particle reaction. S-matrix and density matrix are unified to the notion of M-matrix expressible as a product of square root of density matrix and of unitary S-matrix. Thermodynamics becomes therefore a part of quantum theory.

   One must distinguish M-matrix from U-matrix defined between zero energy states and analogous to S-matrix and characterizing the unitary process associated with quantum jump. U-matrix is most naturally related to the description of intentional action since in a well-defined sense it has elements between physical systems corresponding to different number fields.

2. The notion of measurement resolution represented in terms of inclusions of HFFs is an essential element of the picture. Measurement resolution corresponds to the action of the included sub-algebra creating zero energy states in time scales shorter than the cutoff scale. This algebra effectively replaces complex numbers as coefficient fields and the condition that its action commutes with the M-matrix implies that M-matrix corresponds to Connes tensor product. Thus S-matrix is characterized by the measurement resolution analogous to length scale cutoff of quantum field theories. Together with super-conformal symmetries this fixes possible M-matrices to a very high degree. The amazing conclusion interpreted in terms of asymptotic freedom is that at the never-reachable limit of infinite measurement resolution the S-matrix becomes trivial.

3. An essential difference between TGD and string models is the replacement of stringy diagrams with generalized Feynman diagrams obtained by gluing 3-D light-like surfaces (instead of lines) together at their ends represented as partonic 2-surfaces. This makes the construction of vertices very simple. The notion of number theoretic braid in turn implies discretization having also interpretation in terms of non-commutativity due to finite measurement resolution replacing anti-commutativity along stringy curves with anti-commutativity at points of braids. Braids can replicate at vertices which suggests interpretation in terms of topological quantum computation combined with non-faithful copying and communication of information. The analogs of stringy diagrams have quite different interpretation in TGD: for instance, photons travelling via two different paths in double slit experiment are represented in terms of stringy branching of the photonic 2-surface.

4. Light-likeness of the basic fundamental objects implies that TGD is almost topological QFT so that the formulation in terms of category theoretical notions is expected to work. M-matrices form in a natural manner a functor from the category of cobordisms to the category of pairs of Hilbert spaces and this gives additional strong constraints on the theory.

5. $HO-H$ duality or "number theoretical compactification" [E2] states that one can regard space-time surfaces X^4 either as hyperquaternionic surfaces in the space $HO = M^8$ of hyper-octonions or as preferred extremals of Kähler action in $M^4 \times CP_2$. Hyper-quaternionicity means that the tangent space of X^4 at each point is some hyperquaternionic subspace $HQ = M^4$ of HO. Besides this a preferred plane $M^2 \subset M^8$ identifiable as a plane of non-physical polarizations belongs to the tangent space at each point. This hypothesis provides a purely number theoretic interpretation of gauge conditions and implies a large number of "must-be-trues" of quantum TGD, and together with zero energy ontology leads to a precise view about the realization of zero energy states in terms of causal diamonds allowing to deduce p-adic length scale hypothesis and a general vision about coupling constant evolution in which time scales appear as power of 2 multiples of a basic length scale.

   One important implication is a justification for the coset construction based on the lifting of Super Kac-Moody algebra (SKM) at a given light-like 3-surface to a sub-algebra of super-canonical algebra (SC) lifted from $\delta M^{\pm} \times CP_2$ to algebra in H. Coset construction provides a precise realization for what I used to call 7-3 duality stating that the actions of SC and SKM Virasoro algebras on physical states are identical. The interpretation is in terms of a generalization of Einstein's equations realizing Equivalence Principle in TGD framework. Also a justification for p-adic thermodynamics emerges.

6. The outcome is a generalization of Feynman diagrammatics in which the lines of Feynman diagrams are replaced with 3-D light-like surfaces meeting at 2-D surfaces representing vertices. The contribution of a given Feynman diagram is calculated using the fusion rules of a generalized conformal field theory recursively rather than instead of the ordinary Feynman rules. A new element is symplectically invariant (invariant under symplectic/contact transformations of $\delta M^4_{\pm} \times CP_2$) factor of N-point function and thus expressible in terms of symplectic invariants constructed from the areas assignable to the geodesic triangles defined by the subsets of N points and satisfying fusion rules. Simple argument shows that this factor vanishes if any two arguments of N-point function are identical: this gives excellent hopes that infinities are avoided as general arguments indeed predict. The construction and classification of symplectic QFTs as analogs of conformal field theories becomes a basic mathematical challenge.

   The restriction of the arguments of N-point functions to a discrete set of points at partonic 2-surfaces and defining number theoretical braids is an essential ingredient of the approach making it possible the completion of the theory to real and various p-adic domains. These points correspond to the unique intersection of the hyper-quaternionic (and thus associativ

7. subset $M^4 \subset M^8$ with the partonic 2-surfaces, where M^4 is now a fixed hyper-quaternionic plane of M^8 which should not be confused with the varying hyper-quaternionic plane defining the tangent spaces at points of X^4.

   A structure resembling stringy perturbation theory involving fermionic propagators expressible as inverses of the super-generator G_0 is what one expects. Contrary to original naive beliefs, the fact that G_0 and also ordinary imbedding space gamma matrices γ^k must carry fermion number is not any problem. Even in the case of ordinary Feynman diagrams the interpretation that $p^k\gamma_k$ creates 1-fermion state from vacuum works in massless gauge theories involving no scalar fields (and thus no Higgs field). There is no need for Majorana spinors leading to super string models and imbedding space dimension $D = 8$ works.

1.3 Some general predictions of quantum TGD

TGD is consistent with the symmetries of the standard model by construction although there are definite deviations from the symmetries of standard model. TGD however predicts also a lot of new physics. Below just some examples of the predictions of TGD.

1. Fractal hierarchies meaning the existence of scaled variants of standard model physics is the basic prediction of quantum TGD. p-Adic length scale hypothesis predicts the possibility that elementary particles can have scaled variants with mass scales related by power of $\sqrt{2}$. Dark matter hierarchy predicts the existence of infinite number of scaled variants with same mass spectrum with quantum scales like Compton length scaling like $\hbar$.

2. TGD predicts that standard model fermions and gauge bosons differ topologically in a profound manner. Fermions correspond to light-like wormhole throats associated with topologically condensed CP_2 type extremals whereas gauge bosons correspond to fermion-antifermion states associated with the throats of wormhole contacts connecting two space-time sheets with opposite time orientation. The implication is that Higgs vacuum expectation value cannot contribute to fermion mass: this conforms with the results of p-adic mass calculations. TGD predicts also so called super-canonical quanta and these give dominating contribution to most hadron masses. These degrees of freedom correspond to those of hadronic string and should not reduce to QCD.

3. The most fascinating applications of zero energy ontology are to quantum biology and TGD inspired theory of consciousness. Basic new element

are negative energy photons making possible communications to the direction of geometric past. Here also dark matter hierarchy is involved in an essential manner.

4. In cosmology the mere imbeddability required for Robertson-Walker cosmology implies that critical and over-critical cosmologies are almost unique and characterized by their finite duration. The cosmological expansion is accelerating for them and there is no need to assume cosmological constant. Macroscopic quantum coherence of dark matter in astrophysical scales is a dramatic prediction and allows also to assign periods of accelerating expansion to quantum phase transition changing the value of gravitational Planck constant. The dark matter parts of astrophysical systems are predicted to be quantum systems.

5. The notion of generalized imbedding space suggests that the physics of TGD Universe is universal in the sense that it is possible to engineer a system able to mimic the physics of any consistent gauge theory. Kind of analog of Turing machine would be in question.

2 Physics as geometry of configuration space spinor fields

The construction of the configuration space geometry has proceeded rather slowly. The experimentation with various ideas has however led to the identification of the basic constraints on the configuration space geometry.

2.1 Reduction of quantum physics to the Kähler geometry and spinor structure of configuration space of 3-surfaces

The basic philosophical motivation for the hypothesis that quantum physics could reduce to the construction of configuration space Kähler metric and spinor structure, is that infinite-dimensional Kähler geometric existence could be unique not only in the sense that the geometry of the space of 3-surfaces could be unique but that also the dimension of the space-time is fixed to $D = 4$ by this requirement and $M^4_+ \times CP_2$ is the only possible choice of imbedding space. This optimistic vision derives from the work of Dan Freed with loops spaces demonstrating that they possess unique Kähler geometry and from the fact that in $D > 1$ case the existence of Riemann connection, finiteness of Ricci tensor, and general coordinate invariance poses even stronger constraints.

2.2 Constraints on configuration space geometry

The detailed considerations of the constraints on configuration space geometry suggests that it should possess at least the following properties.

1. Metric should be Kähler metric. This property is necessary if one wants to geometrize the oscillator algebra used in the construction of the physical states and to obtain a well defined divergence free functional integration in the configuration space.

2. Metric should allow Riemann connection, which, together with the Kähler property, very probably implies the existence of an infinite dimensional isometry group as the construction of Kähler geometry for the loop spaces demonstrates [25].

3. The so called symmetric spaces classified by Cartan [26] are Cartesian products of the coset spaces G/H with maximal isometry group G. Symmetric spaces possess G invariant metric and curvature tensor is constant so that all points of the symmetric space are metrically equivalent. Symmetric space structure means that the Lie-algebra of G decomposes as

$$\begin{aligned} & g = h \oplus t \ , \\ & [h,h] \subset h \ , \quad [h,t] \subset t \ , \quad [t,t] \subset h \ , \end{aligned}$$

   where g and h denote the Lie-algebras of G and H respectively and t denotes the complement of h in g. The existence of the $g = t + h$ decomposition poses an extremely strong constraint on the symmetry group G.

   In the infinite-dimensional context symmetric space property would mean a drastic calculational simplification. The most one can hope is that configuration space is expressible as a union $\cup_i (G/H)_i$ of symmetric spaces. Reduction to a union of G/H is the best one can hope since 3-surface of Planck size cannot be metrically equivalent with a 3-surface having the size of galaxy! The coordinates labelling the symmetric spaces in this union do not appear as differentials in the line element of configuration space and are thus zero modes. They correspond to non-quantum fluctuating degrees of freedom in a well defined sense and are identifiable as classical variables of quantum measurement theory.

4. Metric should be Diff^4 (not only $Diff^3$!) invariant and degenerate and the definition of the metric should associate a unique space-time surface $X^4(X^3)$ to a given 3-surface X^3 to act on. This requirement is absolutely crucial for all developments.

5. Divergence cancellation requirement for the functional integral over the configuration space requires that the metric is Ricci flat and thus satisfies vacuum Einstein equations.

2.3 Configuration space as a union of symmetric spaces

In the finite-dimensional context, globally symmetric spaces are of form G/H and connection and curvature are independent of the metric, provided it is left invariant under G. Good guess is that same holds true in the infinite-dimensional context. The task is to identify the infinite-dimensional groups G and H. Only quite recently, more than seven years after the discovery of the candidate for Kähler function defining the metric, it became finally clear that these identifications follow quite nicely from Diff^4 invariance and Diff^4 degeneracy.

The crux of the matter is Diff^4 degeneracy : all 3-surfaces on the orbit of 3-surface X^3 must be physically equivalent so that one can effectively replace all 3-surfaces Z^3 on the orbit of X^3 with a suitably chosen surface Y^3 on the orbit of X^3. The Lorentz and Diff^4 invariant choice of Y^3 is as the intersection of the 4-surface with the set $\delta M^4_+ \times CP_2$, where δM^4_+ denotes the boundary of the light cone: effectively the imbedding space can be replaced with the product $\delta M^4_+ \times CP_2$ as far as vibrational degrees of freedom are considered. More precisely: configuration space has a fiber structure: the 3-surfaces $Y^3 \subset \delta M^4_+ \times CP_2$ correspond to the base space and the 3-surfaces on the orbit of given Y^3 and diffeomorphic with Y^3 correspond to the fiber and are separated by a zero distance from each other in the configuration space metric.

These observations lead to the identification of the isometry group as some subgroup G of the group of the diffeomorphisms of $\delta H = \delta M^4_+ \times CP_2$. These diffeomorphisms indeed act in a natural manner in δCH, the space of the 3-surfaces in δH. Therefore one can identify the configuration space as the union of the coset spaces G/H, where H corresponds to the subgroup of G acting as diffeomorphisms for a given X^3. H depends on the topology of X^3 and since G does not change the topology of the 3-surface, each 3-topology defines a separate orbit of G. Therefore, the union involves the sum over all topologies of X^3 plus possibly other 'zero modes'.

The task is to identify correctly G as a sub-algebra of the diffeomorphisms of δH. The only possibility seems to be that the canonical transformations of δH generated by the function algebra of δH act as isometries of the configuration space. The canonical transformations act nontrivially also in δM^4_+ since δM^4_+ allows Kähler structure and thus also symplectic structure.

2.3.1 The magic properties of the light like 3-surfaces

In case of the Kähler metric, G- and H Lie-algebras must allow a complexification so that the isometries can act as holomorphic transformations. The unique feature of the lightcone boundary δM^4_+, realized already seven years ago, is its metric degeneracy: the boundary of the light cone is metrically 2-dimensional sphere although it is topologically 3-dimensional! This implies that light cone boundary allows an infinite-dimensional group of conformal symmetries generated by an algebra, which is a generalization of the ordinary Virasoro algebra! There is actually also an infinite-dimensional group of isometries (!) isomorphic with the group of the conformal transformations! Even more, in case of δH the groups of the conformal symmetries and isometries are local with respect to CP_2. Furthermore, light cone boundary allows infinite dimensional group of canonical transformations as the symmetries of the symplectic structure automatically associated with the Kähler structure. Therefore 4-dimensional Minkowski space is in a unique position in TGD approach. δM^4_+ allows also complexification and Kähler structure unlike the boundaries of the higher-dimensional light cones so that it becomes possible to define a complexification in the tangent space of the configuration space, too.

The space of the vector fields on $\delta H = \delta M^4_+ \times CP_2$ inherits the complex structure of the light cone boundary and CP_2. The complexification can be induced from the complex conjugation for the functions depending on the radial coordinate of the light cone boundary playing the same role as the time coordinate associated with string space-time sheet. In M^4_+ degrees of freedom complexification works only provided that the radial vector fields posses zero norm as configuration space vector fields (they have also zero norm as vector fields).

The effective two-dimensionality of the light cone boundary allows also to circumvent the no-go theorems associated with the higher-dimensional Abelian extensions. First, in the dimensions $D > 2$ Abelian extensions of the gauge algebra are extensions by an infinite dimensional Abelian group rather than central extensions by the group $U(1)$. In the present case the extension is a symplectic extension analogous to the extension defined by the Poisson bracket $\{p, q\} = 1$ rather than the standard central extension but is indeed 1- dimensional and well defined provided that the configuration space metric is Kähler. Secondly, $D > 2$ extensions possess no unitary faithful representations (satisfying certain well motivated physical constraints) [27]. The point is that light cone boundary is metrically and conformally 2-sphere and therefore the gauge algebra is effectively the algebra associated with the 2-sphere and, as a consequence, also the configuration space metric is Kähler.

There is counter argument against complexifixation. The Kähler structure of the light cone boundary is not unique: various complex structures are parameterized by $SO(3, 1)/SO(3)$ (Lobatchewski space). The definition of the Kähler function as absolute minimum of Kähler action however makes it possible to assign unique space-time surface $X^4(Y^3)$ to any Y^3 on the light cone boundary and the requirement that the group $SO(3)$ specifying the Kähler structure is isotropy group of the classical four-momentum associated with $X^4(Y^3)$, fixes the complex structure uniquely as a function of Y^3. Thus it seems that Kähler action is necessary ingredient of the group theoretical approach.

2.3.2 Symmetric space property reduces to conformal and canonical invariance

The idea about symmetric space is extremely beautiful but it millenium had to change before I was ripe to identify the precise form of the Cartan decomposition. The solution of the puzzle turned out to be amazingly simple.

The inspiration came from the finding that quantum TGD leads naturally to an extension of Super Algebras by combining Ramond and Neveu-Schwartz algebras into single algebra. This led to the introduction Virasoro generators and generators of canonical algebra of CP_2 localized with respect to the light cone boundary and carrying conformal weights with a half integer valued real part. Soon came the realization that the conformal weights $h = -1/2 - i\sum_i y_i$, where $z_i = 1/2 + y_i$ are non-trivial zeros of Riemann Zeta, are excellent candidates for the conformal weights. It took some time to answer affirmatively the question whether also the negatives of the trivial zeros $z = -2n$, $n > 0$ should be included. Thus the conjecture inspired by the work with Riemann hypothesis stating that the zeros of Riemann Zeta appear at the level of basic quantum TGD turned out be correct.

The generators whose commutators define the basis of the entire algebra have conformal weights given by the negatives of the zeros of Rieman Zeta. The algebra is a direct sum $g = g_1 \oplus g_2$ such that g_1 has $h = n$ as conformal weights and g_2 $h = n - 1/2 + iy$, where y is sum over imaginary parts y_i of non-trivial zeros of Zeta. Only $h = 2n$, $n > 1$, and $h = -1/2 - iy + n$, such that n is even (odd) if y is sum of odd (even) number of y_i correspond to the weights labelling the generators of t in the Cartan decomposition $g = h+t$. The resulting super-canonical algebra would quite well be christened as Riemann algebra.

The requirement that ordinary Virasoro and Kac Moody generators annihilate physical states corresponds now to the fact that the generators of h vanish at the point of configuration space, which remains invariant under the action of h. The maximum of Kähler function corresponds naturally to this point and plays also an essential role in the integration over configuration space by generalizing the Gaussian integration of free quantum field theories.

The light cone conformal invariance differs in many respects from the conformal invariance of string theories. Finite-dimensional group defining Kac-Moody group is replaced by an infinite-dimensional canonical group. Conformal weights correspond to zeros of Riemann zeta and suitable superpositions of them in case of trivial zeros, and physical states can have non-vanishing conformal weights just as the representations of color group in CP_2 can have non-vanishing color isospin and hyper charge. The conformal weights have also interpretation as quantum numbers associated with unitary representations of Lorentz group: thus there is no conflict between conformal invariance and Lorentz invariance in TGD framework.

2.4 An educated guess for the Kähler function

The turning point in the attempts to construct configuration space geometry was the realization that four-dimensional $Diff$ invariance (not only 3-dimensional $Diff$ invariance!) of General Relativity must have a counterpart in TGD. In order to realize this symmetry in the space of 3-surfaces, the definition of the configuration space metric should somehow associate to a given 3-surface X^3 a unique space-time surface $X^4(X^3)$ for Diff^4 to act on. Physical considerations require that the metric should be, not only Diff^4 invariant, but also Diff^4 degenerate so that infinitesimal Diff^4 transformations should correspond to zero norm vector fields of the configuration space.

Since Kähler function determines Kähler geometry, the definition of the Kähler function should associate a unique space-time surface $X^4(X^3)$ to a given

3-surface X^3. The natural physical interpretation for this space-time surface is as the classical space-time associated with X^3 so that in TGD classical physics ($X^4(X^3)$) becomes a part of the configuration space geometry and of the quantum theory.

One could try to construct the configuration space geometry by finding the metric for a single representative 3-surface at each orbit of G and extending it by left translations to the entire orbit of G. The metric for this representative should be $Diff^3$ invariant and somehow it should associate a unique space-time surface to the 3-surface in question. The original attempt was however more indirect and based on the realization that the construction of the Kähler geometry reduces to that of finding Kähler function $K(X^3)$ with the property that it associates a unique space-time surface $X^4(X^3)$ to a given 3-surface X^3 and possesses mathematically and physically acceptable properties. The guess for the Kähler function is the following one.

By Diff^4 invariance one can restrict the consideration on the set of 3-surfaces Y^3 on the 'light cone boundary' $\delta H = \delta M^4_+ \times CP_2$ since one can define the space-time surface associated with $X^3 \subset X^4(Y^3)$ to be $X^4(X^3) = X^4(Y^3)$ in case that the initial value problem for X^3 has $X^4(Y^3)$ as its solution. This implies $K(X^3) = K(Y^3)$.

The value of the Kähler function K for a given 3-surface Y^3 on light cone boundary is obtained in the following manner.

1. Consider all possible 4-surfaces $X^4 \subset M^4_+ \times CP_2$ having Y^3 as its sub-manifold: $Y^3 \subset X^4$. If Y^3 has boundary then it belongs to the boundary of X^4: $\delta Y^3 \subset \delta X^4$.

2. Associate to each four surface Kähler action as the Maxwell action for the Abelian gauge field defined by the projection of the CP_2 Kähler form to the four-surface. For a Minkowskian signature of the induced metric Kähler electric field gives a negative contribution to the action density whereas for an Euclidian signature the action density is always non-positive.

3. Define the value of the Kähler function K for Y^3 as the absolute minimum of the Kähler action S_K over all possible four-surfaces having Y^3 as its sub-manifold: $K(Y^3) = Min\{S_K(X^4)|X^4 \supset Y^3\}$.

This definition of the Kähler function has several physically appealing features.

1. Kähler geometry associates with each X^3 a unique four-surface, which will be interpreted as the classical space-time associated with X^3. This means that the so called classical space time (and physics!) in TGD approach is not defined via some approximation procedure (stationary phase approximation of the functional integral) but is an essential part of not only quantum theory, but also of the configuration space geometry, which in turn might be determined by a mere mathematical consistency! Since quantum states are superpositions over these classical space-times, it is

clear that the observed classical space-time is some kind of effective, quantum average space-time, presumably defined as an absolute minimum for the effective action of the theory.

2. The space-time surface associated with a given 3-surface is analogous to a Bohr orbit of the old fashioned quantum theory. The point is that the initial value problem in question differs from the ordinary initial value problem in that although the values of the H coordinates h^k as functions $h^k(x)$ of X^3 coordinates can be chosen arbitrarily, the time derivatives $\partial_t h^k(x)$ at X^3 are uniquely fixed by the principle selecting preferred extremals as generalized Bohr orbits (absolute minimization or something more delicate [E2]) unlike in the ordinary variational problems encountered in the classical physics. This implies something closely analogous to the quantization of the canonical momenta so that the space-time surface can be regarded as a generalized Bohr orbit. The classical quantization of electric charge and mass are possible consequences of the Bohr orbit property.

3. Kähler function is Diff^4 invariant in the sense that the value of the Kähler function is same for all 3-surfaces belonging to the orbit of a given 3-surface. As a consequence, configuration space metric is Diff^4 degenerate. The implications of the Diff^4 invariance have turned out to be decisive, not only for the geometrization of the configuration space, but also for the construction of the quantum theory. For instance, time like vibrational modes tangential to the 4-surface imply tachyonic mass spectrum unless they correspond to the zero modes of the configuration space metric. Diff^4 invariance however guarantees the required kind of degeneracy of the metric.

4. The non-determinism of Kähler action means that the complete reduction to the light cone boundary is not possible. This means a mathematical challenge but is physically a highly desirable feature since otherwise time would be lost as it is lost in the canonically quantized general relativity.

The most general expectation is that configuration space can be regarded as a union of coset spaces: $C(H) = \cup_i G/H(i)$. Index i labels 3-topology and zero modes. The group G, which can depend on 3-surface, can be identified as a subgroup of diffeomorphisms of $\delta M^4_+ \times CP_2$ and H must contain as its subgroup a group, whose action reduces to $Diff(X^3)$ so that these transformations leave 3-surface invariant.

The task is to identify plausible candidate for G and to show that the tangent space of the configuration space allows Kähler structure, in other words that the Lie-algebras of G and $H(i)$ allow complexification. One must also identify the zero modes and construct integration measure for the functional integral in these degrees of freedom. Besides this one must deduce information about the explicit form of configuration space metric from symmetry considerations combined with the hypothesis that Kähler function is determined as absolute minimum of Kähler action.

It will be found that in the case of $M^4_+ \times CP_2$ Kähler geometry, or strictly speaking contact Kähler geometry, characterized by a degenerate Kähler form ($Diff^4$ degeneracy and plus possible other degeneracies) seems possible. Although it seems that this construction must be generalized by allowing all light like 7-surfaces $X^3_l \times CP_2$, at least those for which X^3_l is boundary of light-cone inside M^4_+ or M^4, with the physical interpretation differing dramatically from the original one, the original construction discussed in the sequel involves the most essential aspects of the problem.

2.5 An alternative for the absolute minimization of Kähler action

One can criticize the assumption that extremals correspond to absolute minima, and the number theoretical vision discussed in [E2] indeed favors the separate minimization of magnitudes of positive and negative contributions to the Kähler action.

For this option Universe would do its best to save energy, being as near as possible to vacuum. Also vacuum extremals would become physically relevant: note that they would be only inertial vacua and carry non-vanishing density gravitational energy. The non-determinism of the vacuum extremals would have an interpretation in terms of the ability of Universe to engineer itself.

The 3-surfaces for which CP_2 projection is at least 2-dimensional and not a Lagrange manifold would correspond to non-vacua since conservation laws do not leave any other option. The variational principle would favor equally magnetic and electric configurations whereas absolute minimization of action based on S_K would favor electric configurations. The positive and negative contributions would be minimized for 4-surfaces in relative homology class since the boundary of X^4 defined by the intersections with 7-D light-like causal determinants would be fixed. Without this constraint only vacuum bubbles would result.

The attractiveness of the number theoretical variational principle from the point of calculability of TGD would be that the initial values for the time derivatives of the imbedding space coordinates at X^3 at light-like 7-D causal determinant could be computed by requiring that the energy of the solution is minimized. This could mean a computerizable solution to the construction of Kähler function.

It should be noticed that the considerations of this chapter relate only to the extremals of Kähler action which need not be absolute minima nor more general preferred extremals discussed in [E2] although this is suggested by the high symmetries. The number theoretic approach based on the properties of quaternions and octonions discussed in the chapter [E2] leads to a proposal for the general solution of field equations based on the generalization of the notion of calibration [80] providing absolute minima of volume to that of Kähler calibration. This approach will not be discussed in this chapter.

2.6 The construction of the configuration space geometry from symmetry principles

The gigantic size of the isometry group suggests that it might be possible to deduce very detailed information about the metric of the configuration space by group theoretical arguments. This turns out to be the case. In order to have a Kähler structure, one must define a complexification of the configuration space. Also one should identify the Lie algebra of the isometry group and try to derive explicit form of the Kähler metric using this information. One can indeed construct the metric in this manner but a rigorous proof that the corresponding Kähler function is the one defined by Kähler action does not exist yet although both approaches predict the same general qualitative properties for the metric. The argument stating the equivalence of the two approaches reduces to the hypothesis stating electric-magnetic duality of the theory. For the Bohr orbit like preferred extremals of Kähler action magnetic configuration space Hamiltonians derivable from group theoretical approach are essentially identical with electric configuration space Hamiltonians derivable from Kähler action.

2.6.1 General Coordinate Invariance and generalized quantum gravitational holography

The basic motivation for the construction of configuration space geometry is the vision that physics reduces to the geometry of classical spinor fields in the infinite-dimensional configuration space of 3-surfaces of $M^4_+ \times CP_2$ or of $M^4 \times CP_2$. Hermitian conjugation is the basic operation in quantum theory and its geometrization requires that configuration space possesses Kähler geometry. Kähler geometry is coded into Kähler function.

The original belief was that the four-dimensional general coordinate invariance of Kähler function reduces the construction of the geometry to that for the boundary of configuration space consisting of 3-surfaces on $\delta M^4_+ \times CP_2$, the moment of big bang. The proposal was that Kähler function $K(Y^3)$ could be defined as absolute minimum of so called Kähler action for the unique space-time surface $X^4(Y^3)$ going through given 3-surface Y^3 at $\delta M^4_+ \times CP_2$. For Diff^4 transforms of Y^3 at $X^4(Y^3)$ Kähler function would have the same value so that Diff^4 invariance and degeneracy would be the outcome.

This picture is however too simple.

1. The degeneracy of the absolute minima caused by the classical non-determinism of Kähler action however brings in additional delicacies, and it seems that the reduction to the light cone boundary which in fact corresponds to what has become known as quantum gravitational holography must be replaced with a construction involving more general light like 7-surfaces $X^3_l \times CP_2$.

2. It has also become obvious that the gigantic symmetries associated with $\delta M^4_+ \times CP_2$ manifest themselves as the properties of propagators and vertices, and that M^4 is favored over M^4_+. Cosmological considerations,

Poincare invariance, and the new view about energy favor the decomposition of the configuration space to a union of configuration spaces associated with various 7-D causal determinants. The minimum assumption is that all possible unions of future and past light cone boundaries $\delta M^4_\pm \times CP_2 \subset M^4 \times CP_2$ label the sectors of CH: the nice feature of this option is that the considerations of this chapter restricted to $\delta M^3_+ \times CP_2$ generalize almost trivially. This option is beautiful because the center of mass degrees of freedom associated with the different sectors of CH would correspond to M^4 itself and its Cartesian powers. One cannot exclude the possibility that even more general light like surfaces $X^3_l \times CP_2$ of M^4 are important as causal determinants.

The definition of the Kähler function requires that the many-to-one correspondence $X^3 \rightarrow X^4(X^3)$ must be replaced by a bijective correspondence in the sense that X^3 is unique among all its Diff^4 translates. This also allows physically preferred "gauge fixing" allowing to get rid of the mathematical complications due to Diff4 degeneracy. The internal geometry of the space-time sheet $X^4(X^3)$ must define the preferred 3-surface X^3 and also a preferred light like 7-surface $X^3_l \times CP_2$.

This is indeed possible. The possibility of negative values of Poincare energy(or equivalently inertial energy) inspires the hypothesis that the total quantum numbers and classical conserved quantities of the Universe vanish. This view is consistent with experimental facts if gravitational energy is defined as a difference of Poincare energies of positive and negative energy matter. Space-time surface consists of pairs of positive and negative energy space-time sheets created at some moment from vacuum and branching at that moment. This allows to select X^3 uniquely and define $X^4(X^3)$ as the absolute minimum of Kähler action in the set of 4-surfaces going through X^3. These space-time sheets should also define uniquely the light like 7-surface $X^3_l \times CP_2$, most naturally as the "earliest" surface of this kind. Note that this means that it become possible to assign a unique value of geometric time to the space-time sheet.

The realization of this vision means a considerable mathematical challenge. The effective metric 2-dimensionality of 3-dimensional light-like surfaces X^3_l of M^4 implies generalized conformal and canonical invariances allowing to generalize quantum gravitational holography from light like boundary so that the complexities due to the non-determinism can be taken into account properly.

2.6.2 Light like 3-D causal determinants, 7-3 duality, and effective 2-dimensionality

Thanks to the non-determinism of Kähler action, also light like 3-surfaces X^3_l of space-time surface appear as causal determinants (CDs). Examples are boundaries and elementary particle horizons at which Minkowskian signature of the induced metric transforms to Euclidian one. This brings in a second conformal symmetry related to the metric 2-dimensionality of the 3-D CD. This symmetry is identifiable as TGD counterpart of the Kac Moody symmetry of string models. The challenge is to understand the relationship of this symmetry to configuration space geometry and the interaction between the two conformal symmetries.

The possibility of spinorial shock waves at X^3_l leads to the hypothesis that they correspond to particle aspect of field particle duality whereas the physics in the interior of space-time corresponds to field aspect. More generally, field particle duality in TGD framework states that 3-D light like CDs and 7-D CDs are dual to each other. In particular, super-canonical and Super Kac Moody symmetries are also dually related.

The underlying reason for 7–3 duality be understood from a simple geometric picture in which 3-D light like CDs X^3_l intersect 7-D CDs X^7 along 2-D surfaces X^2 and thus form 2-sub-manifolds of the space-like 3-surface $X^3 \subset X^7$. One can regard either canonical deformations of X^7 or Kac-Moody deformations of X^2 as defining the tangent space of configuration space so that 7–3 duality would relate two different coordinate choices for CH.

The assumption that the data at either X^3 or X^3_l are enough to determine configuration space geometry implies that the relevant data is contained to their intersection X^2. This is the case if the deformations of X^3_l not affecting X^2 and preserving light likeness corresponding to zero modes or gauge degrees of freedom and induce deformations of X^3 also acting as zero modes. The outcome is effective 2-dimensionality. One cannot over-emphasize the importance of this conclusion. It indeed stream lines dramatically the earlier formulas for configuration space metric involving 3-dimensional integrals over $X^3 \subset M^4_+ \times CP_2$ reducing now to 2-dimensional integrals. Most importantly, no data about absolute minima of Kähler are needed to construct the configuration space metric so that the construction is also practical.

The reduction of data to that associated with 2-D surfaces conforms with the number theoretic vision about imbedding space as having hyper-octonionic structure [E2]: the commutative sub-manifolds of $OH = M^8$ have dimension not larger than two and for them tangent space is complex sub-space of hyper-octonion tangent space. Number theoretic counterpart of quantum measurement theory forces the reduction of relevant data to 2-D commutative sub-manifolds of X^3. These points are discussed in more detail in the next chapter whereas in this chapter the consideration will be restricted to $X^3_l = \delta M^4_+$ case which involves all essential aspects of the problem.

2.6.3 Two guesses for configuration space Hamiltonians

The detailed view about configuration space Hamiltonians developed gradually through guesses. The last section of this chapter provides the recent view about the construction of configuration space Hamiltonians based on a fundamental action principle at partonic level. Although the magnetic and electric Hamiltonians discussed below do not represent the last step in this progress they deserve a discussion.

1. *Magnetic Hamiltonians*

Assuming that the elements of the radial Virasoro algebra of δM^4_+ have zero norm, one ends up with an explicit identification of the symplectic structure of the configuration space. There is almost unique identification for the symplectic structure. Configuration space counterparts of $\delta M^4 \times CP_2$ Hamiltonians are defined by the generalized signed and and unsigned Kähler magnetic fluxes

$$Q_m(H_A, X^2) = Z \int_{X^2} H_A J \sqrt{g_2} d^2x \ ,$$

$$Q^+_m(H_A, r_M) = Z \int_{X^2} H_A |J| \sqrt{g_2} d^2x \ ,$$

$$J \equiv \epsilon^{\alpha\beta} J_{\alpha\beta} \ .$$

H_A is CP_2 Hamiltonian multiplied by a function of coordinates of light cone boundary belonging to a unitary representation of the Lorentz group. Z is a conformal factor depending on canonical invariants. The symplectic structure is induced by the symplectic structure of CP_2.

The most general flux is superposition of signed and unsigned fluxes Q_m and Q^+_m.

$$Q^{\alpha,\beta}_m(H_A, X^2) = \alpha Q_m(H_A, X^2) + \beta Q^+_m(H_A, X^2) \ .$$

Thus it seems that symmetry arguments fix the form of the configuration space metric apart from the presence of a conformal factor Z multiplying the magnetic flux and the degeneracy related to the signed and unsigned fluxes.

The notion of 7–3-duality described in the introduction implies that the relevant data about configuration space geometry is contained by 2-D surfaces X^2 at the intersections of 3-D light like CDS and 7-D CDs such as $M^4_+ \times CP_2$. In this case the entire Hamiltonian could be defined as the sum of magnetic fluxes over surfaces $X^2_i \subset X^3$. The maximally optimistic guess would be that it is possible to fix both X^2_i and 7-D CDs freely with X^2_i possibly identified as commutative sub-manifold of octonionic H.

2. Electric Hamiltonians and electric-magnetic duality

Absolute minimization of Kähler action in turn suggests that one can identify configuration space Hamiltonians as classical charges $Q_e(H_A)$ associated with the Hamiltonians of the canonical transformations of the light cone boundary, that is as variational derivatives of the Kähler action with respect to the infinitesimal deformations induced by $\delta M^4_+ \times CP_2$ Hamiltonians. Alternatively, one might simply replace Kähler magnetic field J with Kähler electric field defined by space-time dual $*J$ in the formulas of previous section. These Hamiltonians are analogous to Kähler electric charge and the hypothesis motivated by the experience with the instantons of the Euclidian Yang Mills theories and 'Yin-Yang' principle, as well as by the duality of CP_2 geometry, is that for the absolute minima of the Kähler action these Hamiltonians are affinely related:

$$Q_e(H_A) = Z \left[Q_m(H_A) + q_e(H_A) \right] \ .$$

Here Z and q_e are constants depending on canonical invariants only. Thus the equivalence of the two approaches to the construction of configuration space geometry boils down to the hypothesis of a physically well motivated electric-magnetic duality.

The crucial technical idea is to regard configuration space metric as a quadratic form in the entire Lie-algebra of the isometry group G such that the matrix elements of the metric vanish in the sub-algebra H of G acting as $Diff^3(X^3)$. The Lie-algebra of G with degenerate metric in the sense that H vector fields possess zero norm, can be regarded as a tangent space basis for the configuration space at point X^3 at which H acts as an isotropy group: at other points of the configuration space H is different. For given values of zero modes the maximum of Kähler function is the best candidate for X^3. This picture applies also in symplectic degrees of freedom.

2.6.4 Complexification and explicit form of the metric and Kähler form

The identification of the Kähler form and Kähler metric in canonical degrees of freedom follows trivially from the identification of the symplectic form and definition of complexification. The requirement that Hamiltonians are eigen states angular momentum (and possibly also of Lorentz boost), isospin and hypercharge implies physically natural complexification. In order to fix the complexification completely one must introduce some convention fixing which states correspond to 'positive' frequencies and which to 'negative frequencies' and which to zero frequencies that is to decompose the generators of the canonical algebra to three sets Can_+, Can_- and Can_0. One must distinguish between Can_0 and zero modes, which are not considered here at all. For instance, CP_2 Hamiltonians correspond to zero modes.

The natural complexification relies on the imaginary part of the radial conformal weight whereas the real part defines the $g = t+h$ decomposition naturally. The wave vector associated with the radial logarithmic plane wave corresponds to the angular momentum quantum number associated with a wave in S^1 in the case of Kac Moody algebra. One can imagine three options.

1. It is quite possible that the spectrum of k_2 does not contain $k_2 = 0$ at all so that the sector Can_0 could be empty. This complexification is physically very natural since it is manifestly invariant under $SU(3)$ and $SO(3)$ defining the preferred spherical coordinates. The choice of $SO(3)$ is unique if the classical four-momentum associated with the 3-surface is time like so that there are no problems with Lorentz invariance.

2. If $k_2 = 0$ is possible one could have

$$\begin{aligned} Can_+ &= \{H^a_{m,n,k=k1_+ik_2}, k_2 > 0\} \ , \\ Can_- &= \{H^a_{m,n,k}, k_2 < 0\} \ , \\ Can_0 &= \{H^a_{m,n,k}, k_2 = 0\} \ . \end{aligned} \tag{1}$$

3. If it is possible to $n_2 \neq 0$ for $k_2 = 0$, one could define the decomposition as

$$\begin{aligned} Can_+ &= \{H^a_{m,n,k}, k_2 > 0 \text{ or } k_2 = 0, n_2 > 0\} \ , \\ Can_- &= \{H^a_{m,n,k}, k_2 < 0 \text{ or} k_2 = 0, n_2 < 0\} \ , \\ Can_0 &= \{H^a_{m,n,k}, k_2 = n_2 = 0\} \ . \end{aligned} \tag{2}$$

   In this case the complexification is unique and Lorentz invariance guaranteed if one can fix the $SO(2)$ subgroup uniquely. The quantization axis of angular momentum could be chosen to be the direction of the classical angular momentum associated with the 3-surface in its rest system.

The only thing needed to get Kähler form and Kähler metric is to use the "half Poisson bracket"

$$\begin{aligned} J_f(X(H_A), X(H_B)) &= 2Im\,(iQ_f(\{H_A, H_B\}_{-+})) \ , \\ G_f(X(H_A), X(H_B)) &= 2Re\,(iQ_f(\{H_A, H_B\}_{-+})) \ . \end{aligned} \tag{3}$$

Here the subscript + and − refer to complex isometry current and its complex conjugate in terms of which the "half Poisson bracket" can be expressed.

Symplectic form, and thus also Kähler form and Kähler metric, could contain a conformal factor depending on the isometry invariants characterizing the size and shape of the 3-surface. At this stage one cannot say much about the functional form of this factor.

2.7 Configuration space spinor structure

Quantum TGD should be reducible to the classical spinor geometry of the configuration space. In particular, physical states should correspond to the modes of the configuration space spinor fields. The immediate consequence is that configuration space spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of the configuration space spinor structure there are some important clues.

1. The classical bosonic physics is coded into the definition of the configuration space metric; therefore the classical physics associated with the spinors of the imbedding space should be coded into the definition of the configuration space spinor structure. This means that the generalized massless Dirac equation for the induced spinor fields on $X^4(X^3)$ should be closely related to the definition of the configuration space gamma matrices.

2. Complex probability amplitudes (scalar fields) in the configuration space correspond to the second quantized boson fields in X^4. Hence the spinor

fields of the configuration space should correspond to the second quantized, free, induced spinor fields on X^4. The space of the configuration space spinors should be just the Fock space of the second quantized fermions on X^4!

3. Canonical algebra might generalize to a super canonical algebra and that super generators should be linearly related to the gamma matrices of the configuration space. If this indeed is the case then the construction of the configuration space spinor structure becomes a purely group theoretical problem.

The realization of these ideas is simple in principle. Perform a second quantization for the free induced spinor field in X^4. Express configuration space gamma matrices and canonical super generators as superpositions of the fermionic oscillator operators. This means that configuration space gamma matrices are analogous to spin 3/2 fields and can be regarded as a superpartner of the gravitational field of the configuration space. Deduce the anti-commutation relations of the spinor fields from the requirement of super canonical invariance. Generalize the flux representation for the configuration space Hamiltonians to a spinorial flux representation for their super partners.

2.7.1 Configuration space gamma matrices as super algebra generators

The basic idea is that the space of the configuration space spinors must correspond to the Fock space for the second quantized induced spinor fields. In accordance with this the gamma matrices of the configuration space must be expressible as superpositions of the fermionic oscillator operators for the second quantized induced free spinor fields in X^4 so that they are analogous to spin 3/2 fields. The Dirac equation is fixed from the requirement of super symmetry and has same vacuum degeneracy as Kähler action. A further assumption is that the contractions of the gamma matrices with isometry currents correspond to super charges of the group of isometries of the configuration space so that the construction reduces to group theory. Also the super Kac Moody algebra associated with light like 3-D CDs defines candidates for gamma matrices defining the components of the metric as anti-commutators and the question is whether the two definitions are mutually consistent.

2.7.2 7–3 duality

The failure of the classical non-determinism forces to introduce two kinds of causal determinants (CDs). 7-D light like CDs are unions of the boundaries of future and past directed light cones in M^4 at arbitrary positions (also more general light like surfaces $X^7 = X^3_l \times CP_2$ might be considered). CH is a union of sectors associated with these 7-D CDs playing in a very rough sense the roles of big bangs and big crunches. The creation of pairs of positive and negative energy space-time sheets occurs at $X^3 \subset X^7$ in the sense that negative and positive energy space-time sheet meet at X^3. Negative and positive energy space-time sheets are space-time correlates for bras and kets and the meeting of negative and positive energy space-time sheets is the space-time correlate for their scalar product.

Also 3-D light like causal determinants $X^3_l \subset X^4$ must be introduced: elementary particle horizons provide a basic example of this kind of CDs. The deformations of the 2-surfaces defining X^3_l define Kac Moody type conformal symmetries.

7–3 duality states that the two kind of CDs play a dual role in the construction of the theory and implies that 3-surfaces are effectively two-dimensional with respect to the CH metric in the sense that all relevant data about CH geometry is contained by the two-dimensional intersections $X^2 = X^3_l \cap X^7$ defining 2-sub-manifolds of $X^3 \subset X^7$.

The relationship between super-canonical (SC) and Super Kac-Moody (SKM) symmetries has been one of the central themes in the development of TGD. The progress in the understanding of the number theoretical aspects of TGD gives good hopes of lifting $SKMV$ (V denotes Virasoro) to a subalgebra of SCV so that coset construction works meaning that the differences of SCV and $SKMV$ generators annihilate physical states. This condition has interpretation in terms of Equivalence Principle with coset Super Virasoro conditions defining a generalization of Einstein's equations in TGD framework. Also p-adic thermodynamics finds a justification since the expectation values of SKM conformal weights can be non-vanishing in physical states.

2.7.3 The modified Dirac equation and gamma matrices

The modified Dirac equation is deduced from Kähler action by requiring it to have the same vacuum degeneracy as Kähler action itself. The interpretation of the solutions of the modified Dirac equation is as super gauge symmetry generators whereas physical degrees of freedom corresponds to generalized eigen modes at X^3_l and at space-like 3-surfaces $X^3 \subset X^7$.

The decisive property of the modified Dirac equation is that it allows shock wave solutions restricted to X^3_l: in terms of field-particle duality these shock waves correspond to the click caused by a particle in a detector. This allows to realize quantum gravitational holography and 7–3 duality in the sense that the induced second quantized spinor fields at the intersections $X^2 = X^3_l \cap X^7$ determine the super-generators super-canonical and super Kac Moody algebras invariant under the super gauge symmetries generated by the solutions of the modified Dirac equation.

Both the function algebra and Poisson algebra of X^7 allow super-symmetrization and both N-S and Ramond type representations are possible. For Ramond type representation the modified Dirac operators D_+ and D_-^{-1} associated with the positive and negative energy space-time sheets $X^4_\pm$ meeting at X^3 are present in the expressions of the super generators. NS-type representations correspond to the replacement of these operators with projection operators to the space of spinor modes with non-vanishing eigenvalues of $D_\pm$. Both representations are necessary and correspond to leptonic and quark like representations of configuration space gamma matrices. Similar statements apply to super Kac-Moody representations. These two kinds of representations correspond to super and kappa symmetries of super-string models.

2.7.4 Expressing Kähler function in terms of Dirac determinant

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography.

The simplest option (not the only one [B4] is that Kähler function for a given space-time sheet is the product of Dirac determinants associated with the light-like partonic 3-surfaces associated with with it. p-Adicization requires that for a given prime p the generalized eigenvalues of the modified Dirac operator D belong to an algebraic extension of rationals. The simplest manner to achieve this is to restrict the number of the allowed modes of D to those in the algebraic extension. This restriction would give rise to a purely physical cutoff and define one level in the number theoretical hierarchy of physics. This restriction could also lead automatically to a finite value of the Dirac determinant.

2.7.5 The relationship between super-canonical and super Kac-Moody algebras

The conformal weights of the generating elements of super-canonical representations correspond to the zeros of Riemann Zeta and one can identify the Cartan decomposition of the super-canonical algebra crucial for defining configuration space of 3-surfaces as a union of symmetric Kähler manifolds labelled by zero modes. Super-canonical algebra differs dramatically from super Kac Moody algebra. 7-3 duality however allows to see super-canonical and super Kac-Moody algebras as associated with two different tangent space basis for CH and giving rise to different coordinate systems. Hence both super algebras could give rise to a gamma matrix algebra of CH.

7–3 duality allows to generalize the Olive-Goddard-Kent coset construction. By 7-3 duality the differences of the commuting Virasoro generators of super-canonical and super Kac-Moody algebras must annihilate the physical states. For the same reason the central charges of the two Virasoro algebras must be identical so that the net central charge vanishes. This condition leads to a generalization of stringy mass formula involving besides super Kac-Moody algebra also the super-canonical algebra and allowing continuum mass spectrum for many particle states.

The $N = 4$ super symmetries generated by the solutions of the modified Dirac equation are pure super gauge transformations. All CP_2 spinor harmonics except the covariantly constant right handed neutrino spinor carry color quantum numbers and thus a non-vanishing vacuum conformal weight: hence only an $N = 1$ global super symmetry is in principle possible. Since the Ramond type super-generator corresponding to the covariantly constant neutrino vanishes identically even $N = 1$ global super-symmetry is absent and no sparticles are predicted. This means a decisive difference in comparison with super string models and M-theory.

Physical states satisfy both N-S and Ramond type Super Virasoro conditions separately: note that in super-canonical degrees of freedom Ramond/NS representations super generators involve carry quark/lepton number. The most obvious application of the mass formula would be to hadron physics. The effective 2-dimensionality allows to identify partons as 2-dimensional surfaces X_i^2, and the more than decade old notion of elementary particle vacuum functional finds a first principle justification as a functional in the modular degrees of freedom of X^2.

By quantum classical correspondence is that Virasoro algebra associated with super Kac-Moody algebra acts on the conformal weights of the super-canonical representations as conformal transformations and the generators of the super-canonical algebra can be regarded as conformal fields. This dictates the matrix elements of the algebra to a high degree as function of conformal weights. A connection with braid and quantum groups and II_1 sub-factors of type von Neumann algebras associated with the Clifford algebra of the configuration space emerges.

2.8 What about infinities?

The construction of a divergence free and unitary inner product for the configuration space spinor fields is one of the major challenges. In the sequel constraints on the geometry of the configuration space posed by the finiteness of the inner product are analyzed.

2.8.1 Inner product from divergence cancellation

Forgetting the delicacies related to the non-determinism of the Kähler action, the inner product is given by integrating the usual Fock space inner product defined at each point of the configuration space over the reduced configuration space containing only the 3-surfaces Y^3 belonging to $\delta H = \delta M_+^4 \times CP_2$ ('light cone boundary') using the exponent $exp(K)$ as a weight factor:

$$\begin{aligned} \langle \Psi_1 | \Psi_2 \rangle &= \int \overline{\Psi}_1(Y^3)\Psi_2(Y^3) exp(K)\sqrt{G} dY^3 \ , \\ \overline{\Psi_1}(Y^3)\Psi_2(Y^3) &\equiv \langle \Psi_1(Y^3)|\Psi_2(Y^3)\rangle_{Fock} \ . \end{aligned} \tag{4}$$

The degeneracy for the absolute minima of Kähler action implies additional summation over the degenerate minima associated with Y^3. The restriction of the integration on light cone boundary is Diff^4 invariant procedure and resolves in elegant manner the problems related to the integration over Diff^4 degrees of freedom. A variant of the inner product is obtained dropping the bosonic vacuum functional $exp(K)$ from the definition of the inner product and by assuming that it is included into the spinor fields themselves. Probably it is just a matter of taste how the necessary bosonic vacuum functional is included into the inner product: what is essential that the vacuum functional $exp(K)$ is somehow present in the inner product.

The unitarity of the inner product follows from the unitary of the Fock space inner product and from the unitarity of the standard L^2 inner product defined by configuration space integration in the set of the L^2 integrable scalar functions. It could well occur that $Diff^4$ invariance implies the reduction of the configuration space integration to $C(\delta H)$.

Consider next the bosonic integration in more detail. The exponent of the Kähler function appears in the inner product also in the context of the finite dimensional group representations. For the representations of the noncompact groups (say $SL(2,R)$) in coset spaces (now $SL(2,R)/U(1)$ endowed with Kähler metric) the exponent of Kähler function is necessary in order to get square integrable representations [31]. The scalar product for two complex valued representation functions is defined as

$$(f,g) \quad = \quad \int \overline{f} g exp(nK) \sqrt{g} dV \quad . \tag{5}$$

By unitarity, the exponent is an integer multiple of the Kähler function. In the present case only the possibility $n = 1$ is realized if one requires a complete cancellation of the determinants. In finite dimensional case this corresponds to the restriction to single unitary representation of the group in question.

The sign of the action appearing in the exponent is of decisive importance in order to make theory stable. The point is that the theory must be well defined at the limit of infinitely large system. Minimization of action is expected to imply that the action of infinitely large system is bound from above: the generation of electric Kähler fields gives negative contributions to the action. This implies that at the limit of the infinite system the average action per volume is non-positive. For systems having negative average density of action vacuum functional $exp(K)$ vanishes so that only configurations with vanishing average action per volume have significant probability. On the other hand, the choice $exp(-K)$ would make theory unstable: probability amplitude would be infinite for all configurations having negative average action per volume. In the fourth part of the book it will be shown that the requirement that average Kähler action per volume cancels has important cosmological consequences.

Consider now the divergence cancellation in the bosonic integration. One can develop the Kähler function as a Taylor series around maximum of Kähler function and use the contravariant Kähler metric as a propagator. Gaussian and metric determinants cancel each other for a unique vacuum functional. Ricci flatness guarantees that metric determinant is constant in complex coordinates so that one avoids divergences coming from it. The non-locality of the Kähler function as a functional of the 3-surface serves as an additional regulating mechanism: if $K(X^3)$ were a local functional of X^3 one would encounter divergences in the perturbative expansion.

The requirement that quantum jump corresponds to a quantum measurement in the sense of quantum field theories implies that quantum jump involves localization in zero modes. Localization in the zero modes implies automatically p-adic evolution since the decomposition of the configuration space into sectors D_P labelled by the infinite primes P is determined by the corresponding decomposition in zero modes. Localization in zero modes would suggest that the calculation of the physical predictions does not involve integration over zero modes: this would dramatically simplify the calculational apparatus of the theory. Probably this simplification occurs at the level of practical calculations if U-matrix separates into a product of matrices associated with zero modes and fiber degrees of freedom.

One must also calculate the predictions for the ratios of the rates of quantum transitions to different values of zero modes and here one cannot actually avoid integrals over zero modes. To achieve this one is forced to define the transition probabilities for quantum jumps involving a localization in zero modes as

$$P(x,\alpha \rightarrow y,\beta) = \sum_{r,s} |S(r,\alpha \rightarrow s,\beta)|^2 |\Psi_r(x)|^2 |\Psi_s(y)|^2 \quad ,$$

where x and y correspond to the zero mode coordinates and r and s label a complete state functional basis in zero modes and $S(r,m \rightarrow s,n)$ involves integration over zero modes. In fact, only in this manner the notion of the localization in the zero modes makes mathematically sense at the level of S-matrix. In this case also unitarity conditions are well-defined. In zero modes state function basis can be freely constructed so that divergence difficulties could be avoided. An open question is whether this construction is indeed possible.

Some comments about the actual evaluation of the bosonic functional integral are in order.

1. Since configuration space metric is degenerate and the bosonic propagator is essentially the contravariant metric, bosonic integration is expected to reduce to an integration over the zero modes. For instance, isometry invariants are variables of this kind. These modes are analogous to the parameters describing the conformal equivalence class of the orbit of the string in string models.

2. α_K is a natural small expansion parameter in configuration space integration. It should be noticed that α_K, when defined by the criticality condition, could also depend on the coordinates parameterizing the zero modes.

3. Semiclassical approximation, which means the expansion of the functional integral as a sum over the extrema of the Kähler function, is a natural approach to the calculation of the bosonic integral. Symmetric space property suggests that for the given values of the zero modes there is only single extremum and corresponds to the maximum of the Kähler function. There are theorems stating that semiclassical approximation is exact for

certain systems (for example for integrable systems (Duistermaat-Hecke theorem [32]). Symmetric space property suggests that Kähler function might possess the properties guaranteing the exactness of the semiclassical approximation. This would mean that the calculation of the integral $\int exp(K)\sqrt{G}dY^3$ and even more complex integrals involving configuration space spinor fields would be completely analogous to a Gaussian integration of free quantum field theory. This kind of reduction actually occurs in string models and is consistent with the criticality of the Kähler coupling constant suggesting that all loop integrals contributing to the renormalization of the Kähler action should vanish. Also the condition that configuration space integrals are continuable to p-adic number fields requires this kind of reduction.

2.8.2 Divergence cancellation, Ricci flatness, and symmetric space and Hyper Kähler properties

In the case of the loop spaces left invariance implies that Ricci tensor is a multiple of the metric tensor so that Ricci scalar has an infinite value. Mathematical consistency (essentially the absence of the divergences in the integration over the configuration space) forces the geometry to be Ricci flat: in other words, vacuum Einstein's equations are satisfied. It can be shown that Hyper Kähler property guarantees Ricci flatness. The reason is that the contractions of the curvature tensor appearing in the components of the Ricci tensor transform to traces over Lie algebra generators, which are $SU(\infty)$ generators instead of $U(\infty)$ generators as in case of loop spaces, so that the traces vanish.

Hyper Kähler property requires a quaternionic structure in the tangent space of the configuration space. Since any direction on the sphere S^2 defined by the linear combinations of quaternionic imaginary units with unit norm defines a particular complexification physically, Hyper-Kähler property means the possibility to perform complexification in S^2-fold manners. An interesting possibility raised by the notion of number theoretical compactification [E2] is that hyper Kähler structure could be replaced with what might be called "hyper-hyper-Kähler structure" resulting when quaternionic tangent space is replaced with its hyper-quaternionic variant. This would conform with the Minkowski signature of the space-time surface. In this framework also hyper-octonionic structure might be considered. An interesting question not yet even touched, is whether the conjectured $M^8 - -M^4 \times CP_2$ duality is realized also at the level of the configuration space of 3-surfaces.

Consider now the arguments in favor of Ricci flatness of the configuration space.

1. The canonical algebra of δM^4_+ takes effectively the role of the $U(1)$ extension of the loop algebra. More concretely, the $SO(2)$ group of the rotation group $SO(3)$ takes the role of $U(1)$ algebra. Since volume preserving transformations are in question, the traces of the canonical generators vanish

identically and in finite-dimensional this should be enough for Ricci flatness even if Hyper Kähler property is not achieved.

2. The comparison with CP_2 allows to link Ricci flatness with conformal invariance. The elements of the Ricci tensor are expressible in terms of traces of the generators of the holonomy group $U(2)$ at the origin of CP_2, and since $U(1)$ generator is non-vanishing at origin, the Ricci tensor is non-vanishing. In recent case the origin of CP_2 is replaced with the maximum of Kähler function and holonomy group corresponds to super-canonical generators labelled by integer valued real parts k_1 of the conformal weights $k = k_1 + i\rho$. If generators with $k_1 = n$ vanish at the maximum of the Kähler function, the curvature scalar should vanish at the maximum and by the symmetric space property everywhere. These conditions correspond to Virasoro conditions in super string models.

   A possible source of difficulties are the generators having $k_1 = 0$ and resulting as commutators of generators with opposite real parts of the conformal weights. It might be possible to assume that only the conformal weights $k = k_1 + i\rho$, $k_1 = 0, 1, ...$ are possible since it is the imaginary part of the conformal weight which defines the complexification in the recent case. This would mean that the commutators involve only positive values of k_1.

3. In the infinite-dimensional case the Ricci tensor involves also terms which are non-vanishing even when the holonomy algebra does not contain $U(1)$ factor. It will be found that symmetric space property guarantees Ricci flatness even in this case and the reason is essentially the vanishing of the generators having $k_1 = n$ at the maximum of Kähler function.

There are also arguments in favor of the Hyper Kähler property. In the following argument reader can well consider replacing the attribute "quaternionic" with "hyper-quaternionic".

1. The dimensions of the imbedding space and space-time are 8 and 4 respectively so that the dimension of configuration space in vibrational modes is indeed multiple of four as required by Hyper Kähler property. Hyper Kähler property requires a quaternionic structure in the tangent space of the configuration space. Since any direction on the sphere S^2 defined by the linear combinations of quaternionic imaginary units with unit norm defines a particular complexification physically, Hyper Kähler property means the possibility to perform complexification in S^2-fold manners.

2. S^2-fold degeneracy is indeed associated with the definition of the complex structure of the configuration space. First of all, the direction of the quantization axis for the spherical harmonics or for the eigen states of Lorentz Cartan algebra at $X^2_+ \times CP_2$ can be chosen in S^2-fold manners. Quaternion conformal invariance means Hyper Kähler property almost by definition and the S^2-fold degeneracy for the complexification is obvious in this case.

3. One can see the super-canonical conformal weights as points in a particular complex plane of the quaternionic space and the choice of this plane corresponds to a selection of one configuration space Kähler structure which are parameterized by S^2. The necessity to restrict the conformal weights to a complex plane brings in mind the commutativity constraint on simultaneously measurable quantum observables.

If these naive arguments survive a more critical inspection, the conclusion would be that the effective 2-dimensionality of light like 3-surfaces implying generalized conformal and canonical symmetries would also imply Hyper Kähler property of the configuration space and make the theory well-defined mathematically. This obviously fixes the dimension of space-time surfaces as well as the dimension of Minkowski space factor of the imbedding space.

3 Heuristic picture about massivation

The understanding of particle massivation developed in an inverted manner from p-adic mass calculations to their interpretation and has been a process with several side tracks. The recent picture is that massivation involves two mechanisms. p-Adic thermodynamics gives the dominant contribution to the masses of fermions and involves the contributions from p-adic thermodynamics for Virasoro generator L_0 and thermodynamics in modular degrees of freedom explaining the mass differences between different fermion families. Besides this there is contribution to Higgs particle identified as a wormhole contact carrying net weak isospin assignable to the fermion and antifermion at the lightlike partonic 3-surfaces defining the throats of the wormhole contact. This contribution dominates the masses of gauge bosons.

3.1 The relationship between inertial gravitational masses

It took quite a long time to accept the obvious fact that the relationship between inertial and gravitational masses cannot be quite the same as in General Relativity.

3.1.1 Modification of the Equivalence Principle?

The findings of [D3] combined with the basic facts about imbeddings of Robertson-Walker cosmologies [D5] force the conclusion that inertial mass density vanishes in cosmological length scales. This is possible if the sign of inertial energy depends on time orientation of the space-time sheet. This forces a modification of Equivalence Principle. The modified Equivalence Principle states that gravitational energy corresponds to the absolute value of inertial energy. Since inertial energy can have both signs, this means that gravitational mass is not conserved and is non-vanishing even for vacuum extremals. This differences is dual for the two time times: the experienced time identifiable as a sequence of quantum jumps and geometric time.

More generally, all conserved (that is Noether-) charges of the Universe vanish identically and their densities vanish in cosmological length scales. The simplest generalization of the Equivalence Principle would be that gravitational four-momentum equals to the absolute value of inertial four-momentum and is thus not conserved in general. Gravitational mass density does not vanish for vacuum extremals and, as will be found, one can deduce the renormalization of gravitational constant at given space-time sheet from the requirement that gravitational mass is conserved inside given space-time sheet. The conservation law holds only true inside given space-time sheet.

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since induced color fields are proportional to color Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

3.1.2 # contacts, non-conservation of gauge charges and gravitational four-momentum, and Higgs mechanism

Gravitational # contacts are necessary and if gravitational energy can be regarded in the Newtonian limit as a gauge charge, the contacts feed the gravitational energy regarded as a gauge flux to the lower condensate levels. The non-conservation of gravitational gauge flux means that # contacts can carry gravitational four-momentum and since CP_2 type vacuum extremals are the natural candidates for # contacts, the natural hypothesis is that the non-conserved light-like gravitational four-momentum of # contacts is responsible for the non-conservation of gravitational four-momentum flux. The non-conservation of the light-like gravitational four-momentum of CP_2 type extremals would in turn be responsible for the non-conservation of the net gravitational four-momentum.

contacts could be also carriers of inertial mass which must be conserved in absence of four-momentum exchange between environment and wormhole contact. Therefore Equivalence Principle cannot hold true in a strict sense even at elementary particle level. Equivalence Principle would be satisfied in a weak sense if the inertial four-momentum is equal to the average four-momentum associated with the zitterbewegung motion and corresponds to the center of mass motion for the # contact.

The non-conservation of weak gauge currents for CP_2 type extremals implies a non-conservation of weak charges and the finite range of weak forces. If wormhole contacts correspond to pieces of CP_2 type vacuum extremal, electro-weak gauge currents are not conserved classically unlike color and Kähler current. The non-conservation of weak isospin corresponds to the presence of pairs of right/left handed fermion and left/right handed antifermion at wormhole contacts. These wormhole contacts are excellent candidates for the TGD counterpart of Higgs boson providing the most natural mechanism for the massivation of weak bosons. The dominant contribution to fermion mass would be due to p-adic thermodynamics [F3]. If weak form of Equivalence Principle holds true, inertial mass would result simply as the average of non-conserved light-like gravitational four-momentum.

There would be two contributions to the mass of the elementary particle.

1. Part of the inertial mass is generated in the topological condensation of CP_2 type extremal representing elementary particle involving only single light like elementary particle horizon, say fermion, and would correspond naturally to the contribution to the mass modellable using p-adic thermodynamics. The contribution from primary topological condensation is negligible if the radius of the zitterbewegung orbit is larger than the size of the space-time sheet containing the topologically condensed boson so that the motion is along a light-like geodesic in a good approximation. For gauge bosons this contribution should be very small or vanishing. Systems like superconductors where also photons and even gravitons can become massive [D3] might form an exception in this respect.

2. The space-time sheet representing massless state suffered secondary topological condensation at a larger space-time sheet and viewed as a particle can develop an additional contribution to the mass via Higgs mechanism since the wormhole contacts cannot be regarded as moving along light like geodesics of M^4 in the length and time scale involved. # contacts carrying a net weak isospin would have interpretation as TGD counterparts of neutral Higgs bosons and the formation of coherent state involving a superposition of states with varying number of wormhole contacts would correspond to the generation of a vacuum expectation value of Higgs field. The inertial mass of the wormhole contact must be small, presumably its order of magnitude is given by $1/L_p$, where L_p is the characteristic p-adic length scale associated with a given condensate level.

3.1.3 Gravitational mass is necessarily accompanied by non-vanishing gauge charges

The experience from the study of the extremals of the Kähler action [D1] suggests that for non-vacuum extremals at astrophysical scales Kähler charge doesn't depend on the properties of the condensate and is apart from numerical constant equal to the gravitational mass of the system using Planck mass as unit:

$$Q_K = \epsilon_1 \frac{M_{gr}}{m_{proton}} \ . \tag{6}$$

The condition $\frac{\epsilon_1}{\sqrt{\alpha_K}} < 10^{-19}$ must hold true in astrophysical length scales since the long range gauge force implied by the Kähler charge must be weaker than gravitational interaction at astrophysical length scales. It is not clear whether the 'anomalous' Kähler charge can correspond to a mere Z^0 gauge or em charge or more general combination of weak charges.

Also for the imbedding of Schwartschild and Reissner-Nordström metrics as vacuum extremals non-vanishing gravitational mass implies that some electro-weak gauge charges are non-vanishing [D1]. For vacuum extremals with $sin^2(\theta_W) = 0$ em field indeed vanishes whereas Z^0 gauge field is non-vanishing.

If one assumes that the weak charges are screened completely in electro-weak length scale, the anomalous charge can be only electromagnetic if it corresponds to ordinary elementary particles. This however need not be consistent with field equations. Perhaps the most natural interpretation for the "anomalous" gauge charges is due to the elementary charges associated with dark matter. Since weak charges are expected to be screened in the p-adic length scale characterizing weak boson mass scale, the implication is that scaled down copies of weak bosons with arbitrarily small mass scales and arbitrarily long range of interaction are predicted. Also long ranged classical color gauge fields are unavoidable which forces to conclude that also a hierarchy of scaled down copies of gluons exists.

One can hope that photon and perhaps also Z^0 and color gauge charges in Cartan algebra could be quantized classically at elementary particle length scale ($p \leq M_{127}$, say) and electromagnetic gauge charge in all length scales apart from small renormalization effects. One of the reasons is that classical electromagnetic fields make an essential part in the description of, say, hydrogen atom.

The study of the extremals of Kähler action and of the imbeddings of spherically symmetric metrics [D3, D1] shows that the imbeddings are characterized by frequency type vacuum quantum numbers, which allow to fix these charges to pre-determined values. The minimization of Kähler action for a space-time surface containing a given 3-surface leads to the quantization of the vacuum parameters and hopefully to charge quantization. This motivates the hypothesis that the electromagnetic charges associated with the classical gauge fields of topologically condensed elementary particles are equal to their quantized counterparts. The discussion of dark matter leads to the conclusion that electro-weak and color gauge charges of dark matter can be non-vanishing [J6, F9].

3.1.4 Equivalence Principle as duality between super-canonical and Super Kac-Moody conformal algebras

A precise formulation of Equivalence Principle came from a deeper mathematical understanding of the relationship between super-canonical (SC) and Super Kac-Moody (SKM) symmetries which has been one of the central themes in the development of TGD. The progress in the understanding of the number theoretical aspects of TGD [E2] gives good hopes of lifting $SKMV$ (V denotes Virasoro) to a subalgebra of SCV so that coset construction works meaning that the differences of SCV and $SKMV$ generators annihilate physical states. This condition has interpretation in terms of Equivalence Principle with coset Super Virasoro conditions defining a generalization of Einstein's equations in TGD framework. Rather concretely: the actions of the imbedding space Dirac operator associated with the generator G_0 in for SC and SKM degrees of freedom are identical so that SKM and SC four-momenta and color quantum numbers identifiable as gravitational and inertial variants of these quantum numbers can be identified. Also p-adic thermodynamics finds a justification since the expectation values of SKM conformal weights can be non-vanishing in physical states.

3.2 The identification of Higgs as a weakly charged wormhole contact

Quantum classical correspondence suggests that electro-weak massivation should have simple space-time description allowing also to identify Higgs boson if it exists. This description indeed exists and allows also to understand the precise relationship between gravitational and inertial masses and how Equivalence Principle is weakened in TGD framework.

The basic observation is that gauge and gravitational fluxes flow to larger space-time sheets through # (wormhole) contacts. If gravitational energy can be regarded in the Newtonian limit as a gauge charge, the contacts feed the gravitational energy regarded as a gauge flux to the lower condensate levels. The non-conservation of gravitational gauge flux means that # contacts can carry gravitational four-momentum. Since CP_2 type vacuum extremals are the natural candidates for # contacts, the natural hypothesis is that the non-vanishing light-like gravitational four-momentum of # contacts is responsible for the non-conservation of gravitational four-momentum flux. The non-conservation of the light-like gravitational four-momentum of CP_2 type extremals is in turn responsible for the non-conservation of the net gravitational four-momentum.

contacts can be also carriers of inertial four-momentum which must be conserved in absence of four-momentum exchange between environment and wormhole contact. Therefore Equivalence Principle cannot hold true in strict sense. Equivalence Principle is satisfied in a weak sense if the inertial four-momentum is equal to the average four-momentum associated with the zitterbewegung motion and corresponds to the center of mass motion for the # contact.

The non-conservation of weak gauge currents for CP_2 type extremals implies a non-conservation of weak charges and the finite range of weak forces. If wormhole contacts correspond to pieces of CP_2 type vacuum extremal, electro-weak gauge currents are not conserved classically unlike color and Kähler current. The non-conservation of weak isospin corresponds to the presence of pairs of right/left handed fermion and left/right handed antifermion at wormhole contacts. These wormhole contacts are excellent candidates for the TGD counterpart of Higgs boson providing the most natural mechanism for the massivation of weak bosons. The finding that that CP_2 parts of the induced gamma matrices connect different M^4 chiralities of induced spinor fields provided the original motivation for the belief that Higgs mechanism is realized in some manner in TGD Universe. This coupling must be crucial for the formation of weakly charged wormhole contacts.

There are two contributions to the mass of elementary particle corresponding to the primary and secondary topological condensation.

1. The dominant contribution to the fermion masses would be due to p-adic thermodynamics describing primary topological condensation. If weak form of Equivalence Principle holds true, inertial mass would result simply as the average of non-conserved light-like gravitational four-momentum. This contribution to the inertial mass is generated in the topological condensation of CP_2 type extremal representing elementary particle involving only single light like elementary particle horizon, say fermion, and by randomness of the zitterbewegung corresponds naturally to the contribution given by p-adic thermodynamics.

2. For gauge bosons the contribution from primary condensation should be very small or vanishing if the radius of zitterbewegung orbit is larger than the size of the space-time sheet containing the topologically condensed boson so that the motion is along a light-like geodesic in a good approximation. The space-time sheet representing massless state suffered secondary topologically condensation at a larger space-time sheet and viewed as a particle can develop mass via Higgs mechanism since wormhole contacts cannot be regarded as moving along light like geodesics in the length and time scale involved. # contacts carrying net left handed weak isospin have interpretation as TGD counterparts of neutral Higgs bosons and the formation of a coherent state involving superposition of states with varying number of wormhole contacts corresponds to the generation of a vacuum expectation value of Higgs field.

3.3 General mass formula

One of the blessings of effective 2-dimensionality is that one can treat different 2-surfaces X^2_i as almost independent degrees of freedom. In the case of translations this is not true since independent translations lead the surfaces X^2 outside $\delta M^4_\pm \times CP_2$. Therefore one must consider two options.

1. If one neglects the correlation between the translations and assigns to each X^2_i independent translational degrees of freedom a separate mass formula for each X^2_i would result:

$$M_i^2 \quad = \quad -\sum_i L_{0i}(SKM) + \sum_i L_{0i}(SC) \ . \tag{7}$$

   Here $L_{0i}(SKM)$ contains a CP_2 cm term giving the CP_2 contribution to the mass squared known once the spinorial partial waves associated with super generators used to construct the state are known.

2. Perhaps the only internally consistent option is based on the assignment of the mass squared with the total cm. This would give

$$M^2 \quad = \quad (\sum_i p_i)^2 = \sum_i M_i^2 + 2\sum_{i\neq j} p_i \cdot p_j = -\sum_i L_{0i}(SKM) + \sum_i L_{0i}(SC) \quad . \tag{8}$$

   Here $L_{0i}(SKM)$ contains a CP_2 cm term giving the CP_2 contribution to the mass squared known once the spinorial partial waves associated with super generators used to construct the state are known. $L_0(SC)$ term contains only leptonic or quark oscillator operators unless one allows both the lepto-quark type gamma matrices involving both D^+ and D_-^{-1} and leptonic gamma matrices involving instead of $D_\pm^{\pm 1}$ the projector P to the spinor modes with a non-vanishing eigenvalue of D.

The decomposition of the net four-momentum to a sum of individual momenta can be regarded as subjective unless there is a manner to measure the individual masses. It might be that there is no unique assignment of momenta to individual partons and that this non-uniqueness is part of the gauge symmetry implied by 7–3 duality.

3.3.1 Mass squared as a thermal expectation of super Kac-Moody conformal weight

The general view about particle massivation is based on the generalized coset construction allowing to understand the p-adic thermal contribution to mass squared as a thermal expectation value of the conformal weight for super Kac-Moody Virasoro algebra ($SKMV$) or equivalently super-camonical Virasoro algebra (SCV). Conformal invariance holds true only for the differences of $SKMV$ and SCV generators. In the case of SCV and $SKMV$ only the generators G_n,L_n, $n > 0$, annihilate the physical states. Obviously the actions of super-canonical Virasoro (SCV) generators and Super Kac-Moody Virasoro generators on physical states are identical. The interpretation is in terms of Equivalence Principle. The conditions stating the vanishing of the differences of the generators become the TGD counterpart for Einstein's equations. The mathematical justification for this picture comes from the possibility to lift the SC algebra from $\delta M^4_\pm \times CP_2$ and SKM algebra from the partonic 3-surface X^3 to the level of imbedding space to hyper-complex and perhaps even hyper-quaternionic algebra. Here the basic prerequisite for number theoretic compactification to be discussed later plays a key role.

1. Super-Kac Moody conformal weights must be negative for elementary fermions and this can be understood if the real parts of fermionic conformal weights are equal to -1/2 as required by the scaling invariance of integration measures associated with the light-like coordinate of light-cone boundary.

2. Ground state with negative confromal is generated by applying SC generators (Hamiltonians and their super counterparts) with conformal weights $-1/2+iy$ plus SKM generators. Massless state annihilated by L_n, $n>0$ is is obtained from this by applying Super generators.

3. Massless state is thermalized with respect to SKMV with thermal excitations created by generators L_n, $n>0$.

3.3.2 Mass formula for bound states of partons

The coefficient of proportionality between mass squared and conformal weight can be deduced from the observation that the mass squared values for CP_2 Dirac operator correspond to definite values of conformal weight in p-adic mass calculations. It is indeed possible to assign to partonic 2-surface X^2 CP_2 partial waves correlating strongly with the net electro-weak quantum numbers of the parton so that the assignment of ground state conformal weight to CP_2 partial waves makes sense. In the case of M^4 degrees of freedom it is not possible to talk about momentum eigen states since translations take parton out of δH_+ so that momentum must be assigned with the tip of the light-cone containing the particle.

The additivity of conformal weight means additivity of mass squared at parton level and this has been indeed used in p-adic mass calculations. This implies the conditions

$$(\sum_i p_i)^2 = \sum_i m_i^2 \tag{9}$$

The assumption $p_i^2 = m_i^2$ makes sense only for massless partons moving collinearly. In the QCD based model of hadrons only longitudinal momenta and transverse momentum squared are used as labels of parton states, which would suggest that one has

$$\begin{aligned} p_{i,||}^2 &= m_i^2 \ , \\ -\sum_i p_{i,\perp}^2 + 2\sum_{i,j} p_i \cdot p_j &= 0 \ . \end{aligned} \tag{10}$$

The masses would be reduced in bound states: $m_i^2 \rightarrow m_i^2 - (p_T^2)_i$. This could explain why massive quarks can behave as nearly massless quarks inside hadrons.

3.4 Is also Higgs contribution expressible as p-adic thermal expectation?

The consideration of [A9] concerning explicit microscopic structure of dark variants of elementary particles allow to add some details to the general picture about particle massivation reducing to p-adic thermodynamics plus Higgs mechanism, both of them having description in terms of conformal weight.

3.4.1 General picture

1. The mass squared equals to the p-adic thermal average of the conformal weight. There are two contributions to this thermal average. One from the p-adic thermodynamics for super conformal representations, and one from the thermal average related to the spectrum of generalized eigenvalues λ of the modified Dirac operator D. Higgs expectation value appears in the role of a mass term in the Dirac equation just like λ in the modified Dirac equation. For the zero modes of D λ vanishes.

2. There are good motivations to believe that λ is expressible as a superposition of zeros of Riemann zeta or some more general zeta function. The problem is that λ is complex. Since Dirac operator is essentially the square root of d'Alembertian (mass squared operator), the natural interpretation of λ would be as a complex "square root" of the conformal weight.

Remark: The earlier interpretation of λ as a complex conformal weight looks rather stupid in light of this observation.

This encourages to consider the interpretation in terms of vacuum expectation of the square root of Virasoro generator, that is generators G of super Virasoro algebra, or something analogous. The super generators G of the super-conformal algebra carry fermion number in TGD framework where Majorana condition does not make sense physically. The modified Dirac operators for the two possible choices $t_\pm$ of the light-like vector appearing in the eigenvalue equation $D\Psi = \lambda t^k_\pm \Gamma_k \Psi$ could however define a bosonic algebra resembling super-conformal algebra. In fact, the operators $a_\pm = \lambda t^k_\pm \Gamma_k$ are nilpotent and anti-commute to λ so that the minimal super-algebra would be 3-dimensional.

The p-adic thermal expectation values of contractions of $t^k_- \gamma_k D_+$ and $t^k_+ \gamma_k D_-$ should co-incide with the vacuum expectations of Higgs and its conjugate. Note that D_+ and D_- would be same operator but with different definition of the generalized eigenvalue and hermitian conjugation would map these two kinds of eigen modes to each other. The real contribution to the mass squared would thus come naturally as $\langle |\lambda|^2 \rangle$. Of course, $\langle H \rangle = \langle \lambda \rangle$ is only a hypothesis encouraged by the internal consistency of the physical picture, not a proven mathematical fact.

3.4.2 Questions

This leaves still some questions.

1. Does the p-adic thermal expectation $\langle \lambda \rangle$ dictate $\langle H \rangle$ or vice versa? Physically it would be rather natural that the presence of a coherent state of Higgs wormhole contacts induces the mixing of the eigen modes of D. On the other hand, the quantization of the p-adic temperature T_p suggests that Higgs vacuum expectation is dictated by T_p.

2. Also the phase of $\langle\lambda\rangle$ should have physical meaning. Could the interpretation of the imaginary part of $\langle\lambda\rangle$ make possible the description of dissipation at the fundamental level?

3. Is p-adic thermodynamics consistent with the quantal description as a coherent state? The approach based on p-adic variants of finite temperature QFTs associate with the legs of generalized Feynman diagrams might resolve this question neatly since thermodynamical states would be genuine quantum states in this approach made possible by zero energy ontology.

4 Could also gauge bosons correspond to wormhole contacts?

The developments in the formulation of quantum TGD which have taken place during the period 2005-2007 (see [C2] besides this chapter) suggest dramatic simplifications of the general picture about elementary particle spectrum. p-Adic mass calculations [F3, F4, F5] leave a lot of freedom concerning the detailed identification of elementary particles. The basic open question is whether the *theory is free at parton level* as suggested by the recent view about the construction of S-matrix and by the almost topological QFT property of quantum TGD at parton level [C2]. Or more concretely: do partonic 2-surfaces carry only free many-fermion states or can they carry also bound states of fermions and anti-fermions identifiable as bosons?

What is known that Higgs boson corresponds naturally to a wormhole contact [C5]. The wormhole contact connects two space-time sheets with induced metric having Minkowski signature. Wormhole contact itself has an Euclidian metric signature so that there are two wormhole throats which are light-like 3-surfaces and would carry fermion and anti-fermion number in the case of Higgs. Irrespective of the identification of the remaining elementary particles MEs (massless extremals, topological light rays) would serve as space-time correlates for elementary bosons. Higgs type wormhole contacts would connect MEs to the larger space-time sheet and the coherent state of neutral Higgs would generate gauge boson mass and could contribute also to fermion mass.

The basic question is whether this identification applies also to gauge bosons (certainly not to graviton). This identification would imply quite a dramatic simplification since the theory would be free at single parton level and the only stable parton states would be fermions and anti-fermions. As will be found this identification allows to understand the dramatic difference between graviton and other gauge bosons and the weakness of gravitational coupling, gives a connection with the string picture of gravitons, and predicts that stringy states are directly relevant for nuclear and condensed matter physics as has been proposed already earlier [F8, J1, J2].

4.1 Option I: Only Higgs as a wormhole contact

The only possibility considered hitherto has been that elementary bosons correspond to partonic 2-surfaces carrying fermion-anti-fermion pair such that either fermion or anti-fermion has a non-physical polarization. For this option CP_2 type extremals condensed on MEs and travelling with light velocity would serve as a model for both fermions and bosons. MEs are not absolutely necessary for this option. The couplings of fermions and gauge bosons to Higgs would be very similar topologically. Consider now the counter arguments.

1. This option fails if the theory at partonic level is free field theory so that anti-fermions and elementary bosons cannot be identified as bound states of fermion and anti-fermion with either of them having non-physical polarization.

2. Mathematically oriented mind could also argue that the asymmetry between Higgs and elementary gauge bosons is not plausible whereas asymmetry between fermions and gauge bosons is. Mathematician could continue by arguing that if wormhole contacts with net quantum numbers of Higgs boson are possible, also those with gauge boson quantum numbers are unavoidable.

3. Physics oriented thinker could argue that since gauge bosons do not exhibit family replication phenomenon (having topological explanation in TGD framework) there must be a profound difference between fermions and bosons.

4.2 Option II: All elementary bosons as wormhole contacts

The hypothesis that quantum TGD reduces to a free field theory at parton level is consistent with the almost topological QFT character of the theory at this level. Hence there are good motivations for studying explicitly the consequences of this hypothesis.

4.2.1 Elementary bosons must correspond to wormhole contacts if the theory is free at parton level

Also gauge bosons could correspond to wormhole contacts connecting MEs [D1] to larger space-time sheet and propagating with light velocity. For this option there would be no need to assume the presence of non-physical fermion or anti-fermion polarization since fermion and anti-fermion would reside at different wormhole throats. Only the definition of what it is to be non-physical would be different on the light-like 3-surfaces defining the throats.

The difference would naturally relate to the different time orientations of wormhole throats and make itself manifest via the definition of light-like operator $o = x^k\gamma_k$ appearing in the generalized eigenvalue equation for the modified

Dirac operator [B4]. For the first throat o^k would correspond to a light-like tangent vector t^k of the partonic 3-surface and for the second throat to its M^4 dual $\hat{t}^k$ in a preferred rest system in M^4 (implied by the basic construction of quantum TGD). What is nice that this picture non-asks the question whether t^k or $\hat{t}^k$ should appear in the modified Dirac operator.

Rather satisfactorily, MEs (massless extremals, topological light rays) would be necessary for the propagation of wormhole contacts so that they would naturally emerge as classical correlates of bosons. The simplest model for fermions would be as CP_2 type extremals topologically condensed on MEs and for bosons as pieces of CP_2 type extremals connecting ME to the larger space-time sheet. For fermions topological condensation is possible to either space-time sheet.

4.2.2 What about light-like boundaries and macroscopic wormhole contacts?

Light-like boundaries of the space-time sheet can have macroscopic size and can carry free many-fermion states but not elementary bosons. Number theoretic braids and anyons might be assignable to these structures. Deformations of cosmic strings to magnetic flux tubes with a light-like outer boundary are especially interesting in this respect.

If the ends of a string like object move with light velocity as implied by the usual stringy boundary conditions they indeed define light-like 3-surfaces. Many-fermion states could be assigned at the ends of string. One could also connect in pairwise manner the ends of two time-like strings having opposite time orientation using two space-like strings so that the analog of boson state consisting of two wormhole contacts and analogous to graviton would result. "Wormhole throats" could have arbitrarily long distance in M^4.

Wormhole contacts can be regarded as slightly deformed CP_2 type extremals only if the size of M^4 projection is not larger than CP_2 size. The natural question is whether one can construct macroscopic wormhole contacts at all.

1. The throats of wormhole contacts cannot belong to vacuum extremals. One might however hope that small deformations of macrosopic vacuum extremals could yield non-vacuum wormhole contacts of macroscopic size.

2. A large class of macroscopic wormhole contacts which are vacuum extremals consists of surfaces of form $X_1^2 \times X_2^2 \subset (M^1 \times Y^2) \times E^3$, where Y^2 is Lagrangian manifold of CP_2 (induced Kähler form vanishes) and $M^4 = M^1 \times E^3$ represents decomposition of M^1 to time-like and space-like sub-spaces. X_2^2 is a stationary surface of E^3. Both $X_1^2 \subset M^1 \times CP_2$ and X_2^2 have an Euclidian signature of metric except at light-like boundaries $X_a^1 \times X_2^2$ and $X_b^1 \times X_2^2$ defined by ends of X_1^2 defining the throats of the wormhole contact.

3. This kind of vacuum extremals could define an extremely general class of macroscopic wormhole contacts as their deformations. These wormhole contacts describe an interaction of wormhole throats regarded as closed

strings as is clear from the fact that X^2 can be visualized as an analog of closed string world sheet X_1^2 in $M^1 \times Y^2$ describing a reaction leading from a state with a given number of incoming closed strings to a state with a given number of outgoing closed strings which correspond to wormhole throats at the two space-time sheets involved.

4.2.3 Phase conjugate states and matter- antimatter asymmetry

By fermion number conservation fermion-boson and boson-boson couplings must involve the fusion of partonic 3-surfaces along their ends identified as wormhole throats. Bosonic couplings would differ from fermionic couplings only in that the process would be $2 \rightarrow 4$ rather than $1 \rightarrow 3$ at the level of throats.

The decay of boson to an ordinary fermion pair with fermion and anti-fermion at the same space-time sheet would take place via the basic vertex at which the 2-dimensional ends of light-like 3-surfaces are identified. The sign of the boson energy would tell whether boson is ordinary boson or its phase conjugate (say phase conjugate photon of laser light) and also dictate the sign of the time orientation of fermion and anti-fermion resulting in the decay.

The two space-time sheets of opposite time orientation associated with bosons would naturally serve as space-time correlates for the positive and negative energy parts of the zero energy state and the sign of boson energy would tell whether it is phase conjugate or not. In the case of fermions second space-time sheet is not absolutely necessary and one can imagine that fermions in initial/final states correspond to single space-time sheet of positive/negative time orientation.

Also a candidate for a new kind interaction vertex emerges. The splitting of bosonic wormhole contact would generate fermion and time-reversed anti-fermion having interpretation as a phase conjugate fermion. This process cannot correspond to a decay of boson to ordinary fermion pair. The splitting process could generate matter-antimatter asymmetry in the sense that fermionic anti-matter would consist dominantly of negative energy anti-fermions at space-time sheets having negative time orientation [D5, D7].

This vertex would define the fundamental interaction between matter and phase conjugate matter. Phase conjugate photons are in a key role in TGD based quantum model of living matter. This involves model for memory as communications in time reversed direction, mechanism of intentional action involving signalling to geometric past, and mechanism of remote metabolism involving sending of negative energy photons to the energy reservoir [K1]. The splitting of wormhole contacts has been considered as a candidate for a mechanism realizing Boolean cognition in terms of "cognitive neutrino pairs" resulting in the splitting of wormhole contacts with net quantum numbers of Z^0 boson [J3, M6].

4.3 Graviton and other stringy states

Fermion and anti-fermion can give rise to only single unit of spin since it is impossible to assign angular momentum with the relative motion of wormhole throats. Hence the identification of graviton as single wormhole contact is not possible. The only conclusion is that graviton must be a superposition of fermion-anti-fermion pairs and boson-anti-boson pairs with coefficients determined by the coupling of the parton to graviton. Graviton-graviton pairs might emerge in higher orders. Fermion and anti-fermion would reside at the same space-time sheet and would have a non-vanishing relative angular momentum. Also bosons could have non-vanishing relative angular momentum and Higgs bosons must indeed possess it.

Gravitons are stable if the throats of wormhole contacts carry non-vanishing gauge fluxes so that the throats of wormhole contacts are connected by flux tubes carrying the gauge flux. The mechanism producing gravitons would the splitting of partonic 2-surfaces via the basic vertex. A connection with string picture emerges with the counterpart of string identified as the flux tube connecting the wormhole throats. Gravitational constant would relate directly to the value of the string tension.

The TGD view about coupling constant evolution [C5] predicts $G \propto L_p^2$, where L_p is p-adic length scale, and that physical graviton corresponds to $p = M_{127} = 2^{127} - 1$. Thus graviton would have geometric size of order Compton length of electron which is something totally new from the point of view of usual Planck length scale dogmatism. In principle an entire p-adic hierarchy of gravitational forces is possible with increasing value of G.

The explanation for the small value of the gravitational coupling strength serves as a test for the proposed picture. The exchange of ordinary gauge boson involves the exchange of single CP_2 type extremal giving the exponent of Kähler action compensated by state normalization. In the case of graviton exchange two wormhole contacts are exchanged and this gives second power for the exponent of Kähler action which is not compensated. It would be this additional exponent that would give rise to the huge reduction of gravitational coupling strength from the naive estimate $G \sim L_p^2$.

Gravitons are obviously not the only stringy states. For instance, one obtains spin 1 states when the ends of string correspond to gauge boson and Higgs. Also non-vanishing electro-weak and color quantum numbers are possible and stringy states couple to elementary partons via standard couplings in this case. TGD based model for nuclei as nuclear strings having length of order $L(127)$ [F8] suggests that the strings with light M_{127} quark and anti-quark at their ends identifiable as companions of the ordinary graviton are responsible for the strong nuclear force instead of exchanges of ordinary mesons or color van der Waals forces.

Also the TGD based model of high T_c super-conductivity involves stringy states connecting the space-time sheets associated with the electrons of the exotic Cooper pair [J1, J2]. Thus stringy states would play a key role in nuclear and condensed matter physics, which means a profound departure from stringy wisdom, and breakdown of the standard reductionistic picture.

4.4 Spectrum of non-stringy states

The 1-throat character of fermions is consistent with the generation-genus correspondence. The 2-throat character of bosons predicts that bosons are characterized by the genera (g_1, g_2) of the wormhole throats. Note that the interpretation of fundamental fermions as wormhole contacts with second throat identified as a Fock vacuum is excluded.

The general bosonic wave-function would be expressible as a matrix M_{g_1,g_2} and ordinary gauge bosons would correspond to a diagonal matrix $M_{g_1,g_2} = \delta_{g_1,g_2}$ as required by the absence of neutral flavor changing currents (say gluons transforming quark genera to each other). 8 new gauge bosons are predicted if one allows all 3×3 matrices with complex entries orthonormalized with respect to trace meaning additional dynamical $SU(3)$ symmetry. Ordinary gauge bosons would be $SU(3)$ singlets in this sense. The existing bounds on flavor changing neutral currents give bounds on the masses of the boson octet. The 2-throat character of bosons should relate to the low value $T = 1/n \ll 1$ for the p-adic temperature of gauge bosons as contrasted to $T = 1$ for fermions.

If one forgets the complications due to the stringy states (including graviton), the spectrum of elementary fermions and bosons is amazingly simple and almost reduces to the spectrum of standard model. In the fermionic sector one would have fermions of standard model. By simple counting leptonic wormhole throat could carry $2^3 = 8$ states corresponding to 2 polarization states, 2 charge states, and sign of lepton number giving 8+8=16 states altogether. Taking into account phase conjugates gives 16+16=32 states.

In the non-stringy boson sector one would have bound states of fermions and phase conjugate fermions. Since only two polarization states are allowed for massless states, one obtains $(2+1)\times(3+1) = 12$ states plus phase conjugates giving 12+12=24 states. The addition of color singlet states for quarks gives 48 gauge bosons with vanishing fermion number and color quantum numbers. Besides 12 electro-weak bosons and their 12 phase conjugates there are 12 exotic bosons and their 12 phase conjugates. For the exotic bosons the couplings to quarks and leptons are determined by the orthogonality of the coupling matrices of ordinary and boson states. For exotic counterparts of W bosons and Higgs the sign of the coupling to quarks is opposite. For photon and Z^0 also the relative magnitudes of the couplings to quarks must change. Altogether this makes 48+16+16=80 states. Gluons would result as color octet states. Family replication would extend each elementary boson state into $SU(3)$ octet and singlet and elementary fermion states into $SU(3)$ triplets.

5 Is it possible to understand coupling constant evolution at space-time level?

As this section was written for the first time, it was not yet possible to deduce the length scale evolution of coupling constants from Quantum TGD proper even at the level of principle. Quantum classical correspondence however encouraged the hope that it might be possible to achieve some understanding of the coupling constant evolution by using the classical theory.

The resulting model discussed in the sequel gave support for the earlier speculative picture about gauge coupling constants associated with a given space-time sheet as RG invariants [C5]. It remained an open question whether gravitational coupling constant is RG invariant inside give space-time sheet. Also the discrete p-adic coupling constant evolution replacing in TGD framework the ordinary RG evolution allowed a formulation at space-time level as also does the evolution of $\hbar$ associated with the phase resolution.

The understanding of the coupling constant evolution at the fundamental quantum level emerged much later in the framework of zero energy ontology using the notion of quantum measurement with a finite measurement resolution formulated in terms of the inclusions of von Neumann algebras known as hyper-finite factors of type II_1 emerging naturally in TGD framework. Although this crucial input is completely lacking from the the earlier discussion, I decided to keep it as such and give only a brief summary of the recent picture in the first subsection. A more detailed discussion of coupling constant evolution in the framework of zero energy ontology can be found from [C2].

5.1 General view about coupling constant evolution

The framework provided by zero energy ontology and hyper-finite factors of type II_1 (HFF) led to an almost unique M-matrix (generalization of S-matrix) from the requirement that the included HFF defines measurement resolution in the sense that the states obtained by applying this algebra are not distinguishable from each other in the measurement resolution used. This means that complex rays of state space are replaced with rays defined by the action of the included algebra. The condition that the included algebra behaves like complex numbers leads to the condition that M-matrix corresponds to Connes tensor product.

This leads to a discretization of the continuous coupling constant evolution to that taking place in time scales which are powers of two multiples of a fundamental time scale and to the understanding of p-adic length scale hypothesis with an additional prediction that secondary p-adic time scale defines a fundamental time scale assignable to elementary particles (.1 seconds in the case of electron). The key idea is that zero energy insertions to the positive and negative energy parts of the zero energy state in time scales $T/2^n$, $n > 0$, characterized by time scale T define zero energy radiative corrections. One can imagine also corrections respecting the zero energy state property but affecting the energy of the positive (negative) energy part of correction. The corrections are constrained by the condition that the M-matrix defining time like entanglement coefficients between positive and negative energy parts of the state defines a Connes tensor product.

The causal diamond formed by a pair of future and past directed light-cones with the temporal distance between the tips of light-cones given by secondary p-adic time-scale defines the basic geometric framework behind coupling constant evolution. It is this constraint which is lacking from the discussion below. The recent view about coupling constant evolution is discussed at the end of the chapter.

5.2 The evolution of gauge couplings at single space-time sheet

The renormalization group equations of gauge coupling constants g_i follow from the following idea. The basic observation is that gauge currents have vanishing covariant divergences whereas ordinary divergence does not vanish except in the Abelian case. The classical gauge currents are however proportional to $1/g_i^2$ and if g_i^2 is allowed to depend on the space-time point, the divergences of currents can be made vanishing and the resulting flow equations are essentially renormalization group equations. The physical motivation for the hypothesis is that gauge charges are assumed to be conserved in perturbative QFT. The space-time dependence of coupling constants takes care of the conservation of charges.

A surprisingly detailed view about RG evolution emerges.

1. The UV fixed points of RG evolution correspond to CP_2 type extremals (elementary particles).

2. The Abelianity of the induced Kähler field means that Kähler coupling strength is RG invariant which has indeed been the basic postulate of quantum TGD. The only possible interpretation is that the coupling constant evolution in sense of QFT:s corresponds to the discrete p-adic coupling constant evolution.

3. IR fixed points correspond to space-time sheets with a 2-dimensional CP_2 projection for which the induced gauge fields are Abelian so that covariant divergence reduces to ordinary divergence. Examples are cosmic strings (, which could be also seen as UV fixed points), vacuum extremals, solutions of a sub-theory defined by $M^4\times S^2$, S^2 a homologically non-trivial geodesic sphere, and "massless extremals".

4. At the light-like boundaries of the space-time sheet gauge couplings are predicted to be constant by conformal invariance and by effective two-dimensionality implying Abelianity: note that the 4-dimensionality of the space-time surface is absolutely essential here.

5. In fact, all known extremals of Kähler action correspond to RG fixed points since gauge currents are light-like so that coupling constants are constant at a given space-time sheet. This is consistent with the earlier hypothesis that gauge couplings are renormalization group invariants and coupling constant evolution reduces to a discrete p-adic evolution. As a consequence also Weinberg angle, being determined by a ratio of $SU(2)$ and $U(1)$ couplings, is predicted to be RG invariant. A natural condition fixing its value would be the requirement that the net vacuum em charge

of the space-time sheet vanishes. This would state that em charge is not screened like weak charges.

6. When the flow determined by the gauge current is not integrable in the sense that flow lines are identifiable as coordinate curves, the situation changes. If gauge currents are divergenceless for all solutions of field equations, one can assume that gauge couplings are constant at a given space-time sheet and thus continuous also in this case. Otherwise a natural guess is that the coupling constants obtained by integrating the renormalization group equations are continuous in the relevant p-adic topology below the p-adic length scale. Thus the effective p-adic topology would emerge directly from the hydrodynamics defined by gauge currents.

5.3 RG evolution of gravitational constant at single space-time sheet

Similar considerations apply in the case of gravitational and cosmological constants.

1. In this case the conservation of gravitational mass determines the RG equation (gravitational energy and momentum are not conserved in general).

2. The assumption that coupling cosmological Λ constant is proportional to $1/L_p^2$ (L_p denotes the relevant p-adic length scale) explains the mysterious smallness of the cosmological constant and leads to a RG equation which is of the same form as in the case of gauge couplings.

3. Asymptotic cosmologies for which gravitational four momentum is conserved correspond to the fixed points of coupling constant evolution now but there are much more general solutions satisfying the constraint that gravitational mass is conserved.

4. It seems that gravitational constant cannot be RG invariant in the general case and that effective p-adicity can be avoided only by a smoothing out procedure replacing the mass current with its average over a four-volume 4-volume of size of order p-adic length scale.

5.4 p-Adic evolution of gauge couplings

If RG invariance at given space-time sheet holds true, the question arises whether it is possible to understand p-adic coupling constant evolution at space-time level.

1. Simple considerations lead to the idea that M^4 scalings of the intersections of 3-surfaces defined by the intersections of space-time surfaces with light-cone boundary induce transformations of space-time surface identifiable as RG transformations. If sufficiently small they leave gauge charges

invariant: this seems to be the case for known extremals which form scaling invariant families. When the scaling corresponds to a ratio p_2/p_1, $p_2 > p_1$, bifurcation would become possible replacing p_1-adic effective topology with p_2-adic one.

2. Stability considerations determine whether p_2-adic topology is actually realized and could explain why primes near powers of 2 are favored. The renormalization of coupling constant would be dictated by the requirement that Q_i/g_i^2 remains invariant.

It will be later explained how the construction of M-matrix with a finite measurement resolution defined as a generalization of S-matrix identified as "complex square root" of density matrix characterizing time-like entanglement in zero energy ontology leads to a concrete view about p-adic coupling constant evolution analogous to that in terms of radiative corrections.

5.5 p-Adic evolution in angular resolution and dynamical $\hbar$

The basic philosophical idea behind renormalization group approach is thinning of degrees of freedom when the scale of resolution is reduced. One can also consider renormalization group evolution associated with angle/phase resolution and quantum group phases associated with Beraha numbers defining Jones indices $\mathcal{M} : \mathcal{N}$ are excellent candidates for defining angular resolution in some sense [E9, C7, E10].

The concrete view about Jones inclusions developed in [C8] leads to precise predictions for the scaling factors of Planck constant in M^4 and CP_2 degrees of freedom. M^4 Planck constant is simply the integer n characterizing the quantum phase $q = exp(i\pi/n)$ assignable to the discrete subgroup of $G \subset SU(2)$ acting in CP_2 degrees of freedom. Also the value $n = \infty$ has physical meaning and analogous expression for Planck constants holds true also now. This limit corresponds to a phase in which partonic 2-surfaces have CP_2 projection which is homologically non-trivial geodesic sphere and thus carries topological magnetic charge.

In p-adic TGD these angular resolutions would have very concrete interpretation since only algebraic extensions obtained by introducing phases $q = exp(i\pi/n)$ make it possible to speak about phases in p-adic sense (in cognitive sense). For a finite-dimensional extension of p-adic numbers the number of existing angles is thus always finite. Each Jones inclusion would define algebraic extension of p-adic giving rise to definite angular resolution defined by $\Delta\phi = \pi/n$ and at the limit $n \to \infty$ the resolution would become ideal. The larger the Planck constant the better the angular resolution.

1. The change $n \to n - 1$ of Jones inclusion could be seen as thinning of degrees of freedom associated with angular resolution leading from resolution $\Delta\phi = \pi/n$ to $\Delta\phi = \pi/(n - 1)$. For instance, for $n = 3$ one would obtain a minimal angular resolution with quantum phase equal to

$exp(i\pi/3)$. Only three angles values would be discernible p-adically. These phases would correspond naturally to the phases assignable to the center of SU(3) whose Dynkin diagram indeed corresponds to $n = 3$ inclusion. Color $SU(3)$ would be the most rigid or minimal symmetry and would not reduce to $SU(2)$. Color degrees of freedom would correspond to the color rotational rigid body degrees of freedom of a topologically condensed space-time sheet.

2. Contrary to the original guess it seems that the minimum angular resolution cannot correspond to vacuum extremals: this is also consistent with the assumption that non-perturbative phase is in question. Since $SU(2)$ generates homologically non-trivial geodesic spheres of CP_2, G invariance and the requirement that space-time sheet defines a smooth $N(G)$-fold cover of M^4 probably imply non-vacuum extremal property so that CP_2 projection would have CP_2 dimension $D(CP_2) \geq 2$.

3. Since angular resolution becomes poorer in $n \rightarrow n-1$ transition and the dimension $\mathcal{M} : \mathcal{N}$ of $\mathcal{M}$ as $\mathcal{N}$-module is reduced in the transition, it seems natural to assign angular resolution to the dynamics to these $\mathcal{M} : \mathcal{N}$ degrees of freedom.

4. The natural physical interpretation for the angular resolution would be in terms of multiple cover of M^4 by a subgroup $G \subset SU/2)$. One could perhaps say that $2\pi/n$ rotation in CP_2 would correspond to 2π rotation in M^4 and would be thus have a very concrete representation.

5.6 A revised view about the interpretation and evolution of Kähler coupling strength

The original hypothesis was that Kähler coupling strength is invariant under p-adic coupling constant evolution. Later I gave up this hypothesis and replaced it with the invariance of gravitational coupling since otherwise the prediction would have been that gravitational coupling strength is proportional to p-adic length scale squared. The recent view means return to the roots: Kähler coupling strength is invariant under p-adic coupling constant evolution and has value spectrum dictated by the Chern-Simons coupling k defining the theory at the parton level. Gravitational coupling constant corresponds in this framework to the largest Mersenne prime M_{127} which does not correspond to a completely super-astronomical p-adic length scale.

5.6.1 Formula for Kähler coupling constant

To construct expression for gravitational constant one can use the following ingredients.

1. The exponent $exp(2S_K(CP_2))$ defining the value of Kähler function in terms of the Kähler action $S_K(CP_2)$ of CP_2 type extremal representing elementary particle expressible as

$$S_K(CP_2) = \frac{\pi}{8\alpha_K} . \quad (11)$$

Since CP_2 type extremals suffer topological condensation, one expects that the action is modified:

$$S_K(CP_2) \rightarrow a \times S_K(CP_2) . \quad (12)$$

Naively one would expect reduction of the action so that one would have $a \leq 1$. One must however keep mind open in this respect.

2. The p-adic length scale L_p assignable to the space-time sheet along which gravitational interactions are mediated. Since Mersenne primes seem to characterized elementary bosons and since the Mersenne prime $M_{127} = 2^{127}-1$ defining electron length scale is the largest non-super-astronomical length scale it is natural to guess that M_{127} characterizes these space-time sheets.

The formula for gravitational constant would read as

$$\begin{aligned} G &= L_p^2 \times exp(-2aS_K(CP_2)) . \\ L_p &= \sqrt{p}R . \end{aligned} \quad (13)$$

Here R is CP_2 radius defined by the length $2\pi R$ of the geodesic circle. The relationship boils down to

$$\begin{aligned} \alpha_K &= \frac{a\pi}{4log(pK)} , \\ K &= \frac{R^2}{G} . \end{aligned} \quad (14)$$

The value of K is fixed by the requirement that electron mass scale comes out correctly in the p-adic mass calculations and minimal value of K is factor. The uncertainties related to second order contributions however leave the precise value open.

The earlier calculations contained two errors. First error was related to the value of the parameter $K = R^2/G$ believed to be in good approximation given by the product of primes smaller than 26. Second error was that $1/\alpha_K$ was by a factor 2 too large for $a = 1$. This led first to a conclusion that α_K is very near to fine structure constant and perhaps equal to it. The physically more plausible option turned out to corresponds to $1/\alpha_K = 104$, which corresponds in good approximation to the value of electro-weak U(1) coupling at electron length scale but gave $a > 1$ whereas $a < 1$ would be natural since the action for a wormhole contact formed by a piece of CP_2 type vacuum extremal is expected to be smaller than the full action of CP_2 type vacuum extremal.

The correct calculation gives $a < 1$ for $\alpha_K = 1/104$. From the table one finds that if the parameter a equals to $a = 1/2$ the value of α_K is about 133. It would require $a = .6432$ for $Y_e = 0$ favored by the value of top quark mass. This value of a conforms with the idea that a piece of CP_2 type extremal defining a wormhole contact is in question. Note that a proper choices of value of a can make $K = R^2/G$ rational. The table gives values of various quantities assuming

$$K = 2\times 3\times 5\times 7\times 11\times 13\times 17\times 19\times 23\times 2^{-3}*(15/17) \ . \quad (15)$$

giving simplest approximation as a rational for K producing K_R for $Y_e = 0$ with error of 2.7 per cent which is still marginally consistent with the mass of top quark. This approximation should not be taken too seriously.

Y_e	0	.5	.7798
$(m_0/m_{Pl})10^3$	.2437	.2323	.2266
$K_R\times 10^{-7}$	2.5262	2.7788	2.9202
$(L_R/\sqrt{G})\times 10^{-4}$	3.1580	3.3122	3.3954
$1/\alpha_K$	133.7850	133.9064	133.9696
a_{104}	0.6432	0.6438	0.6441
a_α	0.4881	0.4886	0.4888
$K\times 10^{-7}$	2.4606	2.4606	2.4606
$(L/\sqrt{G})\times 10^{-4}$	3.1167	3.1167	3.1167
$1/\alpha_K$	133.9158	133.9158	133.9158
a_{104}	0.6438	0.6438	0.6438
a_α	.4886	0.4886	0.4886
K_R/K	1.0267	1.1293	1.1868

Table 1. Table gives the values of the ratio $K_R = R^2/G$ and CP_2 geodesic length $L = 2\pi R$ for $Y_e \in \{0, 0.5, 0.7798\}$. Also the ratio of K_R/K, where $K = 2\times 3\times 5\times 7\times 11\times 13\times 17\times 19\times 23\times 2^{-3}*(15/17)$ is rational number producing R^2/G approximately is given*[1]. The values of α_K deduced using the formula above are given for $a = 1/2$ and the values of $a = a_{104}$ giving $\alpha_K = 1/104$ are given. Also the values of $a = a_\alpha$ for which α_K equals to the fine structure constant $1/\alpha_{em} = 137.0360$ are given.

If one assumes that α_K is of order fine structure constant in electron length scale, the value of the parameter a is slightly below 1/2 cannot be far from unity. Symmetry principles do not favor the identification. Later it will be found that rather general arguments predict integer spectrum for $1/\alpha_K$ given by $1/\alpha_K =$

[1]The earlier calculations giving $K = 2\times 3\times 5\times 7\times 11\times 13\times 17\times 19\times 23$ in a good approximation contained an error

$4k$. For this option $\alpha_K = 1/137$ is not allowed whereas the $1/\alpha_K = 104 = 4\times 26$ is.

5.6.2 Formula relating v_0 to α_K and R^2/G

If v_0 is identified as the rotation velocity of distant stars in galactic plane, one can use the Newtonian model for the motion of mass in the gravitational field of long straight string giving $v_0 = \sqrt{TG}$. String tension T can be expressed in terms of Kähler coupling strength as

$$T = \frac{b}{2\alpha_K R^2} \ ,$$

where R is the radius of geodesic circle. The factor $b \leq 1$ would explain reduction of string tension in topological condensation caused by the fact that not entire geodesic sphere contributes to the action.

This gives

$$\begin{aligned} v_0 &= \frac{b}{2\sqrt{\alpha_K K}} \ , \\ \alpha_K(p) &= \frac{a\pi}{4log(pK)} \ , \\ K &= \frac{R^2}{G} \ . \end{aligned} \quad (16)$$

The condition that α_K has the desired value for $p = M_{127} = 2^{127} - 1$ defining the p-adic length scale of electron fixes the value of b for given value of a. The value of b should be smaller than 1 corresponding to the reduction of string tension in topological condensation.

The condition 16 for $v_0 = 2^{-m}$, say $m = 11$, allows to deduce the value of a/b as

$$\frac{a}{b} = \frac{4 * log(pK)}{\pi} \frac{2^{2m-1}}{K} \ . \quad (17)$$

The table gives the values of b calculated assuming $a = a_{104}$ producing $\alpha_K = 1/104$ for various values of Y_e.

Y_e	0	.5	.7798
$b_{9,104}$	0.9266	1.0193	1.0711
$b_{11,104}$	0.0579	0.0637	0.0669
$b_{9,\alpha}$	0.7032	0.7736	0.81291
$b_{11,\alpha}$	0.0440	0.0483	0.050

Table 2. Table gives the values of b for $Y_e \in \{0, .5, .7798\}$ assuming the values $a = a_{104}$ guaranteing $\alpha_K = 1/104$ and $\alpha_K = \alpha_{em}$. $b_{9,...}$ corresponds to $m = 9$ and $b_{11,...}$ corresponds to $m = 11$.

From the table one finds that for $\alpha_K = 1/104$ $m = 9$ corresponds to the almost full action for topological condensed cosmic string. $m = 11$ corresponds to much smaller action smaller by a factor of order 1/16. The interpretation would be that as m increases the action of the topologically condensed cosmic string decreases. This would correspond to a gradual transformation of the cosmic string to a magnetic flux tube.

5.6.3 Does α_K correspond α_{em} or weak coupling strength $\alpha_{U(1)}$ at electron length scale?

The preceding arguments allow the original identification $\alpha_K \simeq 1/137$. On the other hand, group theoretical arguments encourage the identification of α_K as weak $U(1)$ coupling strength $\alpha_{U(1)}$:

$$\begin{aligned} \alpha_K &= \alpha_{U(1)} = \frac{\alpha_{em}}{cos^2(\theta_W)} \simeq \frac{1}{104.1867} \ , \\ sin^2(\theta_W)_{|10\ MeV} &\simeq 0.2397(13) \ , \\ \alpha_{em}(M_{127}) &= 0.00729735253327 \ . \end{aligned} \tag{18}$$

Here Weinberg angle corresponds to 10 MeV energy is reasonably near to the value at electron mass scale. The value $sin^2(\theta_W) = 0.2397(13)$ corresponding to 10 MeV mass scale [88] is used.

Later it will be found that general argument predicts that $1/\alpha_K$ is integer valued: $1/\alpha_K = 4k$. This option excludes identification as $\alpha_{em}(127)$ but encourages strongly the identification as $\alpha_{U(1)}(127)$.

5.6.4 Is gravitational constant an approximate RG invariant?

The original model for the p-adic evolution of α_K was based on the p-adic renormalization group invariance of gravitational constant. The starting point was the observation that on purely dimension analytic basis one can write $G = exp(-2S_K(CP_2))L_p^2$. If α_K is p-adic RG invariant, G scales like L_p^2 which looked completely non-sensible at that time so that the identification $\alpha_K = \alpha_{U(1)}$ was given up. Discrete p-adic evolution for α_K is consistent with RG invariance and quantum criticality at a given p-adic space-time sheet.

This view however leads to problems with the identification $\alpha_K = \alpha_{U(1)}$ since the evolution of α_K dictated by RG invariance of G is much faster than that of $\alpha_{U(1)}$. The condition

$$cos^2(\theta_W)(89) = \frac{log(M_{127}K)}{log(M_{89}K)} \times \frac{\alpha_{em}(M_{127})}{\alpha_{em}(M_{89})} \times cos^2(\theta_W)(127) \ . \tag{19}$$

together with the experimental value $1/\alpha_{em}(M_{89}) \simeq 128$ as predicted by standard model, gives $sin^2(\theta_W)(89) = .0474$ to be compared with the measured valued .23120(15) at intermediate boson mass scale [88]!

Furthermore, if α_K evolves with p then v_0 is predicted to evolve too but $v_0 = 2^{-11}$ is consistent with experimental facts (apart from possible presence of sub-harmonics which can be however understood in TGD framework).

5.6.5 Or is α_K RG invariant?

One is forced to ask whether one must give up the existing scenario for the p-adic evolution of α_K and identify it with the evolution of $\alpha_{U(1)}$ or perhaps even p-adic RG invariance of α_K. The predicted very fast evolution $G \propto L_p^2$ in the approximation that α_K is RG invariant makes sense only if L_p characterizes the space-time sheets carrying gravitational interaction or even to gravitons and if these space-time sheet corresponds to $p = M_{127}$ under normal conditions.

If bosons correspond to Mersenne primes, this would be naturally the case since the Mersenne prime next to M_{127} corresponds to a completely super-astrophysical length scale. In this case p-adic length scale hypothesis would predict $v_0^{-2}(L(k)) = 2^{-22}2^{-k+127}$ if α_K is RG invariant so that it would behave as a power of 2. $\hbar_{gr}$ would scale as 2^{-k+127} and approach rapidly to zero as $L(k)$ increases whereas gravitational force would become strong.

If same p_0 characterizes all ordinary gauge bosons with their dark variants included, one would have $p_0 = M_{89} = 2^{89} - 1$. In this case however the gravitational coupling strength would be weaker by a factor 2^{-38}. M_{127} also defines a dark length scale in TGD inspired quantum model of living matter [F9, J6].

A further nice feature of this identification is that one can also allow the scaling of CP_2 metric and thus R^2 by $\lambda^2 = (\hbar/\hbar_0)^2$ inducing $K \rightarrow \lambda^2 K$. $1/v_0 \rightarrow \lambda/v_0$ implies that $\hbar_{gr}$ scales in the same manner as $\hbar$. Hence it would seem that $\hbar$ corresponds to M^4- and $\hbar_{gr}$ to CP_2 degrees of freedom and the huge value of $\hbar_{gr}$ would mean that there is that cosmology has quantal lattice like structure in cosmological length scales with H_a/G, $G \subset SL(2,C)$, serving as a basic lattice cell (here H_a denotes $a = constant$ hyperboloid of M^4_+). The observed sub-harmonics of v_0 could thus be understood in terms of scalings of CP_2 gravitational constant. This structure is supported also by the quantization of cosmological red shifts [87].

The huge value of h_{gr} assignable to color algebra does not mean that colored states would have huge values of color charges since fractionization of color quantum numbers occurs. It however means that dark color charges are de-localized in huge length scales and cosmological color could be seen as responsible for a macroscopic quantum coherence in astrophysical length scales.

5.6.6 What about color coupling strength?

Classical theory should be also able to say something non-trivial about color coupling strength α_s too at the general level. The basic observations are following.

1. Both classical color YM action and electro-weak U(1) action reduce to Kähler action.

2. Classical color holonomy is Abelian which is consistent also with the fact that the only signature of color that induced spinor fields carry is anomalous color hyper charge identifiable as an electro-weak hyper charge.

Suppose that α_K is a strict RG invariant. The first idea would be that the sum of classical color action and electro-weak U(1) action is RG invariant and thus equals to its asymptotic value obtained for $\alpha_{U(1)} = \alpha_s = 2\alpha_K$. Asymptotically the couplings approach to a fixed point defined by $2\alpha_K$ rather than to zero as in asymptotically free gauge theories.

Thus one would have

$$\frac{1}{\alpha_{U(1)}} + \frac{1}{\alpha_s} = \frac{1}{\alpha_K} \ . \tag{20}$$

The formula implies that the beta functions for color and $U(1)$ degrees of freedom are apart from sign identical and the increase of $U(1)$ coupling compensates the decrease of the color coupling. This gives the formula

$$\alpha_s = \frac{1}{\frac{1}{\alpha_K} - \frac{1}{\alpha_{U(1)}}} \ . \tag{21}$$

At least formally $\alpha_s(QCD)$ could become negative below the confinement length scale so that $\alpha_K < \alpha_{U(1)}$ for M_{127} is consistent with this formula. For M_{89} $\alpha_{em} \simeq 1/127$ gives $1/\alpha_{U(1)}(89) = 1/97.6374$.

1. $\alpha_K = \alpha_{em}(127)$ option does not work. Confinement length scale corresponds to the point at which one has $\alpha_{U(1)} = \alpha_K$ and in principle can be predicted precisely using standard model. In the case that $\alpha_s(107)$ diverges, one has

$$\alpha_{em}(107) = cos^2(\theta_W)\alpha_{U(1)} = cos^2(\theta_W)\alpha_K = \frac{cos^2(\theta_W)}{136} \ .$$

The resulting value of α_{em} is too small and the situation worsens for $k > 107$ since $\alpha_{U(1)}$ decreases. Hence this option is excluded.

2. TGD predicts that also M_{127} copy of QCD should exist and that M_{127} quarks should play a key role in nuclear physics [F8]. Hence one could argue that color coupling strength diverges at M_{127} (the largest not completely super-astrophysical Mersenne prim

3. so that one would have $\alpha_K = \alpha_{U(1)}(M_{127})$. Therefore the precise knowledge of $\alpha_{U(1)}(M_{127})$ in principle fixes the value of parameter $K = R^2/G$ and thus also the second order contribution to the mass of electron. On ther other hand, quite a general argument predicts $\alpha_K = 1/104$ so that an exact prediction for $U(1)$ coupling follows.

The predicted value of $\alpha_s(M_{89})$ follows from $sin^2(\theta_W) = .23120$ and $\alpha_{em} \simeq 1/127$ at intermediate boson mass scale using $\alpha_{U(1)} = \alpha_{em}/cos^2(\theta_W)$ and $1/\alpha_s = 1/\alpha_K - 1/\alpha_{U(1)}$. The predicted value $\alpha_s(89) = 0.1572$ is quite reasonable although somewhat larger than QCD value. For $1/\alpha_K = 108 = 4 \times 27$ one would have $\alpha_s(89) = 0.0965$.

To sum up, the proposed formula would dictate the evolution of α_s from the evolution of the electro-weak parameters without any need for perturbative computations and number theoretical prediction for U(1) coupling at electron length scale would be exact. Although the formula of proposed kind is encouraged by the strong constraints between classical gauge fields in TGD framework, it should be deduced in a rigorous manner from the basic assumptions of TGD before it can be taken seriously.

5.7 Does the quantization of Kähler coupling strength reduce to the quantization of Chern-Simons coupling at partonic level?

Kähler coupling strength associated with Kähler action (Maxwell action for the induced Kähler form) is the only coupling constant parameter in quantum TGD, and its value (or values) is in principle fixed by the condition of quantum criticality since Kähler coupling strength is completely analogous to critical temperature. The quantum TGD at parton level reduces to almost topological QFT for light-like 3-surfaces. This almost TQFT involves Abelian Chern-Simons action for the induced Kähler form.

This raises the question whether the integer valued quantization of the Chern-Simons coupling k could predict the values of the Kähler coupling strength. I considered this kind of possibility already for more than 15 years ago but only the reading of the introduction of the [85] about his new approach to 3-D quantum gravity led to the discovery of a childishly simple argument that the inverse of Kähler coupling strength could indeed be proportional to the integer valued Chern-Simons coupling k: $1/\alpha_K = 4k$. $k = 26$ is forced by the comparison with some physical input. Also p-adic temperature could be identified as $T_p = 1/k$.

5.7.1 Quantization of Chern-Simons coupling strength

For Chern-Simons action the quantization of the coupling constant guaranteeing so called holomorphic factorization is implied by the integer valuedness of the Chern-Simons coupling strength k. As Witten explains, this follows from the quantization of the first Chern-Simons class for closed 4-manifolds plus the requirement that the phase defined by Chern-Simons action equals to 1 for a boundaryless 4-manifold obtained by gluing together two 4-manifolds along their boundaries. As explained by Witten in his paper, one can consider also "anyonic" situation in which k has spectrum Z/n^2 for n-fold covering of the gauge group and in dark matter sector one can consider this kind of quantization.

5.7.2 Formula for the Kähler coupling strength

The quantization argument for k seems to generalize to the case of TGD. What is clear that this quantization should closely relate to the quantization of the Kähler coupling strength appearing in the 4-D Kähler action defining Kähler function for the world of classical worlds and conjectured to result as a Dirac determinant. The conjecture has been that g_K^2 has only single value. With some physical input one can make educated guesses about this value. The connection with the quantization of Chern-Simons coupling would however suggest a spectrum of values. This spectrum is easy to guess.

1. Wick rotation argument

The U(1) counterpart of Chern-Simons action is obtained as the analog of the "instanton" density obtained from Maxwell action by replacing $J \wedge *J$ with $J \wedge J$. This looks natural since for self dual J associated with CP_2 type vacuum extremals Maxwell action reduces to instanton density and therefore to Chern-Simons term. Also the interpretation as Chern-Simons action associated with the classical SU(3) color gauge field defined by Killing vector fields of CP_2 and having Abelian holonomy is possible. Note however that *instanton density is multiplied by imaginary unit in the action exponential of path integral.* One should find justification for this "Wick rotation" not changing the value of coupling strength and later this kind of justification will be proposed.

Wick rotation argument suggests the correspondence $k/4\pi = 1/4g_K^2$ between Chern-Simons coupling strength and the Kähler coupling strength g_K appearing in 4-D Kähler action. This would give

$$g_K^2 = \frac{\pi}{k} \ , \frac{1}{\alpha_K} = 4k \ . \quad (22)$$

The spectrum of $1/\alpha_K$would be integer valued. The result is very nice from the point of number theoretic vision since the powers of α_K appearing in perturbative expansions would be rational numbers (ironically, radiative corrections should vanish by number theoretic universality but this might happen only for these rational values ofα_K!).

2. Are more general values of k possible

Note however that if k is allowed to have values in Z/n^2, the strongest possible coupling strength is scaled to $n^2/4$ unless $\hbar$ is not scaled: already for $n = 2$ the resulting perturbative expansion might fail to converge. In the scalings of $\hbar$ associated with M^4 degrees of freedom $\hbar$ however scales as $1/n^2$ so that the spectrum of α_K would remain invariant.

3. Experimental constraints on α_K

It is interesting to compare the prediction with the experimental constraints on the value of $1/\alpha_K$. As already found, there are two options to consider.

1. $\alpha_K = \alpha_{em}$ option suggests $1/\alpha_K = 137$ inconsistent with $1/\alpha_K = 4k$ condition. $1/\alpha_K = 136 = 4 \times 34$ combined with the formula $1/\alpha_s + 1/\alpha_{U(1)} = 1/\alpha_K$ leads to nonsensical predictions.

2. For $1/\alpha_s + 1/\alpha_{U(1)} = 1/\alpha_K = 104$ option option the basic empirical input is that electro-weak $U(1)$ coupling strength reduces to Kähler coupling at electron length scale. This gives $\alpha_K = \alpha_{U(1)}(M_{127}) \simeq 104.1867$, which corresponds to $k = 26.0467$. $k = 26$ would give $\alpha_K = 104$: the difference would be only .2 per cent and one would obtain exact prediction for $\alpha_{U(1)}(M_{127})$. Together with electro-weak coupling constant evolution this would also explain why the inverse of the fine structure constant is so near to 137 but not quite. Amusingly, $k = 26$ is the critical space-time dimension of the bosonic string model. Also the conjectured formula for the gravitational constant in terms of α_K and p-adic prime p involves all primes smaller than 26.

5.7.3 Justification for Wick rotation

It is not too difficult to believe to the formula $1/\alpha_K = qk$, q some rational. $q = 4$ however requires a justification for the Wick rotation bringing the imaginary unit to Chern-Simons action exponential lacking from Kähler function exponential.

In this kind of situation one might hope that an additional symmetry might come in rescue. The guess is that number theoretic vision could justify this symmetry.

1. To see what this symmetry might be consider the generalization of the [86] obtained by combining theta angle and gauge coupling to single complex number via the formula

$$\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2} \ . \tag{23}$$

   What this means in the recent case that for CP_2 type vacuum extremals [D1] Kähler action and instanton term reduce by self duality to Kähler action obtained by the replacement g^2 with $-i\tau/4\pi$. The first duality $\tau \rightarrow \tau + 1$ corresponds to the periodicity of the theta angle. Second duality $\tau \rightarrow -1/\tau$ corresponds to the generalization of Montonen-Olive duality $\alpha \rightarrow 1/\alpha$. These dualities are definitely not symmetries of the theory in the recent case.

2. Despite the failure of dualities, it is interesting to write the formula for τ in the case of Chern-Simons theory assuming $g_K^2 = \pi/k$ with $k > 0$ holding true for Kac-Moody representations. What one obtains is

$$\tau = 4k(1 - i) \ . \tag{24}$$

The allowed values of τ are integer spaced along a line whose direction angle corresponds to the phase $exp(i2\pi/n)$, $n = 4$. The transformations $\tau \rightarrow \tau + 4(1-i)$ generate a dynamical symmetry and as Lorentz transformations define a subgroup of the group E^2 leaving invariant light-like momentum (this brings in mind quantum criticality!). One should understand why this line is so special.

3. This formula conforms with the number theoretic vision suggesting that the allowed values of τ belong to an integer spaced lattice. Indeed, if one requires that the phase angles are proportional to vectors with rational components then only phase angles associated with orthogonal triangles with short sides having integer valued lengths m and n are possible. The additional condition that the phase angles correspond to *roots of unity*! This leaves only $m = n$ and $m = -n > 0$ into consideration so that one would have $\tau = n(1-i)$ from $k > 0$.

4. Notice that theta angle is a multiple of $8k\pi$ so that a trivial strong CP breaking results and no QCD axion is needed (this of one takes seriously the equivalence of Kähler action to the classical color YM action).

5.7.4 Is p-adicization needed and possible only in 3-D sense?

The action of CP_2 type extremal is given as $S = \pi/8\alpha_K = k\pi/2$. Therefore the exponent of Kähler action appearing in the vacuum functional would be $exp(k\pi)$ - Gelfond's constant - known to be a transcendental number [81]. Also its powers are transcendental. If one wants to p-adicize also in 4-D sense, this raises a problem.

Before considering this problem, consider first the 4-D p-adicization more generally.

1. The definition of Kähler action and Kähler function in p-adic case can be obtained only by algebraic continuation from the real case since no satisfactory definition of p-adic definite integral exists. These difficulties are even more serious at the level of configuration space unless algebraic continuation allows to reduce everything to real context. If TGD is integrable theory in the sense that functional integral over 3-surfaces reduces to calculable functional integrals around the maxima of Kähler function, one might dream of achieving the algebraic continuation of real formulas. Note however that for light-like 3-surface the restriction to a category of algebraic surfaces essential for the re-interpretation of real equations of 3-surface as p-adic equations. It is far from clear whether also preferred extremals of Kähler action have this property.

2. Is 4-D p-adicization the really needed? The extension of light-like partonic 3-surfaces to 4-D space-time surfaces brings in classical dynamical variables necessary for quantum measurement theory. p-Adic physics defines correlates for cognition and intentionality. One can argue that these

are not quantum measured in the conventional sense so that 4-D p-adic space-time sheets would not be needed at all. The p-adic variant for the exponent of Chern-Simons action can make sense using a finite-D algebraic extension defined by $q = exp(i2\pi/n)$ and restricting the allowed light-like partonic 3-surfaces so that the exponent of Chern-Simons form belongs to this extension of p-adic numbers. This restriction is very natural from the point of view of dark matter hierarchy involving extensions of p-adics by quantum phase q.

If one remains optimistic and wants to p-adicize also in 4-D sense, the transcendental value of the vacuum functional for CP_2 type vacuum extremals poses a problem (not the only one since the p-adic norm of the exponent of Kähler action can become completely unpredictable).

1. One can also consider extending p-adic numbers by introducing $exp(\pi)$ and its powers and possibly also π. This would make the extension of p-adics infinite-dimensional which does not conform with the basic ideas about cognition. Note that e^p is not p-adic transcendental so that extension of p-adics by powers e is finite-dimensional and if p-adics are first extended by powers of π then further extension by $exp(\pi)$ is p-dimensional.

2. A more tricky manner to overcome the problem posed by the CP_2 extremals is to notice CP_2 type extremals are necessarily deformed and contain a hole corresponding to the light-like 3-surface or several of them. This would reduce the value of Kähler action and one could argue that the allowed p-adic deformations are such that the exponent of Kähler action is a p-adic number in a finite extension of p-adics. This option does not look promising.

5.7.5 Is the p-adic temperature proportional to the Kähler coupling strength?

Kähler coupling strength would have the same spectrum as p-adic temperature T_p apart from a multiplicative factor. The identification $T_p = 1/k$ is indeed very natural since also g_K^2 is a temperature like parameter. The simplest guess is

$$T_p = \frac{1}{k} . \tag{25}$$

Also gauge couplings strengths are expected to be proportional to g_K^2 and thus to $1/k$ apart from a factor characterizing p-adic coupling constant evolution. That all basic parameters of theory would have simple expressions in terms of k would be very nice from the point of view quantum classical correspondence.

If U(1) coupling constant strength at electron length scales equals $\alpha_K = 1/104$, this would give $1/T_p = 1/26$. This means that photon, graviton, and gluons would be massless in an excellent approximation for say $p = M_{89} =$

$2^{89}-1$, which characterizes electro-weak gauge bosons receiving their masses from their coupling to Higgs boson. For fermions one has $T_p = 1$ so that fermionic light-like wormhole throats would correspond to the strongest possible coupling strength $\alpha_K = 1/4$ whereas gauge bosons identified as pairs of light-like wormhole throats associated with wormhole contacts would correspond to $\alpha_K = 1/104$. Perhaps $T_p = 1/26$ is the highest p-adic temperature at which gauge boson wormhole contacts are stable against splitting to fermion-antifermion pair. Fermions and possible exotic bosons created by bosonic generators of super-canonical algebra would correspond to single wormhole throat and could also naturally correspond to the maximal value of p-adic temperature since there is nothing to which they can decay.

A fascinating problem is whether $k = 26$ defines internally consistent conformal field theory and is there something very special in it. Also the thermal stability argument for gauge bosons should be checked.

What could go wrong with this picture? The different value for the fermionic and bosonic α_K makes sense only if the 4-D space-time sheets associated with fermions and bosons can be regarded as disjoint space-time regions. Gauge bosons correspond to wormhole contacts connecting (deformed pieces of CP_2 type extremal) positive and negative energy space-time sheets whereas fermions would correspond to deformed CP_2 type extremal glued to single space-time sheet having either positive or negative energy. These space-time sheets should make contact only in interaction vertices of the generalized Feynman diagrams, where partonic 3-surfaces are glued together along their ends. If this gluing together occurs only in these vertices, fermionic and bosonic space-time sheets are disjoint. For stringy diagrams this picture would fail.

To sum up, the resulting overall vision seems to be internally consistent and is consistent with generalized Feynman graphics, predicts exactly the spectrum of α_K, allows to identify the inverse of p-adic temperature with k, allows to understand the differences between fermionic and bosonic massivation, and reduces Wick rotation to a number theoretic symmetry. One might hope that the additional objections (to be found sooner or later!) could allow to develop a more detailed picture.

5.8 Why gravitation is so weak as compared to gauge interactions?

The weakness of gravitational interaction in contrast to other gauge interactions is definitely a fundamental test for the proposed picture. The heuristic argument allowing to understand the value of gravitational constant is based on the assumption that graviton exchange corresponds to the exchange of CP_2 type extremal for which vacuum functional implies huge reduction of the gravitational constant from the value $\sim L_p^2$ implied by dimensional considerations based on p-adic length scale hypothesis to a value $G = exp(-2S_K)L_p^2$ which for $p = M_{127}$ gives gravitational constant for $\alpha_K = \pi a/log(M_{127} \times K)$, where a is near unity and $K = 2 \times 3 \times 5.... \times 23$ is a choice motivated by number theoretical arguments. The value of K is fixed rather precisely from electron mass scale and the proposed scenario for coupling constant evolution fixes both α_K and K completely in terms of electron mass (using p-adic mass calculations) and electro-weak $U(1)$ coupling at electron length scale $L_{M_{127}}$ by the formula $\alpha_K = \alpha_{U(1)}$ [A9]. The interpretation would be that gravitational masses are measured using p-adic mass scale $M_p = \pi/L_p$ as a natural unit.

5.8.1 Why gravitational interaction is weak

The first problem is that CP_2 type extremal cannot represent the lowest order contribution to the interaction since otherwise the normalization of the configuration space vacuum functional would give $exp[-2S_K(CP_2)]$ factor cancelling the exponential in the propagator so that one would have $G = L_p^2$. The following observations allow to understand the solution of the problem.

1. As already found, the key feature of CP_2 type extremals distinguishing them from other 3-surfaces is their non-deterministic behavior allowing them to carry off mass shell four-momenta. Other 3-surfaces can give rise only to scattering involving exchange of on mass shell particles and for space-like momentum exchanges there is no contribution.

2. All possible light-like 3-surfaces must be allowed as propagator portions of surfaces X^3_V but in absence of non-determinism they can give rise to massless exchanges which are typically non-allowed.

3. The contributions of CP_2 type extremals are suppressed by $exp[-2NS_K(CP_2)]$ factor in presence of N CP_2 type extremals with maximal action. CP_2 type extremals are vacuum extremals and interact with surrounding world only via the topological condensation generating 3-D CP_2 projection near the throat of the wormhole contact. This motivates the assumption that the sector of the configuration space containing N CP_2 type extremals has the approximate structure $CH(N) = CH(0) \times CP^N$, where $CH(0)$ corresponds to the situation without CP_2 type extremals and CP to the degrees of freedom associated with single CP_2 type extremal. With this assumption the functional integral gives a result of form $X \times exp(-2NS_K(CP_2)$ for N CP_2 type extremals. This factorization allows to forget all the complexities of the world of classical worlds which on the first sight seem to destroy all hopes about calculating something and the normalization factor is in lowest order equal to $X(0)$ whereas single CP_2 type extremal gives $exp[-2S_K(CP_2)]$ factor. This argument generalizes also to the case when CP_2 type extremals are allowed to have varying value of action (the distance travelled by the virtual particle can vary).

Massless extremals (MEs) define a natural candidate for the lowest order contribution since for them Kähler action vanishes. MEs describes a dispersion free on-mass shell propagation of massless modes of both induced gauge fields and metric. Hence they can describe only on mass shell massless exchanges of bosons and gravitons which typically vanishes for kinematical reasons except for collinear scattering in the case of massless particles so that CP_2 type extremals would give the leading contribution to the S-matrix element.

There are however exceptional situations in which exchange of ordinary CP_2 type extremals makes kinematically possible the emission of MEs as brehmstrahlung in turn giving rise to exchange of light-like momentum. Since MEs carry also classical gravitational fields, one can wonder whether this kind of exchanges could make possible strong on mass shell gravitation made kinematically possible by ordinary gauge boson exchanges inside interacting systems.

5.8.2 What differentiates between gravitons and gauge bosons?

The simplest explanation for the difference between gauge bosons and gravitons is that for virtual gauge bosons the volume of CP_2 type extremals is reduced dramatically from its maximal value so that $exp(-2S_K)$ brings in only a small reduction factor. The reason would be that for virtual gauge bosons the length of a typical CP_2 type extremal is far from the value giving rise to the saturation of the Kähler action. For gravitational interactions in astrophysical length scales CP_2 type extremals must indeed be very long.

Gravitational interaction should become strong sufficiently below the saturation length scale with gravitational constant approaching its stringy value L_p^2. According to the argument discussed in [A9], this length scale corresponds to the Mersenne prime M_{127} characterizing gravitonic space-time sheets so that gravitation should become strong below electron's Compton length. This suggests a connection with stringy description of graviton. M_{127} quarks connected by the corresponding strings are indeed a basic element of TGD based model of nuclei [F8]. TGD suggests also the existence of lepto-hadrons as bound state of color excited leptons in length scale M_{127} [F7]. Also gravitons corresponding to smaller Mersenne primes are possible but corresponding forces are much weaker than ordinary gravitation. On the other hand, M_{127} is the largest Mersenne prime which does not give rise to super-astronomical p-adic length scale so that stronger gravitational forces are not be predicted in experimentally accessible length scales.

More generally, the saturation length scale should relate very closely to the p-adic length scale L_p characterizing the particle. The amount of zitterbewegung determines the amount dS_K/dl of Kähler action per unit length along the orbit of virtual particle. L_p would naturally define the length scale below which the particle moves in a good approximation along M^4 geodesic. The shorter this length scale is, the larger the value of dS_K/dl is.

If the Kähler action of CP_2 type extremal increases linearly with the distance (in a statistical sense at least), an exponential Yukawa screening results at distances much shorter than saturation length. Therefore CP_2 extremals would provide a fundamental description of particle massivation at space-time level. p-Adic thermodynamics would characterize what happens for a topologically condensed CP_2 type extremal carrying given quantum numbers at the resulting light-like CD. Besides p-adic length scale also the quantized value $T_p = 1/n$ of the p-adic temperature would be decisive. For weak bosons Mersenne prime

M_{89} would define the saturation length scale. For photons the p-adic length scale defining the Yukawa screening should be rather long. An n-ary p-adic length scale $L_{M_{89}}(n) = p^{(n-1)/2} L_{M_{89}}$ would most naturally be in question so that the p-adic temperature associated with photon would be $T_p = 1/n$, $n > 1$ [F3]. In the case of gluons confinement length scale should be much shorter than the scale at which the Yukawa screening becomes visible. If also gluons correspond to $n > 1$ this is certainly the case.

All gauge interactions would give rise to ultra-weak long ranged interactions, which are extremely weak compared to the gravitational interaction: the ratio for the strengths of these interactions would be of order $\alpha Q_1 Q_2 m_e^2 / M_1 M_2$ and very small for particles whose masses are above electron mass. Note however that MEs give rise to arbitrarily unscreened long ranged weak and color interactions restricted to light-like momentum transfers and these interactions play a key role in the TGD based model of living matter [J6, M3]. This prediction is in principle testable.

6 Von Neumann algebras and TGD

The work with TGD inspired model [E9] for topological quantum computation [50] led to the realization that von Neumann algebras [60, 61, 62, 63], in particular so called hyper-finite factors of type II_1 [65], seem to provide the mathematics needed to develop a more explicit view about the construction of S-matrix. I have already discussed a vision for how to achieve this [C7]. In this chapter I will discuss various aspects of type II_1 factors and their physical interpretation in TGD framework. The lecture notes of R. Longo [64] give a concise and readable summary about the basic definitions and results related to von Neumann algebras and I have used this material freely in this chapter.

6.1 Philosophical ideas behind von Neumann algebras

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1 [65].

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless.

6.2 Von Neumann, Dirac, and Feynman

The association of algebras of type I with the standard quantum mechanics allowed to unify matrix mechanism with wave mechanics. Note however that the assumption about continuous momentum state basis is in conflict with separability but the particle-in-box idealization allows to circumvent this problem (the notion of space-time sheet brings the box in physics as something completely real).

Because of the finiteness of traces von Neumann regarded the factors of type II_1 as fundamental and factors of type III as pathological. The highly pragmatic and successful approach of Dirac [66] based on the notion of delta function, plus the emergence of s [69], the possibility to formulate the notion of delta function rigorously in terms of distributions [67, 68], and the emergence of path integral approach [70] meant that von Neumann approach was forgotten by particle physicists.

Algebras of type II_1 have emerged only much later in conformal and topological quantum field theories [51, 52] allowing to deduce invariants of knots, links and 3-manifolds. Also algebraic structures known as bi-algebras, Hopf algebras, and ribbon algebras [39, 40] relate closely to type II_1 factors. In topological quantum computation [50] based on braid groups [56] modular S-matrices they play an especially important role.

In algebraic quantum field theory [53] defined in Minkowski space the algebras of observables associated with bounded space-time regions correspond quite generally to the type III_1 hyper-finite factor [54, 55].

6.3 Factors of type II_1 and quantum TGD

For me personally the realization that TGD Universe is tailored for topological quantum computation [E9] led also to the realization that hyper-finite (ideal for numerical approximations) von Neumann algebras of type II_1 have a direct relevance for TGD.

The basic facts about hyper-finite von Neumann factors of type II_1 suggest a more concrete view about the general mathematical framework needed.

1. The effective 2-dimensionality of the construction of quantum states and configuration space geometry in quantum TGD framework makes hyper-finite factors of type II_1 very natural as operator algebras of the state space. Indeed, the generators of conformal algebras, the gamma matrices of the configuration space, and the modes of the induced spinor fields are labelled by discrete labels. Hence the tangent space of the configuration space is a separable Hilbert space and its Clifford algebra is a hyper-finite type II_1 factor. Super-symmetry requires that the bosonic algebra generated by configuration space Hamiltonians and the Clifford algebra of configuration space both correspond to hyper-finite type II_1 factors.

2. Four-momenta relate to the positions of tips of future and past directed light cones appearing naturally in the construction of S-matrix. In fact, configuration space of 3-surfaces can be regarded as union of big-bang/big crunch type configuration spaces obtained as a union of light-cones parameterized by the positions of their tips. The algebras of observables associated with bounded regions of M^4 are hyper-finite and of type III_1 in algebraic quantum field theory [54]. The algebras of observables in the space spanned by the tips of these light-cones are not needed in the construction of S-matrix so that there are good hopes of avoiding infinities coming from infinite traces.

3. Many-sheeted space-time concept forces to refine the notion of sub-system. Jones inclusions $\mathcal{N} \subset \mathcal{M}$ for factors of type II_1 define in a generic manner to imbed interacting sub-systems to a universal II_1 factor which now naturally corresponds to the infinite Clifford algebra of the tangent space of configuration space of 3-surfaces and contains interaction as $\mathcal{M} : \mathcal{N}$-dimensional analog of tensor factor. Topological condensation of space-time sheet to a larger space-time sheet, the formation of bound states by the generation of join along boundaries bonds, interaction vertices in which space-time surface branches like a line of Feynman diagram: all these situations might be described by Jones inclusion [58, 59] characterized by the Jones index $\mathcal{M} : \mathcal{N}$ assigning to the inclusion also a minimal conformal field theory and quantum group in case of $\mathcal{M} : \mathcal{N} < 4$ and conformal theory with $k = 1$ Kac Moody for $\mathcal{M} : \mathcal{N} = 4$ [57].

4. von Neumann's somewhat artificial idea about identical a priori probabilities for states could replaced with the finiteness requirement of quantum

theory. Indeed, it is traces which produce the infinities of quantum field theories. That $\mathcal{M} : \mathcal{N} = 4$ option is not realized physically as quantum field theory (it would rather correspond to string model type theory characterized by a Kac-Moody algebra instead of quantum group), could correspond to the fact that dimensional regularization works only in $D = 4-\epsilon$. Dimensional regularization with space-time dimension $D = 4-\epsilon \rightarrow 4$ could be interpreted as the limit $\mathcal{M} : \mathcal{N} \rightarrow 4$. $\mathcal{M}$ as an $\mathcal{M} : \mathcal{N}$-dimensional $\mathcal{N}$-module would provide a concrete model for a quantum space with non-integral dimension as well as its Clifford algebra. An entire sequence of regularized theories corresponding to the allowed values of $\mathcal{M} : \mathcal{N}$ would be predicted.

6.4 Does quantum TGD emerge from local version of HFF?

There are reasons to hope that the entire quantum TGD emerges from a version of HFF made local with respect to $D \leq 8$ dimensional space H whose Clifford algebra $Cl(H)$ raised to an infinite tensor power defines the infinite-dimensional Clifford algebra. Bott periodicity meaning that Clifford algebras satisfy the periodicity $Cl(n+k8) \equiv Cl(n) \otimes Cl(8k)$is an essential notion here [C8, A9]. The points m of M^k can be mapped to elements $m^k\gamma_k$ of the finite-dimensional Clifford algebra $Cl(H)$ appearing as an additional tensor factor in the localized version of the algebra.

The requirement that the local version of HFF is not isomorphic with HFF itself is highly non-trivial. The only manner to achieve non-triviality is to multiply the algebra with a non-associative tensor factor representing the space of hyper-octonions M^8 identifiable as sub-space of complexified octonions with tangent space spanned by real unit and octonionic imaginary unit multiplied by commuting imaginary unit (for a good review about properties of octonions see [74]) .

Space-times could be regarded equivalently as surfaces in M^8 or in $M^4 \times CP_2$ and the dynamics would reduce to associativity (hyper-quaternionicity) or co-associativity condition. It is rather remarkable that CP_2 forced by the standard model symmetries has also a purely number theoretic interpretation as parameterizing hyper-quaternionic four-planes containing a preferred hyper-octonionic imaginary unit defining hyper-complex structure in M^8. Physically this choice corresponds to a choice of Cartan algebra of Poincare algebra for which the system is at rest so that a connection with quantum measurement theory is suggestive. Color group is identifiable as a subgroup of octonionic automorphism group G_2 respecting this choice.

6.5 Quantum measurement theory with finite measurement resolution

Jones inclusions $\mathcal{N} \subset \mathcal{M}$ [58, 71] of these algebras lead to quantum measurement theory with a finite measurement resolution characterized by $\mathcal{N}$ [C8, A9]. Quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ interpreted as $\mathcal{N}$-module creates physical states modulo measurement resolution. Complex rays of the state space resulting in the ordinary state function reduction are replaced by $\mathcal{N}$-rays and the notions of unitarity, hermiticity, and eigenvalue generalize [A9, C2].

Non-commutative physics would be interpreted in terms of a finite measurement resolution rather than something emerging below Planck length scale. An important implication is that a finite measurement sequence can never completely reduce quantum entanglement so that entire universe would necessarily be an organic whole.

At the level of conscious experience, the entanglement below measurement resolution would give rise to a pool of shared and fused mental images giving rise to "stereo consciousness" (say stereovision) [H1] so that contents of consciousness would not be something completely private as usually believed. Also fuzzy logic emerges naturally since ordinary spinors are replaced by quantum spinors for which the discrete spectrum of the eigenvalues of the moduli of its spinor components can be interpreted as probabilities that corresponding belief is true is universal [C8].

6.6 Cognitive consciousness, quantum computations, and Jones inclusions

Large $\hbar$ phases provide good hopes of realizing topological quantum computation. There is an additional new element. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with 'true' and 'false'. The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and decoherence is not a problem as long as it does not induce this transition.

6.7 Fuzzy quantum logic and possible anomalies in the experimental data for the EPR-Bohm experiment

The experimental data for EPR-Bohm experiment [90] excluding hidden variable interpretations of quantum theory. What is less known that the experimental data indicates about possibility of an anomaly challenging quantum mechanics [92]. The obvious question is whether this anomaly might provide a test for the notion of fuzzy quantum logic inspired by the TGD based quantum measurement theory with finite measurement resolution.

The experimental situation involves emission of two photons from spin zero system so that photons have opposite spins. What is measured are polarizations of the two photons with respect to polarization axes which differ from standard choice of this axis by rotations around the axis of photon momentum characterized by angles α and β. The probabilities for observing polarizations (i, j), where i,j is taken Z_2 valued variable for a convenience of notation are $P_{ij}(\alpha,\beta)$, are predicted to be $P_{00} = P_{11} = cos^2(\alpha-\beta)/2$ and $P_{01} = P_{10} = sin^2(\alpha-\beta)/2$.

Consider now the discrepancies.

1. One has four identities $P_{i,i} + P_{i,i+1} = P_{ii} + P_{i+1,i} = 1/2$ having interpretation in terms of probability conservation. Experimental data of [90] are not consistent with this prediction [93] and this is identified as the anomaly.

2. The QM prediction $E(\alpha,\beta) = \sum_i (P_{i,i} - P_{i,i+1}) = cos(2(\alpha-\beta)$ is not satisfied neither: the maxima for the magnitude of E are scaled down by a factor $\simeq .9$. This deviation is not discussed in [93].

Both these findings raise the possibility that QM might not be consistent with the data. It turns out that fuzzy quantum logic predicted by TGD and implying that the predictions for the probabilities and correlation must be replaced by ensemble averages, can explain anomaly 2) but not anomaly a). A "mundane" explanation for anomaly 1) can be imagined [C8].

7 Can TGD predict the spectrum of Planck constants?

The quantization of Planck constant has been the basic them of TGD during last years. The breakthrough became with the realization that standard type Jones inclusions lead to a detailed understanding of what is involved and predict very simple spectrum for Planck constants associated with M^4 and CP_2 degrees of freedom.

This requires a generalization of the notion of imbedding space by gluing together copies of H defining singular discrete bundle structures along common points belonging to 4-D sub-manifold $M^2 \times S^2$ of H defined by the choice of quantization axes. This picture allows to understand gravitational Planck constant and coupling constant evolution and leads also to the understanding of ADE correspondences (index $\beta \leq 4$ and $\beta = 4$) from the point of view of Jones inclusions.

Later a further generalization of the notion of imbedding space emerged allowing also products of singular G_a bundles over $M^4\backslash M^2$ with G_b bundles over $CP_2\backslash S^2$ emerged allowing to understand charge fractionization. The original and generalized view about H will be discussed separately in the sequel.

7.1 The first generalization of the notion of imbedding space concept

Quantum classical correspondence suggests that Jones inclusions [58] have space-time correlates [C8, A9]. There is a canonical hierarchy of Jones inclusions labelled by finite subgroups of SU(2)[71]. This leads to a generalization of the imbedding space obtained by gluing an infinite number of copies of H regarded as singular bundles over $H/G_a \times G_b$, where $G_a \times G_b$ is a subgroup of $SU(2) \times SU(2) \subset SL(2,C) \times SU(3)$. Gluing occurs along a factor for which the group is same.

The groups in question define in a natural manner the direction of quantization axes for for various isometry charges and this hierarchy seems to be an essential element of quantum measurement theory. Ordinary Planck constant, as opposed to Planck constants $\hbar_a = n_a\hbar_0$ and $\hbar_b = n_b\hbar_0$ appearing in the commutation relations of symmetry algebras assignable to M^4 and CP_2, is naturally quantized as $\hbar = (n_a/n_b)\hbar_0$, where n_i is the order of maximal cyclic subgroup of G_i. The hierarchy of Planck constants is interpreted in terms of dark matter hierarchy [A9]. What is also important is that $(n_a/n_b)^2$ appear as a scaling factor of M^4 metric so that Kähler action via its dependence on induced metric codes for radiative corrections coming in powers of ordinary Planck constant: therefore quantum criticality and vanishing of radiative corrections to functional integral over WCW does not mean vanishing of radiative corrections.

G_a would correspond directly to the observed symmetries of visible matter induced by the underlying dark matter [A9]. For instance, in living matter molecules with 5- and 6-cycles could directly reflect the fact that free electron pairs associated with these cycles correspond to $n_a = 5$ and $n_a = 6$ dark matter possibly responsible for anomalous conductivity of DNA [A9, J1] and recently reported strange properties of graphene [94]. Also the tedrahedral and icosahedral symmetries of water molecule clusters could have similar interpretation [100, F9].

A further fascinating possibility is that the observed indications for Bohr orbit quantization of planetary orbits [102] could have interpretation in terms of gigantic Planck constant for underlying dark matter [D7] so that macroscopic and -temporal quantum coherence would be possible in astrophysical length scales manifesting itself in many manners: say as preferred directions of quantization axis (perhaps related to the CMB anomaly) or as anomalously low dissipation rates.

7.1.1 Jones inclusions and quantization of Planck constant

Jones inclusions combined with simple anyonic arguments turned out to be the key to the unification of existing heuristic ideas about the quantization of Planck constant.

1. The new view allows to understand how and why Planck constant is quantized and gives an amazingly simple formula for the separate Planck constants assignable to M^4 and CP_2 and appearing as scaling constants of their metrics. This in terms of a mild generalizations of standard Jones inclusions. The emergence of imbedding space means only that the scaling of these metrics have spectrum: their is no landscape.

2. In ordinary phase Planck constants of M^4 and CP_2 are same and have their standard values. Large Planck constant phases correspond to situ-

ations in which a transition to a phase in which quantum groups occurs. These situations correspond to standard Jones inclusions in which Clifford algebra is replaced with a sub-algebra of its G-invariant elements. G is product $G_a \times G_b$ of subgroups of $SL(2,C)$ and $SU(2)_L \times \times U(1)$ which also acts as a subgroup of SU(3). Space-time sheets are $n(G_b)$-fold coverings of M^4 and $n(G_a)$-fold coverings of CP_2 generalizing the picture which has emerged already. An elementary study of these coverings fixes the values of the scaling factors of M^4 and CP_2 Planck constants to orders of the maximal cyclic sub-groups: $\hbar(M^4) = n_a$ and $\hbar(CP_2) = n_b$ whereas scaling factors of M^4 and CP_2 metrics are n_b^2 and n_a^2 respectively.

At the level of Schrödinger equation this means that Planck constant $\hbar$ correspodns to the effective Planck consant $\hbar_{eff} = (\hbar(M^4)/\hbar(CP_2))\hbar_0 = (n_a/n_b)\hbar_0$, which thus can have all possible positive rational values. For some time I believed on the scaling of metrics of M^4 *resp.* CP_2 as n_b^2 *resp.* n_a^2: this would imply invariance of Schrödinger equation under the scalings but would not be consistent with the explanation of the quantization of radii of planetary orbits requiring huge Planck constant [D7]. Poincare invariance is however achieved in the sense that mass spectrum is invariant under the scalings of Planck constants. That the ratio n_a/n_b defines effective Planck constant conforms with the fact that the value of Kähler action involves only this ratio (quantum-classical correspondence). Also the value of gravitational constant is invariant under the scalings of Planck constant since one has $G \propto g_K^2 R^2$, R radius of CP_2 for $n_a = 1$.

3. This predicts automatically arbitrarily large values of effective Planck constant n_a/n_b and they correspond to coverings of CP_2 points by large number of M^4 points which can have large distance and have precisely correlated behavior due to the G_a symmetry. One can assign preferred values of Planck constant to quantum phases $q = exp(i2\pi/n)$ expressible in terms of iterated square roots of rationals: these correspond to polygons obtainable by compass and ruler construction. In particular, experimentally favored values of $\hbar$ in living matter seem to correspond to these special values of Planck constants. This model reproduces also the other aspects of the general vision. The subgroups of $SL(2,C)$ in turn can give rise to re-scaling of $SU(3)$ Planck constant. The most general situation can be described in terms of Jones inclusions for fixed point subalgebras of number theoretic Clifford algebras defined by $G_a \times G_b \subset SL(2,C) \times SU(2)$.

4. These inclusions (apart from those for which G_a contains infinite number of elements) are represented by ADE or extended ADE diagrams depending on the value of index. The group algebras of these groups give rise to additional degrees of freedom which make possible to construct the multiplets of the corresponding gauge groups. For $\beta \leq 4$ the gauge groups A_n, D_{2n}, E_6, E_8 are possible so that TGD seems to be able to mimick these gauge theories. For $\beta = 4$ all ADE Kac Moody groups are possible and again mimicry becomes possible: TGD would be kind of universal physics

emulator but it would be anyonic dark matter which would perform this emulation.

7.1.2 The values of gravitational Planck constant

The understanding of large Planck constants led to the detailed interpretation of what is involved with the emergence of gigantic gravitational Planck constant. The detailed spectrum for Planck constants gives very strong constraints to the values of $\hbar_{gr} = GMm/v_0$ if ones assumes that favored values of Planck constant correspond to the Jones inclusions for which quantum phase corresponds to a simple algebraic number expressible in terms of iterated square roots of rationals. These phases correspond to n-polygons with n equal to a product of power of two and Fermat primes, which are all different. The ratios of planetary masses obey the predictions with an accuracy of 3 percent and GMm/v_0 for Sun-Earth system is consistent with $v_0 = 2^{-11}$ if the fraction of visible matter of all matter is about 6 per cent in solar system to be compared with the accepted cosmological value of 4 per cent.

Gravitational Planck constant $\hbar_{gr}$ can be interpreted as effective Planck constant $\hbar_{eff} = (n_a/n_b)\hbar_0$ so that the Planck constant associated with M^4 degrees of freedom (rather than CP_2 degrees of freedom as in the original wrong picture) must be very large in this kind of situation.

If so, its huge value implies that also the von Neumann inclusions associated with M^4 degrees of freedom are involved meaning that dark matter cosmology has quantal lattice like structure with lattice cell given by H_a/G, H_a the $a = constant$ hyperboloid of M^4_+ and G subgroup of SL(2,C). The quantization of cosmic redshifts provides support for this prediction.

There is however strong objection based on the observation that the radius of CP_2 would become gigantic. Surprisingly, this need not have any dramatic implications as will be found. It is also quite possible that the biomolecules subgroups of rotation group as symmetries could correspond to $n_a > 1$. For instance, the tedrahedral and icosahedral molecular structures appearing in water would correspond to E_6 with $n_a = 3$ and E_8 with $n_a = 5$. Note that $n_a = 5$ is minimal value of n_a allowing universal topological quantum computation.

7.1.3 Large values of Planck constant and coupling constant evolution

Kähler coupling constant is the only coupling parameter in TGD. The original great vision is that Kähler coupling constant is analogous to critical temperature and thus uniquely determined. Later I concluded that Kähler coupling strength could depend on the p-adic length scale. The reason was that the prediction for the gravitational coupling strength was otherwise non-sensible. This motivated the assumption that gravitational coupling is RG invariant in the p-adic sense.

The expression of the basic parameter $v_0 = 2^{-11}$ appearing in the formula of $\hbar_{gr} = GMm/v_0$ in terms of basic parameters of TGD leads to the unexpected conclusion that α_K in electron length scale can be identified as electro-weak

$U(1)$ coupling strength $\alpha_{U(1)}$. This identification is what group theory suggests but I had given it up since the resulting evolution for gravitational coupling was $G \propto L_p^2$ and thus completely un-physical. However, if gravitational interactions are mediated by space-time sheets characterized by Mersenne prime, the situation changes completely since M_{127} is the largest non-super-astrophysical p-adic length scale.

The second key observation is that all classical gauge fields and gravitational field are expressible using only CP_2 coordinates and classical color action and U(1) action both reduce to Kähler action. Furthermore, electroweak group U(2) can be regarded as a subgroup of color SU(3) in a well-defined sense and color holonomy is abelian. Hence one expects a simple formula relating various coupling constants. Let us take α_K as a p-adic renormalization group invariant in strong sense that it does not depend on the p-adic length scale at all.

The relationship for the couplings must involve $\alpha_{U(1)}$, α_s and α_K. The formula $1/\alpha_{U(1)} + 1/\alpha_s = 1/\alpha_K$ states that the sum of $U(1)$ and color actions equals to Kähler action and is consistent with the decrease of the color coupling and the increase of the U(1) coupling with energy and implies a common asymptotic value $2\alpha_K$ for both. The hypothesis is consistent with the known facts about color and electro-weak evolution and predicts correctly the confinement length scale as p-adic length scale assignable to gluons. The hypothesis reduces the evolution of α_s to the calculable evolution of electro-weak couplings: the importance of this result is difficult to over-estimate.

7.2 A further generalization of the notion of imbedding space

The hypothesis that Planck constant is quantized having in principle all possible rational values but with some preferred values implying algebraically simple quantum phases has been one of the main ideas of TGD during last years. The mathematical realization of this idea leads to a profound generalization of the notion of imbedding space obtained by gluing together infinite number of copies of imbedding space along common 4-dimensional intersection. The hope was that this generalization could explain charge fractionization but this does not seem to be the case. This problem led to a further generalization of the imbedding space and this is what I want to discussed below.

7.2.1 The original view about generalized imbedding space

The original generalization of imbedding space was basically following. Take imbedding space $H = M^4 \times CP_2$. Choose submanifold $M^2 \times S^2$, where S^2 is homologically non-trivial geodesic sub-manifold of CP_2. The motivation is that for a given choice of Cartan algebra of Poincare algebra (translations in time direction and spin quantization axis plus rotations in plane orthogonal to this plane plus color hypercharge and isospin) this sub-manifold remains invariant under the transformations leaving the quantization axes invariant.

Form spaces $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ and their Cartesian product. Both spaces have a hole of co-dimension 2 so that the first homotopy group is Z. From these spaces one can construct an infinite hierarchy of factor spaces $\hat{M}^4/G_a$ and $\hat{CP}_2/G_b$, where G_a is a discrete group of SU(2) leaving quantization axis invariant. In case of Minkowski factor this means that the group in question acts essentially as a combination reflection and to rotations around quantization axes of angular momentum. The generalized imbedding space is obtained by gluing all these spaces together along $M^2 \times S^2$.

The hypothesis is that Planck constant is given by the ratio $\hbar/hbar_0 = (n_/n_b)$, where n_i is the order of maximal cyclic subgroups of G_i. The hypothesis states also that the covariant metric of the Minkowski factor is scaled by the factor $(n_a/n_b)^2$. One must take care of this in the gluing procedure. One can assign to the field bodies describing both self interactions and interactions between physical systems definite sector of generalized imbedding space characterized partially by the Planck constant. The phase transitions changing Planck constant correspond to tunnelling between different sectors of the imbedding space.

7.2.2 Fractionization of quantum numbers is not possible if only factor spaces are allowed

The original idea was that the proposed modification of the imbedding space could explain naturally phenomena like quantum Hall effect involving fractionization of quantum numbers like spin and charge. This does not however seem to be the case. $G_a \times G_b$ implies just the opposite if these quantum numbers are assigned with the symmetries of the imbedding space. For instance, quantization unit for orbital angular momentum becomes n_a where Z_{n_a} is the maximal cyclic subgroup of G_a.

One can however imagine of obtaining fractionization at the level of imbedding space for space-time sheets, which are analogous to multi-sheeted Riemann surfaces (say Riemann surfaces associated with $z^{1/n}$ since the rotation by 2π understood as a homotopy of M^4 lifted to the space-time sheet is a non-closed curve. Continuity requirement indeed allows fractionization of the orbital quantum numbers and color in this kind of situation.

7.3 Both covering spaces and factor spaces are possible

The observation above stimulates the question whether it might be possible in some sense to replace H or its factors by their multiple coverings.

1. This is certainly not possible for M^4, CP_2, or H since their fundamental groups are trivial. On the other hand, the fixing of quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is a geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold

is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. Zero energy ontology forces to modify this picture somewhat. In zero energy ontology causal diamonds (CDs) defined as the intersections of future and past directed light-cones are loci for zero energy states containing positive and negative energy parts of state at the two light-cone boundaries. The location of CD in M^4 is arbitrary but p-adic length scale hypothesis suggests that the temporal distances between tips of CD come as powers of 2 using CP_2 size as unit. Thus M^4 is replaces by CD and $\hat{M}^4$ is replaced with $\hat{CD}$ defined in obvious manner.

3. H_4 represents a straight cosmic string inside CD. Quantum field theory phase corresponds to Jones inclusions with Jones index $\mathcal{M} : \mathcal{N} < 4$. Stringy phase would by previous arguments correspond to $\mathcal{M} : \mathcal{N} = 4$. Also these Jones inclusions are labeled by finite subgroups of $SO(3)$ and thus by Z_n identified as a maximal Abelian subgroup.

   One can argue that cosmic strings are not allowed in QFT phase. This would encourage the replacement $\hat{CD} \times \hat{CP}_2$ implying that surfaces in $CD \times S^2$ and $(M^2 \cap CD) \times CP_2$ are not allowed. In particular, cosmic strings and CP_2 type extremals with M^4 projection in M^2 and thus light-like geodesic without zitterwebegung essential for massivation are forbidden. This brings in mind instability of Higgs=0 phase.

4. The covering spaces in question would correspond to the Cartesian products $\hat{CD}_{n_a} \times \hat{CP}_{2n_b}$ of the covering spaces of $\hat{CD}$ and $\hat{CP}_2$ by Z_{n_a} and Z_{n_b} with fundamental group is $Z_{n_a} \times Z_{n_b}$. One can also consider extension by replacing $M^2 \cap CD$ and S^2 with its orbit under G_a (say tedrahedral, octahedral, or icosahedral group). The resulting space will be denoted by $\hat{CD} \hat{\times} G_a$ *resp.* $\hat{CP}_2 \hat{\times} G_b$.

5. One expects the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular submanifolds $M^2 \cap CD$ or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of $M^2 \cap CD$ the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tedrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

6. Also the orbifolds $\hat{CD}/G_a \times \hat{CP}_2/G_b$ can be allowed as also the spaces $\hat{CD}/G_a \times (\hat{CP}_2 \hat{\times} G_b)$ and $(\hat{CD} \hat{\times} G_a) \times \hat{CP}_2/G_b$. Hence the previous framework would generalize considerably by the allowance of both coset spaces and covering spaces.

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $(M^2 \cap CD) \times CP_2$ takes place? It would seem that the covariant metric of M^4 factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of M^4 metric can make sense. This is consistent with the identical vanishing of Chern-Simons action in $M^2 \times S^2$.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in CD degrees of freedom. This is not the case. Light-likeness in $(M^2 \cap CD) \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset (M^2 \cap CD) \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $(M^2 \cap CD) \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of $(M^2 \cap CD)$ as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunneling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) and classical light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

7.4 Do factor spaces and coverings correspond to the two kinds of Jones inclusions?

What could be the interpretation of these two kinds of spaces?

1. Jones inclusions appear in two varieties corresponding to $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$ and one can assign a hierarchy of subgroups of $SU(2)$ with both of them. In particular, their maximal Abelian subgroups Z_n label

these inclusions. The interpretation of Z_n as invariance group is natural for $\mathcal{M} : \mathcal{N} < 4$ and it naturally corresponds to the coset spaces. For $\mathcal{M} : \mathcal{N} = 4$ the interpretation of Z_n has remained open. Obviously the interpretation of Z_n as the homology group defining covering would be natural.

2. $\mathcal{M} : \mathcal{N} = 4$ should correspond to the allowance of cosmic strings and other analogous objects. Does the introduction of the covering spaces bring in cosmic strings in some controlled manner? Formally the subgroup of $SU(2)$ defining the inclusion is $SU(2)$ would mean that states are $SU(2)$ singlets which is something non-physical. For covering spaces one would however obtain the degrees of freedom associated with the discrete fiber and the degrees of freedom in question would not disappear completely and would be characterized by the discrete subgroup of $SU(2)$.

   For anyons the non-trivial homotopy of plane brings in non-trivial connection with a flat curvature and the non-trivial dynamics of topological QFTs. Also now one might expect similar non-trivial contribution to appear in the spinor connection of $\hat{CD}\hat{\times}G_a$ and $\hat{CP}_2\hat{\times}G_b$. In conformal field theory models non-trivial monodromy would correspond to the presence of punctures in plane.

3. For factor spaces the unit for quantum numbers like orbital angular momentum is multiplied by n_a *resp.* n_b and for coverings it is divided by this number. These two kind of spaces are in a well defined sense obtained by multiplying and dividing the factors of $\hat{H}$ by G_a *resp.* G_b and multiplication and division are expected to relate to Jones inclusions with $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$, which both are labeled by a subset of discrete subgroups of SU(2).

4. The discrete subgroups of $SU(2)$ with fixed quantization axes possess a well defined multiplication with product defined as the group generated by forming all possible products of group elements as elements of $SU(2)$. This product is commutative and all elements are idempotent and thus analogous to projectors. Trivial group G_1, two-element group G_2 consisting of reflection and identity, the cyclic groups Z_p, p prime, and tedrahedral, octahedral, and icosahedral groups are the generators of this algebra.

By commutativity one can regard this algebra as an 11-dimensional module having natural numbers as coefficients ("rig"). The trivial group G_1, two-element group G_2¡ generated by reflection, and tedrahedral, octahedral, and icosahedral groups define 5 generating elements for this algebra. The products of groups other than trivial group define 10 units for this algebra so that there are 11 units altogether. The groups Z_p generate a structure analogous to natural numbers acting as analog of coefficients of this structure. Clearly, one has effectively 11-dimensional commutative algebra in 1-1 correspondence with the 11-dimensional "half-lattice" N^{11} (N denotes natural numbers). Leaving away reflections, one obtains N^7. The projector representation suggests a connection with Jones inclusions. An interesting question concerns the possible Jones inclusions assignable to the subgroups containing infinitely manner elements. Reader has of course already asked whether dimensions 11, 7 and their difference 4 might relate somehow to the mathematical structures of M-theory with 7 compactified dimensions. One could introduce generalized configuration space spinor fields in the configuration space labelled by sectors of H with given quantization axes. By introducing Fourier transform in N^{11} one would formally obtain an infinite-component field in 11-D space.

The question how do the Planck constants associated with factors and coverings relate is far from trivial and I have considered several options.

1. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of metric allowed by Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains $r^2 \equiv \hbar^2/\hbar_0^2\hbar^2(M^4)/\hbar^2(CP_2)$. This puts M^4 and CP_2 in a very symmetric role and allows much more flexibility in the identification of symmetries associated with large Planck constant phases.

2. Algebraist would argue that Planck constant must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa. This gives two options.

3. Option I: $r(X) = n$ for covering and $r(X) = 1/n$ for factor space gives $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$. This gives $r = n_a/n_b$ for $\hat{H}/G_a \times G_b$ option and $r = n_b/n_a$ for $\hat{H}\hat{times}(G_a \times G_b)$ option with obvious formulas for hybrid cases.

4. Option II: $r(X) = 1/n$ for covering and $r(X) = n$ for factor space gives $r = r(CP_2)/r(M^4)$. This gives $r = n_b/n_a$ for $\hat{H}/G_a \times G_b$ option and $r = n_a/n_b$ for $\hat{H}\hat{times}(G_a \times G_b)$ option with obvious formulas for the hybrid cases.

5. At quantum level the fractionization would come from the modification of fermionic anti-commutation (bosonic commutation) relations involving $\hbar$ at the right hand side so that particle number becomes a multiple of $1/n$ or n. If one postulates that the total number states is invariant in the transition, the increase in the number of sheets is compensated by the increase of the fundamental phase space volume proportional to $\hbar$. This would give $r(X) \rightarrow r(X)/n$ for factor space and $r(X) \rightarrow nr(X)$ for the covering space to compensate the n-fold reduction/increase of states. This would favor Option II.

6. The second manner to distinguish between these two options is to apply the theory to concrete physical situations. Since G_a and G_b act as symmetries in CD and CP_2 degrees of freedom, one might of being able to distinguish between the two options if it is possible to distinguish between the action of G as symmetry of quantum states associated with covering

and factor space. Also the quantization of the orbital spin quantum number at single particle level as multiples of n can be distinguished from that in multiples of $1/n$.

7.5 A simple model of fractional quantum Hall effect

The generalization of the imbedding space suggests that it could possible to understand fractional quantum Hall effect [97] at the level of basic quantum TGD. This section represents the first rough model of QHE constructed for a couple of years ago is discussed. Needless to emphasize, the model represents only the basic idea and involves ad hoc assumption about charge fractionization.

Recall that the formula for the quantized Hall conductance is given by

$$\begin{aligned} \sigma &= \nu \times \frac{e^2}{h} \ , \\ \nu &= \frac{n}{m} \ . \end{aligned} \quad (26)$$

Series of fractions in $\nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15..., 2/3, 3/5, 4/7, 5/9, 6/11, 7/13...,$ $5/3, 8/5, 11/7, 14/9...4/3, 7/5, 10/7, 13/9..., 1/5, 2/9, 3/13..., 2/7, 3/11..., 1/7....$ with odd denominator have been observed as are also $\nu = 1/2$ and $\nu = 5/2$ states with even denominator [97].

The model of Laughlin [96] cannot explain all aspects of FQHE. The best existing model proposed originally by Jain is based on composite fermions resulting as bound states of electron and even number of magnetic flux quanta [98]. Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

The generalization of the notion of imbedding space suggests the possibility to interpret these states in terms of fractionized charge, spin, and electron number. There are four combinations of covering and factors spaces of CP_2 and three of them can lead to the increase of Planck constant. Besides this there are two options for the formula of Planck constant so that which the very meager theoretical background one can make only guesses. On the following just for fun consideration option I is considered although the conservation of number of states in the phase transition changing $\hbar$ favors option II.

1. The easiest manner to understand the observed fractions is by assuming that both M^4 and CP_2 correspond to covering spaces so that both spin and electric charge and fermion number are fractionized. This means that e in electronic charge density is replaced with fractional charge. Quantized magnetic flux is proportional to e and the question is whether also here fractional charge appears. Assume that this does not occur.

2. With this assumption the expression for the Planck constant becomes for Option II as $r = \hbar/\hbar_0 = n_a/n_b$ and charge and spin units are equal to $1/n_b$

and $1/n_a$ respectively. This gives $\nu = nn_a/n_b$. The values $m = 2, 3, 5, 7, ..$ are observed. Planck constant can have arbitrarily large values. There are general arguments stating that also spin is fractionized in FQHE.

3. The appearance of $\nu = 5/2$ has been observed [99]. The fractionized charge is $e/4$ in this case. Since $n_i > 3$ holds true if coverings are correlates for Jones inclusions, this requires to $n_b = 4$ and $n_a = 10$. n_b predicting a correct fractionization of charge. The alternative option would be $n_b = 2$ that also Z_2 would appear as the fundamental group of the covering space. Filling fraction $1/2$ corresponds in the composite fermion model and also experimentally to the limit of zero magnetic field [98]. $n_b = 2$ is however inconsistent with the observed fractionization of electric charge and with the vision inspired by Jones inclusions.

4. A possible problematic aspect of the TGD based model is the experimental absence of even values of n_b except $n_b = 2$ (Laughlin's model predicts only odd values of n). A possible explanation is that by some symmetry condition possibly related to fermionic statistics (as in Laughlin model) n_a/n_b must reduce to a rational with an odd denominator for $n_b > 2$. In other words, one has $n_a \propto 2^r$, where 2^r the largest power of 2 divisor of n_b.

5. Large values of n_a emerge as B increases. This can be understood from flux quantization. One has $e \int BdS = n\hbar(M^4) = nn_a\hbar_0$. By using actual fractional charge $e_F = e/n_b$ in the flux factor would give $e_F \int BdS = n(n_a/n_b)\hbar_0 = n\hbar$. The interpretation is that each of the n_a sheets contributes one unit to the flux for e. Note that the value of magnetic field in given sheet is not affected so that the build-up of multiple covering seems to keep magnetic field strength below critical value.

6. The understanding of the thermal stability is not trivial. The original FQHE was observed in 80 mK temperature corresponding roughly to a thermal energy of $T \sim 10^{-5}$ eV. For graphene the effect is observed at room temperature. Cyclotron energy for electron is (from $f_e = 6 \times 10^5$ Hz at $B = .2$ Gauss) of order thermal energy at room temperature in a magnetic field varying in the range 1-10 Tesla. This raises the question why the original FQHE requires so low temperature. The magnetic energy of a flux tube of length L is by flux quantization roughly $e^2B^2S \sim E_c(e)m_eL$ ($\hbar_0 = c = 1$) and exceeds cyclotron roughly by a factor L/L_e, L_e electron Compton length so that thermal stability of magnetic flux quanta is not the explanation. A possible explanation is that since FQHE involves several values of Planck constant, it is quantum critical phenomenon and is characterized by a critical temperature. The differences of the energies associated with the phase with ordinary Planck constant and phases with different Planck constant would characterize the transition temperature.

As already noticed, it is possible to imagine several other options and the identification of charge unit is rather ad hoc. Therefore this model can be taken only as a warm-up exercise.

7.6 Phase transitions changing the value of Planck constant

There are two basic kinds of phase transitions changing the value of Planck constant inducing a leakage between sectors of imbedding space. There are three cases to consider corresponding to

1. leakage in M^4 degrees of freedom changing G_a: the critical manifold is $R_+ \times CP_2$;

2. leakage in CP_2 degrees of freedom changing G_b: the critical manifold is $\delta M^4_+ \times S^2_{II}$;

3. leakage in both degrees of freedom changing both G_a and G_b: the critical manifold is $R_+ \times S^2_{II}$. This is the non-generic case

For transitions of type 2) and 3) X^2 must go through vacuum extremal in the classical picture about transition.

Covering space can also change to a factor space in both degrees of freedom or vice versa and in this case G can remain unchanged as a group although its interpretation changes.

The phase transitions satisfy also strong group theoretical constraints. For the transition $G_1 \rightarrow G_2$ either $G_1 \subset G_2$ or $G_2 \subset G_1$ must hold true. For maximal cyclic subgroups Z_n associated with quantization axes this means that n_1 must divide n_2 or vice versa. Hence nice number theoretic view about transitions emerges [A9].

One can classify the points of critical manifold according to the degree of criticality. Obviously the maximally critical points corresponds to fixed points of G_i that its points $z = 0, \infty$ of the spheres S^2_r and S^2_{II}. In the case of δM^4_+ the points $z = 0$ and ∞ correspond to the light-like rays R_+ in opposite directions. This ray would define the quantization direction of angular momentum. Quantum phase transitions changing the value of M^4 Planck constant could occur anywhere along this ray (partonic 2-surface would have 1-D projection along this ray). At the level of cosmology this would bring in a preferred direction. Light-cone dip, the counterpart of big bang, is the maximally quantum critical point since it remains invariant under entire group $SO(3,1)$.

Interesting questions relate to the groups generated by finite discrete subgroups of $SO(3)$. As noticed the groups generated as products of groups leaving R_+ invariant and three genuinely 3-D groups are infinite discrete subgroups of SO(3) and could also define Jones inclusions. In this case orbifold is replaced with orbifold containing infinite number of rotated versions of R_+. These phases could be important in elementary particle length scales or in early cosmology.

7.7 The identification of number theoretic braids

Number theoretic braids should be known once partonic surface and corresponding p-adic prime is known. Braid should belong to the intersection of real and p-adic variant of partonic 2-surface and the definition of should automatically give rise to a finite braid in case of non-vacuum extremals. Quantum criticality suggests that there are two kinds of braids. First kind of braid would relate to phase transitions changing G_a and would correspond to intersection of X^2 with R_+ and for given point in intersection would consist of points of CP_2 with same R_+ projection. Second kind of braid would relate to phase transitions changing G_b and correspond to intersection of X^2 with S^2_{II} and would consist for given point of points of S^2_r with same S^2_{II} coordinates.

7.7.1 Why a discrete set of points of partonic 2-surface must be selected?

As already noticed, p-adicization might provide a deeper motivation for the selection of discrete subset of points of partonic 2-surface in the construction of S-matrix elements in the case of non-diagonal transitions between different number fields.

1. The fusion of p-adic variants of TGD with real TGD, could be possible by algebraic continuation. This however requires the restriction of n-point functions to a finite set of algebraic points of X^2 with the usual stringy formula formula for S-matrix elements involving an integral over a circle of X^2 replaced with a sum over these points.

2. The same universal formula would give not only ordinary S-matrix elements but also those for p-adic-to-real transitions describing transformation intentions to actions. Quite generally, the formula would express S-matrix elements for transitions between two arbitrary number fields as algebraic numbers so that p-adicization of the theory would become trivial.

3. The interpretation of this finite set of points as a braid suggests a connection with the representation of Jones inclusions in terms of a hierarchy of braids [C8, E9] with the increasing number of strands meaning a continually improved finite-dimensional approximation of the hyper-finite factor of type II_1 identifiable as the Clifford algebra for the configuration space. The hierarchy of approximations for the hyper-finite factor would correspond to a genuine physical hierarchy of S-matrices corresponding to increasing dimension of algebraic extension of various p-adic numbers. This hierarchy would also define a cognitive hierarchy.

What could then be this discrete set of points having interpretation as a braid?

1. Number theoretical vision suggests that quantum TGD involves the sequence hyper-octonions → hyper-quaternions→ complex numbers→ reals → finite field $G(p, 1)$ or of its algebraic extension. These reductions would define number theoretical counterparts of dimensional reductions. The points in the finite field $G(p, 1)$ could be defined by p-adic integers modulo p so that a connection with p-adic numbers would emerge. Also more general algebraic extensions of p-adic numbers are allowed.

2. Number theoretical braids must belong to the intersection of real partonic 2-surface and its p-adic counterpart and thus the points must be algebraic points. Besides this a natural cutoff determined by X^2 itself is needed in order to have only finite number of points.

3. The generalization of the imbedding space inspired by the hierarchy of Planck constants suggest a very concrete identification of number theoretic braids in terms of intersections of partonic 2-surface and critical manifolds R_+ and S^2_{II} involving no ad hoc assumptions and giving braids having finite number of points.

7.7.2 Precise definition of number theoretical braid

The precise definition of number theoretic braids has been a challenge for long time. The generalization of the notion of imbedding space however leads to good guess for the identification of number theoretical braids.

What is clear that the points of number theoretic braid belong to the intersection of the real and p-adic variant of partonic 2-surfaces consisting of rationals and algebraic points in the extension used for p-adic numbers. The points of braid have same projection on an algebraic point of a geodesic sphere of $S^2 \subset CP_2$ belonging to the algebraic extension of rationals considered (the reader willing to understand the details can consult [C1]).

There are two different geodesic spheres in CP_2 and the homologically trivial geodesic sphere S^2_{II} is the most natural choice from the point of view of the generalized imbedding space since $M^2 \times S^2_{II}$, which defines the intersection of all sectors of H, is a vacuum extremal so that the ill-definedness of Planck constant does not matter. Note that also the M^4 part of the metric is discontinuous at $M^2 \times S^2_{II}$.

One can argue that algebraicity condition is not strong enough and gives too many points unless one introduces a cutoff in some manner. Since TQFT like theory can naturally assigned with the partonic 2-surfaces in $M^2 \times S^2_{II}$, the natural identification of the intersection points of number theoretical braids with $\delta M^4_\pm \times CP_2$ would be as the intersection of the 2-D CP_2 projection of the partonic 2-surface in $\delta M^4_\pm \times CP_2$ with S^2_{II}. In the generic case the intersection would consist of discrete points and for non-vacuum extremals this would certainly be the case. The intersection should consist of algebraic points allowing also p-adic interpretation: the condition that CP_2 projection is an algebraic surface is a necessary condition for this.

The points of braid are obtained as solutions of polynomial equation and thus one can assign to them a Galois group permuting the points of the braid. In this case finite Galois group could be realized as left or right translation or conjugation in S_∞ or in braid group.

To make the notion of number theoretic braid more concrete, suppose that the complex coordinate w of $\delta M^4_\pm$ is expressible as a polynomial of the complex coordinate z of CP_2 geodesic sphere and the radial light-like coordinate r of $\delta M^4_\pm$ is obtained as a solution of polynomial equation $P(r,z,w)=0$. By substituting w as a polynomial $w=Q(z,r)$ of z and r this gives polynomial equation $P(r,z,Q(z,r))=0$ for r for a given value of z. Only real roots can be accepted. Local Galois group (in a sense different as it is used normally in literature) associated with the algebraic point of S^2 defining the number theoretical braid is thus well defined.

If the partonic 2-surface involves all roots of an irreducible polynomial, one indeed obtains a braid for each point of the geodesic sphere $S^2 \subset CP_2$. In this case the action of Galois group is naturally a braid group action realized as the action on induced spinor fields and configuration space spinors.

The choice of the points of braid as points common to the real and p-adic partonic 2-surfaces would be unique so that the obstacle created by the fact that the finite Galois group as function of point of S^2 fluctuates wildly (when some roots become rational Galois group changes dramatically: the simplest example is provided by $y-x^2=0$ for which Galois group is Z_2 when y is not a square of rational and trivial group if y is rational).

This picture looks nice but a closer inspection show that it is not quite correct. The identification of Higgs field as a purely geometrical object leads to the identification of intersection points as unstable extrema of negative valued Higgs potential for which Higgs vanishes. The stable minima correspond to the extrema in the vicinity of these maxima and correspond to non-vanishing Higgs field (the nearest valley for a peak of 2-D landscape defined on sphere). The minima (bottoms of valleys) define both physically and mathematically natural candidate for the number theoretical braids. At quantum criticality these braids approach to zero braids. Thus one can say that the original identification is correct apart from Higgs mechanism.

7.7.3 What is the fundamental braiding operation?

The basic quantum dynamics of TGD could define the braiding operation for the braid defined by a discrete set of points of X^2 satisfying the algebraicity conditions. I have considered several candidates for braiding operation and the situation is still partially unsettled.

One promising candidate for the braiding operation is found by observing that both Kähler gauge potential and Kähler magnetic field define flows at light-like partonic 3-surface. The dual of the induced Kähler form defines a conserved topological current, whose flow lines are field lines of the Kähler magnetic field in the light like direction. This flow is incompressible. Vector potential defines also a flow in the interior of space-time surface, and Chern-

Simons action at partonic 3-surface defines a topological invariant of this flow known as helicity in hydrodynamics. The non-gauge invariance of helicity is not a problem since symplectic transformations of CP_2 do not define gauge degeneracy but spin glass degeneracy. The flow defined by the vector potential is perhaps the most attractive option but one cannot exclude the possibility that the braids defined by both flows play a role in the definition of S-matrix. Number theoretical braid (tangle if flow line fuse or split) would correspond to the unique orbit for the points of the number theoretic braid at the initial partonic 2-surface. The points of the braid would be algebraic only in suitably chosen discrete time slices but this would not lead to a loss of uniqueness. Hence cobordism would become discrete. This picture makes sense also for macroscopic 2-surfaces defining outer boundaries of physical systems (quantum Hall effect and topological quantum computation [E9]). This picture makes sense also for macroscopic 2-surfaces defining outer boundaries of physical systems (quantum Hall effect and topological quantum computation [E9]).

The second candidate for the braiding operation emerges naturally when one identifies the points defining the number theoretic braid in terms of minima of Higgs field defined on X^2 (the details of this identification are discussed later). In this case time evolution takes minima to minima and can induce braiding for the projections of the points of braid to S^2_{II} *resp.* S^2_r. One might hope that the braiding associated with S^2_r *resp.* S^2_{II} is topologically equivalent with the braiding defined by Kähler gauge potential *resp.* Kähler magnetic field.

8 Does the modified Dirac action define the fundamental action principle?

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. The Dirac determinant associated with the modified Dirac action is an excellent candidate in this respect.

8.1 Modified Dirac equation

In the following the problems of the ordinary Dirac action are discussed and the notion of modified Dirac action is introduced. In particular, the following problems are discussed.

1. Try to guess general formula for the spectrum of the modified Dirac operator and for super-canonical conformal weights by assuming that the eigenvalues are expressible in terms of the data assignable to the two kinds of of number theoretical braids and that the product of vacuum functional expressible as exponent of Kahler function and of the exponent of Chern-Simons action is identifiable as Dirac determinant expressible as

product of M^4 and CP_2 parts. Since Kähler function is isometry invariant only the Dirac determinant defined by M^4 braid can contribute to it. Chern-Simons action is not isometry invariant and can be identified as the Dirac determinant associated with CP_2 braid.

2. Try to understand whether the zeta functions involved can be identified as Riemann Zeta or some zeta coding geometric data about partonic 2-surface. Try to understand whether the assignment of a fixed prime p to a partonic 2-surface implies that the zeta function is actually an analog for basic building block of Riemann Zeta.

8.1.1 Problems associated with the ordinary Dirac action

Minimal 2-surface represents a situation in which the representation of surface reduces to a complex-analytic map. This implies that induced metric is hermitian so that it has no diagonal components in complex coordinates $(z, \overline{z})$ and the second fundamental form has only diagonal components of type H^k_{zz}. This implies that minimal surface is in question since the trace of the second fundamental form vanishes. At first it seems that the same must happen also in the more general case with the consequence that the space-time surface is a minimal surface. Although many basic extremals of Kähler action are minimal surfaces, it seems difficult to believe that minimal surface property plus extremization of Kähler action could really boil down to the absolute minimization of Kähler action or a more general principle selecting preferred extremals as Bohr orbits [E2].

This brings in mind a similar long-standing problem associated with the Dirac equation for the induced spinors. The problem is that right-handed neutrino generates super-symmetry only provided that space-time surface and its boundary are minimal surfaces. Although one could interpret this as a geometric symmetry breaking, there is a strong feeling that something goes wrong. Induced Dirac equation and super-symmetry fix the variational principle but this variational principle is not consistent with Kähler action.

One can also question the implicit assumption that Dirac equation for the induced spinors is consistent with the super-symmetry of the configuration space geometry. Super-symmetry would obviously require that for vacuum extremals of Kähler action also induced spinor fields represent vacua. This is however not the case. This super-symmetry is however assumed in the construction of the configuration space geometry so that there is internal inconsistency.

8.1.2 Super-symmetry forces modified Dirac equation

The above described three problems have a common solution. Nothing prevents from starting directly from the hypothesis of a super-symmetry generated by covariantly constant right-handed neutrino and finding a Dirac action which is consistent with this super-symmetry. Field equations can be written as

$$D_\alpha T^\alpha_k = 0 \ ,$$
$$T^\alpha_k = \frac{\partial}{\partial h^k_\alpha} L_K \ . \quad (27)$$

If super-symmetry is present one can assign to this current its super-symmetric counterpart

$$J^{\alpha k} = \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l \Psi \ ,$$
$$D_\alpha J^{\alpha k} = 0 \ . \quad (28)$$

having a vanishing covariant divergence. The isometry currents currents and super-currents are obtained by contracting T^{α_k} and J^{α_k} with the Killing vector fields of super-symmetries. Note also that the super current

$$J^\alpha = \overline{\nu_R} T^\alpha_l \Gamma^l \Psi \quad (29)$$

has a vanishing divergence.

By using the covariant constancy of the right-handed neutrino spinor, one finds that the divergence of the super current reduces to

$$D_\alpha J^{\alpha k} = \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l D_\alpha \Psi \ . \quad (30)$$

The requirement that this current vanishes is guaranteed if one assumes that modified Dirac equation

$$\hat{\Gamma}^\alpha D_\alpha \Psi = 0 \ ,$$
$$\hat{\Gamma}^\alpha = T^\alpha_l \Gamma^l \ . \quad (31)$$

This equation must be derivable from a modified Dirac action. It indeed is. The action is given by

$$L = \overline{\Psi} \hat{\Gamma}^\alpha D_\alpha \Psi \ . \quad (32)$$

Thus the variational principle exists. For this variational principle induced gamma matrices are replaced with effective induced gamma matrices and the requirement

$$D_\mu \hat{\Gamma}^\mu = 0 \quad (33)$$

guaranteing that super-symmetry is identically satisfied if the bosonic field equations are satisfied. For the ordinary Dirac action this condition would lead to the minimal surface property. What sounds strange that the essentially hydrodynamical equations defined by Kähler action have fermionic counterpart: this is very far from intuitive expectations raised by ordinary Dirac equation and something which one might not guess without taking super-symmetry very seriously.

8.1.3 How can one avoid minimal surface property?

These observations suggest how to avoid the emergence of the minimal surface property as a consequence of field equations. It is not induced metric which appears in field equations. Rather, the effective n metric appearing in the field equations is defined by the anti-commutators of $\hat{\gamma}_\mu$

$$\hat{g}_{\mu\nu} = \{\hat{\Gamma}_\mu, \hat{\Gamma}_\nu\} = 2T^k_\mu T_{\nu k} \ . \qquad (34)$$

Here the index raising and lowering is however performed by using the induced metric so that the problems resulting from the non-invertibility of the effective metric are avoided. It is this dynamically generated effective metric which must appear in the number theoretic formulation of the theory.

Field equations state that space-time surface is minimal surface with respect to the effective metric. Note that a priori the choice of the bosonic action principle is arbitrary. The requirement that effective metric defined by energy momentum tensor has only non-diagonal components except in the case of non-light-like coordinates, is satisfied for the known solutions of field equations.

8.1.4 Does the modified Dirac action define the fundamental action principle?

There is quite fundamental and elegant interpretation of the modified Dirac action as a fundamental action principle discussed also in [E2]. In this approach vacuum functional can be defined as the Grassmannian functional integral associated with the exponent of the modified Dirac action. This definition is invariant with respect to the scalings of the Dirac action so that theory contains no free parameters.

An alternative definition is as a Dirac determinant which might be calculated in TGD framework without applying the poorly defined functional integral. There are good reasons to expect that the Dirac determinant exponent of Kähler function for a preferred Bohr orbit like extremal of the Kähler action with the value of Kähler coupling strength coming out as a prediction. Hence the dynamics of the modified Dirac action at light-like partonic 3-surfaces X^3_l, even when restricted to almost-topological dynamics induced by Chern-Simons action, would dictate the dynamics at the interior of the space-time sheet.

The knowledge of the canonical currents and super-currents, together with the anti-commutation relations stating that the fermionic super-currents S_A

and S_B associated with Hamiltonians H_A and H_B anti-commute to a bosonic current $H_{[A,B]}$, allows in principle to deduce the anti-commutation relations satisfied by the induced spinor field. In fact, these conditions replace the usual anti-commutation relations used to quantize free spinor field. Since the normal ordering of the Dirac action would give Kähler action,

Kähler coupling strength would be determined completely by the anti-commutation relations of the super-canonical algebra. Kähler coupling strength would be dynamical and the selection of preferred extremals of Kähler action would be more or less equivalent with quantum criticality because criticality corresponds to conformal invariance and the hyper-quaternionic version of the super-conformal invariance results only for the extrema of Kähler action. p-Adic (or possibly more general) coupling constant evolution and quantum criticality would come out as a prediction whereas in the case that Kähler action is introduced as primary object, the value of Kähler coupling strength must be fixed by quantum criticality hypothesis.

The mixing of the M^4 chiralities of the imbedding space spinors serves as a signal for particle massivation and breaking of super-conformal symmetry. The induced gamma matrices for the space-time surfaces which are deformations of M^4 indeed contain a small contribution from CP_2 gamma matrices: this implies a mixing of M^4 chiralities even for the modified Dirac action so that there is no need to introduce this mixing by hand.

8.2 The association of the modified Dirac action to Chern-Simons action and explicit realization of super-conformal symmetries

Super Kac-Moody symmetries should correspond to solutions of modified Dirac equation which are in some sense holomorphic. The discussion below is based on the same general ideas but differs radically from the previous picture at the level of details. The additional assumption inspired by the considerations of this section is that the action associated with the partonic 3-surfaces is non-singular and therefore Chern-Simons action for the induced Kähler gauge potential.

This means that TGD is at the fundamental level almost-topological QFT: only the light-likeness of the partonic 3-surfaces brings in the induced metric and gravitational and gauge interactions and induces the breaking of scale and super-conformal invariance. The resulting theory possesses the expected super Kac-Moody and super-canonical symmetries albeit in a more general form than suggested by the considerations of this section. A connection of the spectrum of the modified Dirac operator with the zeros or Riemann Zeta is suggestive and provides support for the earlier number theoretic speculations concerning the spectrum of super-canonical conformal weights. One can safely say, that if this formulation is correct, TGD could not differ less from a physically trivial theory.

8.2.1 Zero modes and generalized eigen modes of the modified Dirac action

Consider net the zero modes and generalized eigen modes for the modified Dirac operator.

1. The modified gamma matrices appearing in the modified Dirac equation are expressible in terms of the Lagrangian density L assignable to the light-like partonic 3-surface X^43_l as

$$\hat{\Gamma}^\alpha = \frac{\partial L}{\partial_\alpha h^k}\Gamma_k \quad , \tag{35}$$

   where Γ_k denotes gamma matrices of imbedding space. The modified Dirac operator is defined as

$$D = \hat{\Gamma}^\alpha D_\alpha \quad , \tag{36}$$

   where D_α is the covariant derivative defined by the induced spinor connection. Modified gamma matrices satisfy the condition

$$D_\alpha \hat{\Gamma}^\alpha = 0 \tag{37}$$

   if the field equations associated with L are satisfied. This guarantees that one indeed obtains the analog of the massless Dirac equation. Zero modes of the modified Dirac equation should define the conformal supersymmetries.

2. The generalized eigenvalues and eigen solutions of the modified Dirac operator are defined as

$$\begin{aligned} D\Psi &= \lambda N \Psi \quad , \\ N &= n^k \Gamma_k \quad . \end{aligned} \tag{38}$$

   Here n^k denotes a light-like vector which must satisfy the integrability condition

$$\left[D, n^k\Gamma_k\right] = 0 \quad . \tag{39}$$

if the analog $D^2\Psi = 0$ for the square of massless Dirac equation is to hold true. n^k should be determined by the field equations associated with L somehow and commutativity condition could fix n more or less uniquely.

If the commutativity condition holds true then any generalized eigen mode Ψ_λ gives rise to a zero mode as $\Psi = N\Psi_\lambda$. One can add to a given non-zero mode any superposition of zero modes without affecting the generalized eigen mode property.

The commutativity condition can be satisfied if the tangent space at each point of X^4 contains preferred plane M^2 guaranteing $HO-H$ duality and having interpretation as a preferred plane of non-physical polarizations. In this case n can be chosen to be constant light-like vector in M^2.

3. The hypothesis is that Kähler function is expressible in terms of the Dirac determinant of the modified Dirac operator defined as the product of the generalized eigenvalues. The Dirac determinant must carry information about the interior of the space-time surface determined as preferred extremal of Kähler action or (as the hypothesis goes) as hyper-quaternionic or co-hyper-quaternionic 4-surface of M^8 defining unique 4-surface of $M^4 \times CP_2$. The assumption that X_L^3 is light-like brings in an implicit dependence on the induced metric. The simplest but non-necessary assumption is that n^k is a light-like vector field tangential to X_l^3 so that the knowledge of X_l^3 fixes completely the dynamics.

4. If the action associated with the partonic light-like 3-surfaces contains induced metric, the field equations become singular and ill-defined unless one defines the field equations at X_l^3 via a limiting procedure and poses additional conditions on the behavior of Ψ at X_l^3. Situation changes if the action does not contain the induced metric. The classical field equations are indeed well-defined at light-like partonic 3-surfaces for Chern-Simons action for the induced Kähler gauge potential

$$L = L_{C-S} = k\epsilon^{\alpha\beta\gamma}J_{\alpha\beta}A_\gamma \ . \quad (40)$$

One obtains the analog of WZW model with gauge field replaced with the induced Kähler form. This action does not depend on the induced metric explicitly so that in this sense a topological field theory results. There is no dependence on M^4 gamma matrices so that local Lorentz transformations act as super-conformal symmetries of both classical field equations and modified Dirac equation and $SL(2,C)$ defines the analog of the $SU(2)$ Kac-Moody algebra for $N = 4$ SCA.

The facts that the induced metric is light-like for X_l^3, that the modified Dirac equation contains information about this and therefore about induced metric, and that Dirac determinant is the product of the non-vanishing eigen values of the modified Dirac operator, imply the failure of topological field theory property at the level of Kähler function identified as the logarithm of the Dirac determinant.

A more complicated option would be that the modified Dirac action contains also interior term corresponding to the Kähler action. This alternative would break super-conformal symmetries explicitly and almost-topological QFT property would be lost. This option is not consistent with the idea that quantum-classical correspondence relates the partonic dynamics at X_l^3 with the classical dynamics in the interior of space-time providing first principle justification for the basic assumptions of the quantum measurement theory.

The classical field equations defined by L_{C-S} read as

$$\begin{aligned} D_\mu \frac{\partial L_{C-S}}{\partial_\mu h^k} &= 0 \ , \\ \frac{\partial L_{C-S}}{\partial_\mu h^k} &= \epsilon^{\mu\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k)\right] \ . \end{aligned} \tag{41}$$

From the explicit form of equations it is obvious that the most general solution corresponds to a X_l^3 with at most 2-dimensional CP_2 projection.

Although C-S action vanishes, the color isometry currents are in general non-vanishing. One can assign currents also to super-Kac Moody and super-canonical transformations using standard formulas and the possibility that the corresponding charges define configuration space Hamiltonians and their super-counterparts must be considered seriously.

Suppose that the CP_2 projection is 2-dimensional and not a Lagrange manifold. One can introduce coordinates for which the coordinates for X^2 are same as those for CP_2 projection. For instance, complex coordinates $(z, \overline{z})$ of a geodesic sphere could be used as local coordinates for X^2. One can also assign one M^4 coordinate, call it r, with M^4 projection X^1 of X_l^3. Locally this coordinate can be taken to be one of the standard M^4 coordinates. The remaining five H-coordinates can be expressed in terms of $(r, z, \overline{z})$ and light-likeness condition boils down to the vanishing of the metric determinant:

$$det(g_3) = 0 \ . \tag{42}$$

All diffeomorphisms of H respecting the light-likeness condition are symmetries of the solution ansatz.

Consider some special cases serve as examples.

1. The simplest situation results when X_l^4 is of form $X^1 \times X^2$, where X^1 is light-like random curve in M^4 as for CP_2 type vacuum extremals. In this case light-likeness boils down to Virasoro conditions with real parameter r playing the role analogous to that of a complex coordinate: this conformal symmetry is dynamical and must be distinguished from conformal symmetries assignable to X^2. A plausible guess is that light-likeness condition quite generally reduces to the classical Virasoro conditions.

2. A solution in which CP_2 projection is dynamical is obtained by assuming that for a given value of M^4 time coordinate CP_2- and M^4- projections are one-dimensional curves. For instance, CP_2 projection could be the circle $\Theta = \Theta(m^0 \equiv t)$ whereas M^4 projection could be the circle $\rho = \sqrt{x^2+y^2} = \rho(m^0)$. Light-likeness condition reduces to the condition $g_{tt} = 1 - R^2\partial_t\Theta^2 - \partial_t\rho^2 = 0$.

8.2.2 Classical field equations for the modified Dirac equation defined by Chern-Simons action

The modified Dirac operator is given by

$$\begin{aligned} D &= \frac{\partial L_{C-S}}{\partial_\mu h^k}\Gamma_k D_\mu \\ &= \epsilon^{\mu\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_\mu \ , \\ \hat{\epsilon}^{\alpha\beta\gamma} &= \epsilon^{\alpha\beta\gamma}\sqrt{g_3} \ . \end{aligned} \quad (43)$$

Note $\hat{\epsilon}^{\alpha\beta\gamma} =$ does not depend on the induced metric. The operator is non-trivial only for 3-surfaces for which CP_2 projection is 2-dimensional non-Lagrangian sub-manifold. The modified Dirac operator reduces to a one-dimensional Dirac operator

$$D = \hat{\epsilon}^{r\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_r \ . \quad (44)$$

The solutions of the modified Dirac equation are obtained as spinors which are covariantly constant with respect to the coordinate r:

$$D_r\Psi = 0 \ . \quad (45)$$

Non-vanishing spinors $\Psi_1 = \partial_r\Psi$ satisfying $\Gamma_r\Psi_1 = 0$ are not possible. Ψ defines super-symmetry for the generalized eigen modes if the additional condition

$$\Psi = N\Psi_0 \quad (46)$$

is satisfied. The interpretation as super-conformal symmetries makes sense if the Fourier coefficients of zero modes and their conjugates are anticommuting Grassmann numbers. The zero modes which are not of this form do not generate super-conformal symmetries and might correspond to massless particles. TGD based vision about Higgs mechanism suggest the interpretation of n^k as a non-conserved gravitational four-momentum whose time average defines inertial four-momentum of parton. The sum of the partonic four-momenta would be identified as the classical four-momentum associated with the interior of the space-time sheet.

The covariant derivatives D_α involve only CP_2 spinor connection and the metric induced from CP_2. D_r involves CP_2 spinor connection unless X_l^3 is of form $X^1 \times X^2 \subset M^4 \times CP_2$. The eigen modes of D correspond to the solutions of

$$D\Psi = \lambda N\Psi \qquad (47)$$

The first guess is that $N = n^k\gamma_k$ corresponds to the tangential light-like vector $n^k = \Phi\partial_r h^k$ where Φ is a normalization factor which can depend on position.

The obvious objection is that with this assumption it is difficult to understand how Dirac determinant can correspond to an absolute extremum of Kähler action for 4-D space-time sheet containing partonic 3-surfaces as causal determinants ($\sqrt{g_4} = 0$). However, if one can select a unique M^4 time coordinate, say as that associated with the rest system for the average four-momentum defined as Chern-Simons Noether charge, then one can assign to n^k a unique dual obtained by changing the sign of its spatial components. The condition that this vector is tangential to the 4-D space-time sheet would provide information about the space-time sheet and bring in 4-dimensionality. At this stage one must however leave the question about the choice of n^k open.

One should be able to fix Φ apart from overall normalization. First of all, the requirement that zero modes defines super symmetries implies the condition $[D, n^k\Gamma_k]\Psi = 0$ for zero modes. This condition boils down to the requirement

$$D_r(\Phi\partial_r h^k\Gamma_k)\Psi = 0 \ . \qquad (48)$$

This in turn boils down to a condition

$$D_r\partial_r h^k + \frac{\partial_r\Phi}{\Phi}\partial_r h^k = 0 \ . \qquad (49)$$

These conditions in turn guarantee that the condition

$$D_r(h_{kl}\partial_r h^k\partial_r h^l) = 0 \qquad (50)$$

implied by the light-likeness condition are satisfied. Since Φ is determined apart from a multiplicative constant from the light-likeness condition the system is internally consistent. The conditions above are not general coordinate invariant so that the coordinate r must correspond to a physically preferred coordinate perhaps defined by the conditions above.

One can express the eigenvalue equation in the form

$$\begin{aligned}
\partial_r\Psi &= \lambda O\Psi \ , \\
O &= (\hat{\Gamma}^r)^{-1}N \ , \\
(\hat{\Gamma}^r)^{-1} &= \frac{\hat{\Gamma}^r}{a^k a^l h_{kl}} \ , \ \hat{\Gamma}^r \equiv a^k\Gamma_k \ .
\end{aligned} \qquad (51)$$

This equation defines a flow with r in the role of a time parameter. The solutions of this equation can be formally expressed as

$$\Psi(r,z,\overline{z}) \quad = \quad Pe^{\lambda \int O(r,z,\overline{z})dr}\Psi_0(z,\overline{z}) \ . \qquad (52)$$

Here P denotes the ordered exponential needed because the operators $O(r,z\overline{z})$ need not commute for different values of r.

8.2.3 Can one allow light-like causal determinants with 3-D CP_2 projection?

The standard quantum field theory wisdom would suggest that light-like partonic 3-surfaces which are extremals of the Chern-Simons action correspond only to what stationary phase approximation gives when vacuum functional is the product of exponent of Kähler function resulting from Dirac determinant and an imaginary exponent of Chern-Simons action whose coefficient is proportional to the central charge of Kac-Moody algebras associated with CP_2 degrees of freedom.

One cannot exclude the possibility that 3-D light-like causal determinants might be required by the general consistency of the theory. The identification of the exponent of Kähler function as Dirac determinant remains a viable hypothesis for this option. "Off mass shell" breaking of super-conformal symmetries is implied since modified Dirac equation implies the conservation of super conformal currents only when CP_2 projection is at most 2-dimensional.

8.2.4 Some problems of TGD as almost-topological QFT and their resolution

There are some problems involved with the precise definition of the quantum TGD as an almost-topological QFT at the partonic level and the resolution of these problems leads to an unexpected connection between cosmology and parton level physics.

1. Three problems

The proposed view about partonic dynamics is plagued by three problems.

1. The definition of supercanonical and super-Kac-Moody charges in M^4 degrees of freedom poses a problem. These charges are simply vanishing since M^4 coordinates do not appear in field equations.

2. Classical field equations for the C-S action imply that this action vanishes identically which would suggest that the dynamics does not depend at all on the value of k. The central extension parameter k determines the over-all scaling of the eigenvalues of the modified Dirac operator. $1/k$-scaling occurs for the eigenvalues so that Dirac determinant scales by a finite power k^N if the number N of the allowed eigenvalues is finite for the

algebraic extension considered. A constant $Nlog(k)$ is added to the Kähler function and its effect seems to disappear completely in the normalization of states.

3. The general picture about Jones inclusions and the possibility of separate Planck constants in M^4 and CP_2 degrees of freedom suggests a close symmetry between M^4 and CP_2 degrees of freedom at the partonic level. Also in the construction of the geometry for the world of classical worlds the symplectic and Kähler structures of both light-cone boundary and CP_2 are in a key role. This symmetry should be somehow coded by the Chern-Simons action.

2. A possible resolution of the problems

A possible cure to the above described problems is based on the modification of Kähler gauge potential by adding to it a gradient of a scalar function Φ with respect to M^4 coordinates.

1. This implies that super-canonical and super Kac-Moody charges in M^4 degrees of freedom are non-vanishing.

2. Chern-Simons action is non-vanishing if the induced CP_2 Kähler form is non-vanishing. If the imaginary exponent of C-S action multiplies the vacuum functional, the presence of the central extension parameter k is reflected in the properties of the physical states.

3. The function Φ could code for the value of $k(M^4)$ via a proportionality constant

$$\Phi = \frac{k(M^4)}{k(CP_2)}\Phi_0 \ , \tag{53}$$

   Here $k(CP_2)$ is the central extension parameter multiplying the Chern-Simons action for CP_2 Kähler gauge potential. This tricks does just what is needed since it multiplies the Noether currents and super currents associated with M^4 degrees of freedom with $k(M^4)$ instead of $k(CP_2)$.

The obvious breaking of $U(1)$ gauge invariance looks strange at first but it conforms with the fact that in TGD framework the canonical transformations of CP_2 acting as $U(1)$ gauge symmetries do not give to gauge degeneracy but to spin glass degeneracy since they act as symmetries of only vacuum extremals of Kähler action.

3. How to achieve Lorentz invariance?

Lorentz invariance fixes the form of function Φ uniquely as the following argument demonstrates.

1. Poincare invariance would be broken in any case for a given light-cone in the decomposition $CH = \cup_m CH_m$ of the configuration space to sub-configuration spaces associated with light-cones at various locations of M^4 but since the functions Φ associated with various light cones would be related by a translation, translation invariance would not be lost.

2. The selection of Φ should not break Lorentz invariance. If Φ depends on the Lorentz proper time a only, this is partially achieved. Momentum currents would be proportional to m^k and become light like at the boundary of the light-cone. This fits very nicely with the interpretation that the matter emanates from the tip of the light cone in Robertson-Walker cosmology.

Lorentz invariance poses even stronger conditions on Φ.

1. Partonic four-momentum defined as Chern-Simons Noether charge is definitely not conserved and must be identified as gravitational four-momentum whose time average corresponds to the conserved inertial four-momentum assignable to the Kähler action [D3, D5]. This identification is very elegant since also gravitational four-momentum is well-defined although not conserved.

2. Lorentz invariance implies that mass squared is constant of motion. Hence it is interesting to look what expression for Φ results if the gravitational mass defined as Noether charge for C-S action is conserved. The components of the four-momentum for Chern-Simons action are given by

$$P^k = \frac{\partial L_{C-S}}{\partial(\partial_\alpha a)} m^{kl}\partial_{m^l} a \quad .$$

Chern-Simons action is proportional to $A_\alpha = A_a \partial_\alpha a$ so that one has

$$P^k \propto \partial_a \Phi \partial_{m^k} a = \partial_a \Phi \frac{m^k}{a} \quad .$$

The conservation of gravitational mass gives $\Phi \propto a$. Since CP_2 projection must be 2-dimensional, M^4 projection is 1-dimensional so that mass squared is indeed conserved.

Thus one could write

$$\Phi = \frac{k(M^4)}{k(CP_2)} x\theta(a)\frac{a}{R} \quad , \tag{54}$$

where R the radius of geodesic sphere of CP_2 and x a numerical constant which could be fixed by quantum criticality of the theory. Chern-Simons action density does not depend on a for this choice and this independence guarantees that the earlier ansatz satisfies field equations. The presence of the step function $\theta(a)$ tells that Φ is non-vanishing only inside light-cone and gives to the gauge potential delta function term which is non-vanishing only at the light-cone boundary and makes possible massless particles.

3. If M^4 projection is 1-dimensional, only homologically charged partonic 3-surfaces can carry gravitational four-momentum. This is not a problem since M^4 projection can be 2-dimensional in the general case. For CP_2 type extremals, ends of cosmic strings, and wormhole contacts the non-vanishing of homological charge looks natural. For wormhole contacts 3-D CP_2 projection suggests itself and is possible only if one allows also quantum fluctuations around light-like extremals of Chern-Simons action. The interpretation could be that for a vanishing homological charge boundary conditions force X^4 to approach vacuum extremal at partonic 3-surfaces.

This picture does not fit completely with the picture about particle massivation provided by CP_2 type extremals. Massless partons must correspond to 3-surfaces at light-cone boundary in this picture and light-likeness allows only linear motion so that inertial mass defined as average must vanish.

5. Comment about quantum classical correspondence

The proposed general picture allows to define the notion of quantum classical correspondence more precisely. The identification of the time average of the gravitational four-momentum for C-S action as a conserved inertial four-momentum associated with the Kähler action at a given space-time sheet of a finite temporal duration (recall that we work in the zero energy ontology) is the most natural definition of the quantum classical correspondence and generalizes to all charges.

In this framework the identification of gravitational four-momentum currents as those associated with 4-D curvature scalar for the induced metric of X^4 could be seen as a phenomenological manner to approximate partonic gravitational four-momentum currents using macroscopic currents, and the challenge is to demonstrate rigorously that this description emerges from quantum TGD.

For instance, one could require that at a given moment of time the net gravitational four-momentum of $Int(X^4)$ defined by the combination of the Einstein tensor and metric tensor equals to that associated with the partonic 3-surfaces. This identification, if possible at all, would certainly fix the values of the gravitational and cosmological constants and it would not be surprising if cosmological constant would turn out to be non-vanishing.

8.2.5 The eigenvalues of D as complex square roots of conformal weight and connection with Higgs mechanism?

An alternative interpretation for the eigenvalues of D emerges from the TGD based description of particle massivation. The eigenvalues could be interpreted as complex square roots of conformal weights in the sense that $|\lambda|^2$ would have interpretation as a conformal weight. There is of course the possibility of numerical constant of proportionality.

The physical motivation for the interpretation is that λ is in the same role as the mass term in the ordinary Dirac equation and thus indeed square root of mass squared proportional to the conformal weight. The vacuum expectation of Higgs would correspond to that for λ and Higgs contribution to the mass squared would correspond to the p-adic thermodynamical expectation value $\langle|\lambda|^2\rangle$ [A9]. Additional contributions to mass squared would come from super conformal and modular degrees of freedom. The interpretation of the generalized eigenvalue as a Higgs field is also natural because the generalized eigen values of the modified Dirac operator can depend on position.

8.2.6 Super-conformal symmetries

The topological character of the solutions spectrum makes possible the expected and actually even larger conformal symmetries in X^2 degrees of freedom. Arbitrary diffeomorphisms of CP_2, including local SU(3) and its holomorphic counterpart, act as symmetries of the non-vacuum solutions. Also the canonical transformations of CP_2 inducing a U(1) gauge transformation are symmetries. More generally, the canonical transformations of $\delta M^4_\pm \times CP_2$ define configuration space symmetries.

Diffeomorphisms of M^4 respecting the light-likeness condition define Kac-Moody symmetries. In particular, holomorphic deformations of X^3_l defined in E^2 factor of $M^2 \times E^2$ compensated by a hyper-analytic deformation in M^2 degrees taking care that light-likeness is not lost, act as symmetry transformations. This requires that M^2 and E^2 contributions of the deformation to the induced metric compensate each other.

The fact that the modified Dirac equation reduces to a one-dimensional Dirac equation allows the action of Kac-Moody algebra as a symmetry algebra of spinor fields. In M^4 degrees of freedom X^2-local SL(2,C) acts as super-conformal symmetries and extends the SU(2) Kac-Moody algebra of $N = 4$ super-conformal algebra to $SL(2,C)$. The reduction to SU(2) occurs naturally. These symmetries act on all spinor components rather than on the second spinor chirality or right handed neutrinos only. Also electro-weak $U(2)$ acts as X^2-local Kac-Moody algebra of symmetries. Hence all the desired Kac-Moody symmetries are realized.

The action of Super Kac-Moody symmetries corresponds to the addition of a linear combination of zero modes of D to a given eigen mode. This defines a symmetry if zero modes satisfy the additional condition $N\Psi = 0$ implied by $\Psi = N\Psi_0$ in turn guaranteed by the already described conditions. These symmetries are super-conformal symmetries with respect to z and $\overline{z}$.

The radial conformal symmetries generalize the dynamical conformal symmetries characterizing CP_2 type vacuum extremals and could be regarded as dynamical conformal symmetries defining the spectrum of super-canonical conformal weights assigned originally to the radial light-like coordinate of $\delta M^4_\pm$. It deserves to be emphasized that the topological QFT character of TGD at fundamental level broken only by the light-likeness of X_l^3 carrying information about H metric makes possible these symmetries.

$N = 4$ super-conformal symmetry corresponding to the maximal representation with the group $SU(2) \times SU(2) \times U(1)$ acting as rotations and electro-weak symmetries on imbedding space spinors is in question. This symmetry is broken for light-like 3-surfaces not satisfying field equations. It seems that rotational SU(2) can be extended to the full Lorentz group.

8.2.7 How the super-conformal symmetries of TGD relate to the conventional ones?

The representation of super-symmetries as an addition of anticommuting zero modes to the second quantized spinor field defined by the superposition of non-zero modes of the modified Dirac equation differs radically from the standard realization based on the replacement of the world sheet or target space coordinates with super-coordinates. Also the fundamental role of the generalized eigen modes of the modified Dirac operator is something new and absolutely essential for the understanding of how super-conformal invariance is broken: the breaking of super-symmetries is indeed the basic problem of the super-string theories.

Since the spinor fields in question are not Majorana spinors the standard super-field formalism cannot work in TGD context. It is however interesting to look to what extent this formalism generalizes and whether it allows some natural modification allowing to formally integrate the notions of the bosonic action and corresponding modified Dirac action.

1. One can consider the formal introduction of super fields by replacing of X_l^3 coordinates by super-coordinates requiring the introduction of anticommuting parameters θ and $\overline{\theta}$ transforming as H-spinors of definite chirality, which is not consistent with Majorana condition. Using real coordinates x^α for X_l^3, one would have

$$x^\alpha \rightarrow X^\alpha = x^\alpha + \overline{\theta}\hat{\Gamma}^\alpha \Psi + \overline{\Psi}\hat{\Gamma}^\alpha \theta \ ,$$

   Super-conformal symmetries would add to θ a zero mode with Grassmann number valued coefficient. The replacement $z^\alpha \rightarrow X^\alpha$ for the arguments of CP_2 and M^4 coordinates would super-symmetrize the field C-S action density. As a matter fact, the super-symmetrization is non-trivial only in radial degree of freedom since only $\hat{\Gamma}^r$ is non-vanishing.

2. Also imbedding space coordinates could be formally replaced with super-fields using a similar recipe and super-symmetries would act on them. The topological character of Chern-Simons action would allow the super-symmetries induced by the translation of θ by an anticommuting zero mode as formal symmetries at the level of the imbedding space. In both cases it is however far from clear whether the formal super-symmetrization has any real physical meaning.

3. The notion of super-surface suggests itself and would mean that imbedding space Θ parameters are functions of single θ parameter assignable with X^3_l. A possible representation of super-part of the imbedding is a generalization of ordinary imbedding in terms of constraints $H_{i)}(h^k) = 0$, $i = 1,2....$ Symmetries allow only linear functions so that one would have

$$c^{\alpha}_{i)}(r,z,\overline{z})\Theta_{\alpha} = 0 \ .$$

A hyper-plane in the space of theta parameters is obtained. Since only single theta parameter is possible in integral the number of constraints is seven and one obtains the modified Dirac action from the super-space imbedding.

Consider next the basic difficulty and its resolution.

1. The super-conformal symmetries do not generalize to the level of action principle in the standard sense of the word and the reason is the failure of the Majorana property forced by the separate conservation of quark and lepton numbers so that the standard super-space formalism remains empty of physical content.

2. One can however consider the modification of the integration measure $\prod_i d\theta_i d\bar{\theta}_i$ over Grassmann parameters by replacing the product of bilinears with

$$d\bar{\theta}\gamma_1 d\theta d\bar{\theta}\gamma_2 d\theta ...$$

analogous to the product $dx^1 \wedge dx^2 ...$ (where γ^k would be gamma matrices of the imbedding spac

3. transforming like a pseudoscalar. It seems that the replacement of product with wedge product leads to a trivial theory. This formalism could work for super fields obeying Weyl condition instead of Majorana condition and it would be interesting to find what kind of super-symmetric field theories it would give rise to.

   The requirement that the number of Grassmann parameters given by $2D$ is the number of spinor components of definite chirality (counting also conjugates) given by $2 \times 2^{D/2-1}$ gives critical dimension $D = 8$, which suggest that this kind of quantum field theory might exist. As found, the zero modes which are not of form $\Psi = N\Psi_0$ do not generate super-conformal symmetries in the strict sense of the word and might correspond to light particles. One could ask whether chiral SUSY in $M^4 \times CP_2$ might describe the low energy dynamics of corresponding light parton states. General arguments do not however support space-time super-symmetry.

4. Because of the light-likeness the super-symmetric variant of C-S action should involve the modified gamma matrices $\hat{\Gamma}^\alpha$ instead of the ordinary ones. Since only $\hat{\Gamma}^r$ is non-vanishing for the extremals of C-S action and since super-symmetrization takes place for the light-like coordinate r only, the integration measure must be defined as $d\overline{\theta}\hat{\Gamma}_r d\theta$, with θ perhaps assignable to a fixed covariantly constant right-handed neutrino spinor and $\hat{\Gamma}_r$ the inverse of $\hat{\Gamma}^r$. This action gives rise to the modified Dirac action with the modified gamma matrices emerging naturally from the Taylor expansion of the C-S action in powers of super-coordinate.

8.3 Why the cutoff in the number superconformal weights and modes of D is needed?

Two kinds of cutoffs are necessary in the number theoretic approach involving a hierarchy of algebraic extensions of rationals with increasing algebraic dimension.

8.3.1 Spatial cutoff realized in terms of number theoretical braids

The first cutoff corresponds to a spatial discretization selecting a subset of algebraic points of the partonic 2-surface X^2 as a subset of the points common to the real and p-adic variants of X^2 obeying the same algebraic equations. Almost topological field theory property allows to assume algebraic equations and also quantum criticality and generalization of the imbedding space concept are crucial for achieving the cutoff as a completely inherent property of X^2.

8.3.2 Cutoff in the number of super-canonical conformal weights

It is not quite clear whether the number of radial conformal weights should be finite or not. The assumption HFF property is realized also in configuration space degrees of freedom would motivate finiteness for the number of conformal weights and would effectively replace the world of the classical worlds with a finite-D space. Also super-symmetry suggests the same. Finiteness would be guaranteed if the ζ function involved characterizes partonic 2-surface and is labelled by p-adic prime: this would also guarantee that zeros of ζ are algebraic numbers. If the zeta function in question characterizes the spectrum of modified Dirac operator and the number of eigenvalues is finite then this goal is achieved. In the case of Riemann Zeta one would be forced to use cutoff due related to the algebraic extension of p-adic numbers used and to assume that zeros and even more general arguments are algebraic numbers.

8.3.3 Cutoff in the number of generalized eigenvalues of the modified Dirac operator

Second cutoff corresponds to a cutoff in the number of generalized eigenvalues of the modified Dirac operator and also now almost TQFT provides the needed flexibility.

1. If the generalized eigenvalues are interpreted as Higgs field then the number of eigenvalues is just one and also orthogonality condition for the modes is achieved without posing ad hoc correlations between longitudinal and transversal degrees of freedom.

2. A priori the dependence of the eigenmodes on transversal degrees of freedom of X^2 is arbitrary. This looks strange on basis of experience with quantum field theory and would imply non-stringy anti-commutation relations. Holomorphic dependence however leads to stringy anti-commutations.

3. Anti-commutativity at braid points only would be highly satisfactory since it would allow to avoid delta functions but would require that the transverse degrees of freedom reduce to a finite number of modes. The reduction of this cutoff to inherent properties of X^2 remains to be understood. What is clear is that the number of conformal modes in transversal degrees of freedom corresponds essentially to the number of points in the braid and the precise realization of this cutoff remains to be understood. Since this cutoff relates to finite measurement resolution, the idea that non-commutative S^2_{II} coordinates provides an elegant manner to realize the anti-commutativity at finite number of points.

It is natural to choose the modes to be S^2_{II} partial waves with a well defined color isospin quantum numbers I, I_3. The Abelianity of the color holonomy group of induced spinor connection suggests also color confinement in weak sense meaning vanishing of I_3 and Y for the physical states.

Since cutoff hierarchy must relate closely to the hierarchy of quantum phases, it seems natural to assume that for given value of $q = exp(i2\pi/n_b)$ only the angular momentum values $I \leq n_b$ are allowed. Here n_b is the order of the maximal cyclic subgroup of G_b involved with the Jones inclusion. In the similar manner one can introduce cutoff for S^2 partial waves in $\delta M^4_\pm$ as cutoff $l \leq n_b$ for angular momentum. Both cutoffs are needed in the definition of configuration space Hamiltonians and super-Hamiltonians allowing to approximate configuration space with a finite-dimensional space which is obviously in spirit with the hyper-finiteness.

Cutoffs imply that n-point functions are finite and non-trivial since the anti-commutators of second quantized induced spinor fields are non-local and delta function singularity is smoothed out. Non-locality implies that vertices are non-trivial and pair creation becomes possible. It is of course essential that the dynamics of the space-time interior induces correlations between different partonic 2-surfaces.

That this picture can give rise to the basic vertices of quantum theory seems clear. For instance, suppose that bosons can be assigned to the fermionic representation of Hamiltonians and fermions by super Hamiltonians. The idea would be that right handed neutrino represents vacuum state to which imbedding space gamma matrices act like creation operators. The vertex for the emission of boson would involve sum of vacuum expectation values for the product of the operators $\overline{\Psi} J_A \Psi(x)$, $\overline{\nu} J_B \Psi(y)$ $\overline{\Psi} J_C \nu(z)$, $J_A = j^k_A \Gamma_k$ with various choices of arguments. Anticommutation relations would give sum over the values of the quantity $\overline{\nu}J_A(x)J_B(y)J_C(z)\nu$ multiplied by "wave functions" coming the modes of Ψ. Summation would be over the discrete set of points of the number theoretical braid. A discretized version of stringy scattering amplitude would be in question.

8.3.4 Attempt to form an overall view

This approach leads to both a hierarchy of discretized theories and continuum theory. Continuum theory indeed seems to be completely well defined and would correspond to string theory with free fermions with $N = 4$ super-conformal symmetry as far vertices are considered.

The interpretation encouraged by Jones inclusion hierarchy is that the limit $n \rightarrow \infty$ for quantum phase $q = exp(i2\pi/n)$ is not equivalent with the exact real theory based on stringy amplitudes defined using 1-D integrals over the inverse image of the image of the critical line. The natural interpretation for the stringy option without discretization could be in terms of Jones inclusions with group $SU(2)$ and classified by extended ADE diagrams relating to the monodromies of the theory. This interpretation would also conform with the full Kac-Moody invariance whereas for quantum version infinite-dimensional symmetries are reduced to finite-dimensional ones. Note that quantum trace should be equivalent with the condition that the trace of the unit matrix is unity for hyper-finite factors of type II_1.

The number theoretic cutoff hierarchy for the allowed zeros of ζ relates closely to the hierarchy of finite-dimensional extensions of p-adic numbers and to the quantum criticality realized in terms of the generalized imbedding space. This hierarchy of extensions defines a hierarchy of number theoretic braids with an increasing number of strands since the number of points in the intersection between real and corresponding p-adic surface increases and does also the number of allowed zeros. Also the hierarchy of finite-dimensional approximations for the inclusions of hyperfinite factors of type II_1 can be visualized in terms of a hierarchy of braid inclusions with increasing number of braids and is described in terms of Temperley-Lieb algebras. This hierarchy of approximate representations of the inclusion means the replacement of the Beraha number $B_n = 4cos^2(\pi/n)$ by a rational number defining the ratio of dimensions of two subsequent finite-dimensional algebras in the hierarchy. Hence the number theoretic braid hierarchy could provide a concrete representation for the hierarchy of approximations for the hyper-finite factors of type II_1 and their Jones inclusions in terms of inclusions of Temperley Lieb algebras assignable to the number theoretic braids. Physics itself would define this sequence of approximations via p-adicization which basically means space-time realization of cognitive representations.

8.4 The spectrum of Dirac operator and radial conformal weights from physical and geometric arguments

The identification of the generalized eigenvalues of the modified Dirac operator as Higgs field suggests the possibility of understanding the spectrum of D purely geometrically by combining physical and geometric constraints.

The standard zeta function associated with the eigenvalues of the modified Dirac action is the best candidate concerning the interpretation of super-canonical conformal weights as zeros of ζ. This ζ should have very concrete geometric and physical interpretation related to the quantum criticality if these eigenvalues have geometric meaning based on geometrization of Higgs field.

Before continuing it its convenient to introduce some notations. Denote the complex coordinate of a point of X^2 w, its $H = M^4 \times CP_2$ coordinates by $h = (m, s)$, and the H coordinates of its $R_+ \times S^2_{II}$ projection by $h_c = (r_+, s_{II})$.

8.4.1 Generalized eigenvalues

The generalized eigenvalue equation defined by the modified Dirac equation is a differential equation involving only the derivative with respect to r. Hence the eigenvalues λ can depend on X^2 coordinate w and on the coordinates of the critical manifold $R_+ \times S^2_{II}$ via the dependence of w these. As a function of $R_+ \times S^2_{II}$ coordinates they would be many-valued functions of these coordinates since several points of X^2 can project at given point of $R_+ \times S^2_{II}$.

The replacement of the ordinary eigenvalues with continuous functions would be a space-time analog for generalized eigenvalues identified as Hermitian operators (or equivalently, their spectra inspired by the quantum measurement theory based on inclusions of hyper-finite factors of type II_1 [C8]. The replacement of these functions with their values in a discrete set defined by number theoretic braid would in turn be the counterpart for the finite measurement resolution.

The interpretation of eigenvalue as a complex Higgs field gives the most concrete interpretation for the generalized eigenvalues. Of course, only single eigenvalue would be realized in this kind of situation. Also the requirement that different modes are orthogonal with respect to the inner product at the partonic 2-surface allows only single generalized eigenvalue. Hence the modes in transversal degrees of freedom would code for physics as in the usual QFT.

This interpretation does not kill the idea about eigenvalues as inverses of zeta function $\lambda = \zeta^{-1}(z)$, S^2_{II}. The point is that one can regard X^2 as a covering of S^2 and assign different branches of ζ^{-1} to the different sheets of covering. Different branches of $\zeta^{-1}(z)$, call them $\zeta_k^{-1}(z)$, would combine to single function of the coordinate w of X^2. In the case of Riemann zeta the corresponding construction would replaced complex plane with its infinite-fold covering.

8.4.2 General definition of Dirac determinant

The first guess is that Dirac determinant can defined as a product of determinants assignable to the points $z = z_k$ of the number theoretic braids:

$$det(D) = \prod_{z_k)} det(D(z_k)) \ . \tag{55}$$

The determinant $det(D(z))$ at point z of S^2 would be defined as the product of the eigenvalues $\lambda(z)$ at points associated with the number theoretic braids.

$$det(D)(z_k) = \left[\prod_i \zeta_i^{-1}(z_k)\right]^{n(z_k)} \ , \tag{56}$$

$n(z_k)$ is the number of strands in the number theoretical braid of associated with z_k. Higgs interpretation would imply that only single value of Higgs contributes for a given point of X^2. Dirac determinant must be an algebraic number. This is the case if the total number of points of number theoretic braids involved is finite. It turns out that this guess is quite not general enough: it turns out that actual Dirac determinant must be identified as a ratio of two determinants.

8.4.3 Interpretation of eigenvalues of D as Higgs field

The eigenvalues of the modified Dirac operator have a natural interpretation as Higgs field which vanishes for the unstable extrema of Higgs potential. These unstable extrema correspond naturally to quantum critical points resulting as intersection of M^4 *resp.* CP_2 projection of the partonic 2-surface X^2 with R_+ *resp.* S^2_{II}.

Quantum criticality suggests that the counterpart of Higgs potential could be identified as the modulus square of ζ:

$$V(H(s)) = -|H(s)|^2 \ . \tag{57}$$

which indeed has the points s with $V(H(s)) = 0$ as extrema which would be unstable in accordance with quantum criticality. The fact that for ordinary Higgs mechanism minima of V are the important ones raises the question whether number theoretic braids might more naturally correspond to the minima of V rather than intersection points with S^2. This turns out to be the case. It will also turn out that the detailed form of Higgs potential does not matter: the only thing that matters is that $|V|$ is monotonically decreasing function of the distance from the critical manifold.

8.4.4 Purely geometric interpretation of Higgs

Geometric interpretation of Higgs field suggests that critical points with vanishing Higgs correspond to the maximally quantum critical manifold $R_+ \times S^2_{II}$. The value of H should be determined once $h(w)$ and $R_+ \times S^2_{II}$ projection $h_c(w)$ are known. $|H|$ should increase with the distance between these points. The question is whether one can assign to a given point pair $(h(w), h_c(w))$ naturally a value of H. The first guess is that value of H is most determined by the shortest piece of the geodesic line connecting the points which is a product of geodesics of δM^4_+ and CP_2.

This guess need not be quite correct. An alternative guess is that M^4 projection is indeed geodesic but that CP_2 projection extremizes its length subject to the constraint that the absolute value of the phase defined by the one-dimensional Kähler action $\int A_\mu dx^\mu$ is minimized: this point will be discussed below.

The value should be in general complex and invariant under the isometries of δH affecting h and h_c. The minimal distance $d(h, h_c)$ between the two points constrained by extremal property of phase would define the first candidate for the modulus of H.

The phase factor should relate close to the Kähler structure of CP_2 and one possibility would be the non-integrable phase factor $U(s, s_{II})$ defined as the integral of the induced Kähler gauge potential along the geodesic line in question. Hence the first guess for the Higgs would be as

$$\begin{aligned} H(w) &= d(h, h_c) \times U(s, s_{II}) \ , \\ d(h, h_c) &= \int_h^{h_c} ds \ \ , \ \ U(s, s_{II}) = exp\left[i \int_s^{s^1} A_k ds^k \right] \ . \end{aligned} \tag{58}$$

This gives rise to a holomorphic function in X^2 the local complex coordinate of X^2 is identified as $w = d(h, h_s)U(s, s_{II})$ so that one would have $H(w) = w$ locally. This view about H would be purely geometric.

One can ask whether one should include to the phase factor also the phase obtained using the Kähler gauge potential associated with S^2_r having expression $(A_\theta, A_\phi) = (k, cos(\theta))$ with k even integer from the requirement that the non-integral phase factor at equator has the same value irrespective of whether it is calculated with respect to North or South pole. For $k = 0$ the contribution would be vanishing. The value of k might correlate directly with the value of quantum phase. The objection against inclusion of this term is that Kähler action defining Kähler function should contain also M^4 part if this term is included. If this inclusion is allowed then internal consistency requires also the extremization of $\int A_\mu dx^\mu$ so that geodesic lines are not allowed.

In each coordinate patch Higgs potential could be simply the quadratic function $V = -w\overline{w}$. Negative sign is required by quantum criticality. As noticed any monotonically increasing function of V works as well since the minima of the potential remain unaffected. Potential could indeed have minima as minimal distance of X^2 point from $R_+ \times S^2_{II}$. Earth's surface with zeros as tops of mountains and bottoms of valleys as minima would be a rather precise visualization of the situation for given value of r_+. Mountains would have a shape of inverted rotationally symmetry parabola in each local coordinate system.

8.4.5 Consistency with the vacuum degeneracy of Kähler action and explicit construction of preferred extremals

An important constraint comes from the condition that the vacuum degeneracy of Kähler action should be understood from the properties of the Dirac determinant. In the case of vacuum extremals Dirac determinant should have unit modulus.

Suppose that the space-time sheet associated with the vacuum parton X^2 is indeed vacuum extremal. This requires that also X_l^3 is a vacuum extremal: in this case Dirac determinant must be real although it need not be equal to unity. The CP_2 projection of the vacuum extremal belongs to some Lagrangian sub-manifold Y^2 of CP_2. For this kind of vacuum partons the ratio of the product of minimal H distances to corresponding $M^4_\pm$ distances must be equal to unity, in other words minima of Higgs potential must belong to the intersection $X^2 \cap S^2_{II}$ or to the intersection $X^2 \cap R_+$ so that distance reduces to M^4 or CP_2 distance and Dirac determinant to a phase factor. Also this phase factor should be trivial.

It seems however difficult to understand how to obtain non-trivial phase in the generic case for all points if the phase is evaluated along geodesic line in CP_2 degrees of freedom. There is however no deep reason to do this and the way out of difficulty could be based on the requirement that the phase defined by the Kähler gauge potential is evaluated along a curve either minimizing the absolute value of the phase modulo 2π.

One must add the condition that curve is not shorter than the geodesic line between points. For a given curve length s_0 the action must contain as a Lagrange multiplier the curve length so that the action using curve length s as a coordinate reads as

$$S = \int A_s ds + \lambda(\int ds - s_0) \ . \tag{59}$$

This gives for the extremum the equation of motion for a charged particle with Kähler charge $Q_K = 1/\lambda$:

$$\begin{aligned} \frac{D^2 s^k}{ds^2} + \frac{1}{\lambda} \times J^k_{\ l} \frac{ds^l}{ds} &= 0 \ , \\ \frac{D^2 m^k}{ds^2} &= 0 \ . \end{aligned} \tag{60}$$

The magnitude of the phase must be further minimized as a function of curve length s.

If the extremum curve in CP_2 consists of two parts, first belonging to X^2_{II} and second to Y^2, the condition is certainly satisfied. Hence if $X^2_{CP_2} \times Y^2$ is not empty, the phases are trivial. In the generic case 2-D sub-manifolds of CP_2 have intersection consisting of discrete points (note again the fundamental role of 4-dimensionality of CP_2). Since S^2_{II} itself is a Lagrangian sub-manifold, it has especially high probably to have intersection points with S^2_{II}. If this is not the case one can argue that X^3_l cannot be vacuum extremal anymore.

Radial conformal invariance of $\delta M^4_\pm$ raises the question whether $\delta M^4_\pm$ geodesics should be defined by allowing $r_M(s)$ to be arbitrary rather than constant. The minimization of $\delta M^4_\pm$ distance would favor geodesics for which $r_M(s)$ decreases as fast as possible so that one has a light-like geodesics going directly to the tip of $\delta M^4_\pm$. Therefore this option does not seem to work.

The construction gives also a concrete idea about how the 4-D space-time sheet $X^4(X^3_l)$ becomes assigned with X^3_l. The point is that the construction extends X^2 to 3-D surface by connecting points of X^2 to points of S^2_{II} using the proposed curves. This process can be carried out in each intersection of X^3_l and M^4_+ shifted to the direction of future. The natural conjecture is that the resulting space-time sheet defines the 4-D preferred extremum of Kähler action.

The most obvious objection is that this construction might not work for cosmic strings of form $X^2 \times S^2_I$, where S^2_I is a homologically non-trivial geodesic sphere of CP_2. In this case X^2 would correspond to string ends, copies of S^2_I at different points of $\delta M^4_\pm$. There seems to be however no real problem. If $S^2_I \cap S^2_{II}$ is not empty, the orbits representing motion in the induced Kähler gauge field could simply define a flow at S^2_I connecting the points of S^2_I to one of the intersection points. Since geodesic manifold is in question one expects that the orbits indeed belong to S^2_I and cosmic string is obtained. Also a flow with several sources and sinks is possible. Situation should be the same for complex 2-sub-manifolds of CP_2. The 3-D character of the resulting surface would be due to the fact that $\delta M^4_\pm$ projections of the orbits are not points. If the second end of the string is at R_+ string and has the same value of r_M coordinate, single string would result. Otherwise one would obtain two strings with second end point at R_+ with the same value of r_M.

8.4.6 About the definition of the Dirac determinant and number theoretic braids

The definition of Dirac determinant should be independent of the choice of complex coordinate for X^2 and local complex coordinate implied by the definition of Higgs is a unique choice for this coordinate. The physical intuition based on Higgs mechanism suggests that apart from normalization factor the Dirac determinant should be defined simply as product of the eigenvalues of D, that is those of Higgs field, associated with the number theoretic braids.

If only single kind of braid is allowed then the minima of Higgs field define the points of the braid very naturally. The points in $R_+ \times S^2_{II}$ cannot contribute to the Dirac determinant since Higgs vanishes at the critical manifold. Note that at S^2_{II} criticality Higgs values become real and the exponent of Kähler action should become equal to one. This is guaranteed if Dirac determinant is normalized by dividing it with the product of $\delta M^4_\pm$ distances of the extrema from R_+. The value of the determinant would equal to one also at the limit $R_+ \times S^2_{II}$.

One would define the Dirac determinant as the product of the values of Higgs field over all minima of local Higgs potential

$$det(D) \quad = \quad \frac{\prod_k H_(w_k)}{\prod_k H_0(w_k)} = \prod_k \frac{w_k}{w_k^0} \quad . \tag{61}$$

Here w_k^0 are M^4 distances of extrema from R_+. Equivalently: one can identify the values of Higgs field as dimensionless numbers w_k/w_k^0. The modulus of Higgs field would be the ratio of H and $M^4_\pm$ distances from the critical sub-manifold. The modulus of the Dirac determinant would be the product of the ratios of H and M^4 depths of the valleys.

This definition would be general coordinate invariant and independent of the topology of X^2. It would also introduce a unique conformal structure in X^2 which should be consistent with that defined by the induced metric. Since the construction used relies on the induced metric this looks natural. The number of eigen modes of D would be automatically finite and eigenvalues would have purely geometric interpretation as ratios of distances on one hand and as masses on the other hand. The inverse of CP_2 length defines the natural unit of mass. The determinant is invariant under the scalings of H metric as are also Kähler action and Chern-Simons action. This excludes the possibility that Dirac determinant could also give rise to the exponent of the area of X^2.

Number theoretical constraints require that the numbers w_k are algebraic numbers and this poses some conditions on the allowed partonic 2-surfaces unless one drops from consideration the points which do not belong to the algebraic extension used.

8.4.7 About the detailed definition of number theoretic braids

Consider now the detailed definition of number theoretic braids. One can define a pile X^2_t of cross sections of $X^3_l \cap (\delta M^4_{\pm,t} \times CP_2)$, where $\delta M^4_{\pm,t}$ represents $\delta M^4_\pm$ shifted by t in a preferred time direction defined by M^2. In the same manner one can decompose M^2 to a pile of light-like geodesics $R_{+,t}$ defining the quantization axis of angular momentum. For each value of t one obtains a collection of minima of the "Higgs field" λ_t in 3-dimensional space $R_{+,t} \times S^2_{II}$. The minima define orbits $\gamma(t)$: $(r_{+,i}(t), s_{II}(t))$ in $M^2 \times S^2_{II}$ space.

One can consider braidings (or more generally tangles, two minima can disappear in collision or can be created from vacuum) both in X^3_l and at the level of imbedding space.

1. Braids in X^3_l

A braid in X^3_l is obtained by considering the fate of points of $X^2 t = 0$ in X^3_l and by assigning a braiding to the minima of Higgs field in X^3_l. Also the field lines of Kähler magnetic field or of Kähler gauge potential on X^3_l going through the initial positions of Higgs minima can be considered. Since the construction of the Higgs field involves induced Kähler gauge potential in an essential manner, the braiding defined by the Kähler gauge potential could be consistent with the time evolution for the positions of the minima of Higgs.

Recall that only topological rather than point-wise equivalence of the braids is required. It is not clear how much these definition depend on the coordinates used for X_l^3. For instance, could one trivialize the braid by making a time dependent coordinate change for X^2? This requires that it is possible to define global time coordinate whose coordinate lines correspond to field lines. This is possible only if the flow satisfies additional integrability conditions [D1].

2. Braidings defined by imbedding space projections

One can define braidings also by the projections to the heavenly spheres S^2_{II} of CP_2 and S^2_r of $\delta M^4_\pm$. A linear braid like structure is also obtained by considering the projections of Higgs minima in M^2.

1. The simplest option is the identification of the braid as the projections of the orbits of the minima of Higgs field to S^2_{II} or S^2_r (for various values of t). This seems to be the most elegant choice. One could decompose the braid to sub-braids such that each initial value $r_{+,i}(0)$ would define its own braid in S^2_{II} or S^2_r. Also each point of S^2_{II} or S^2_r could define its own sub-braid.

2. Factoring quantum field theories defined in M^2 [40, 83] suggest a further definition of a braid like structure based on the projections of Higgs minima to M^2. The braid like structure would result from the motion of braid points with different velocities so that they would pass by each other. This kind of pattern with constant velocities of particles describes scattering in factoring quantum field theories defined in M^2. The M^2 velocities of particles would not be constant now. S-matrix is almost trivial inducing only a permutation of the initial state momenta and S-matrix elements are mere phases. The interpretation is that each pass-by process induces a time lag. At the limit when the velocities approach to zero or infinity such that their ratios remain constant, S-matrix reduces to a braiding S-matrix.

The Higgs minima contributing to the elements of S-matrix (or at least U-matrix) should correspond to algebraic points of braids. This suggests that the information about the braids comes from the minima of Higgs in X_l^3 rather than X_t^2 so that only some values of t at each strand $\gamma(t)$ give rise to physically relevant braid points. The condition that the resulting numbers are algebraic poses restrictions on X_l^3 as does also the condition that X_l^3 have also p-adic counterparts. This does not of course mean the loss of braids. Note that the discretization allows to assign Dirac determinant and zeta function to any 3-surface X_l^3 rather than only those corresponding to the maxima of Kähler function.

8.4.8 The identification of zeta function

The proposed picture supports the identification of the eigenvalues of D in terms of a Higgs fields having purely geometric meaning. It also seems that number theoretic braids must be identified as minima of Higgs potential in X^2. Furthermore, the braiding operation could be defined for all intersections of X^3_l defined by shifts $M^4_\pm$ as orbits of minima of Higgs potential. Second option is braiding by Kähler magnetic flux lines.

The question is how to understand super-canonical conformal weights for which the identification as zeros of a zeta function of some kind is highly suggestive. The natural answer would be that the normalized eigenvalues of D defines this zeta function as

$$\zeta(s) = \sum_k (\frac{H(w_k)}{H_0(w_k)})^{-s} . \quad (62)$$

The number of eigenvalues contributing to this function would be finite and $H(w_k)/H_0(w_k)$ should be rational or algebraic at most. ζ function would have a precise meaning consistent with the usual assignment of zeta function to Dirac determinant.

The case of Riemann Zeta inspires the question whether one should allow only the moduli of the eigenvalues in the zeta or allow only real and positive eigenvalues. The moduli of eigenvalues are not smaller than unity as is the case also for Riemann Zeta. Real eigenvalues correspond to vanishing phase and thus vanishing Chern-Simons action and unit eigenvalues to the quantum critical points of S^2_{II}.

The ζ function would directly code the basic geometric properties of X^2 since the moduli of the eigenvalues characterize the depths of the valleys of the landscape defined by X^2 and the associated non-integrable phase factors. The degeneracies of eigenvalues would in turn code for the number of points with same distance from a given zero intersection point.

The zeros of the ζ function in turn define natural candidates for the super-canonical conformal weights and their number would thus be finite in accordance with the idea about inherent cutoff present also in configuration space degrees of freedom. Super-canonical conformal weights would be functionals of X^2. The scaling of λ by a constant depending on p-adic prime factors out from the zeta so that zeros are not affected: this is in accordance with the renormalization group invariance of both super-canonical conformal weights and Dirac determinant.

The zeta function should exist also in p-adic sense. This requires that the numbers λ^s at the points s of S^2_{II} which corresponds to the number theoretic braid are algebraic numbers. The freedom to scale λ could help to achieve this.

The conformal weights defined by the zeros of zeta would be constant. One could however consider also the generalization of the super-canonical conformal weights to functions of S^2_{II} or S^2_r coordinate although this is not necessary and would spoil the simple group theoretical properties of the δH Hamiltonians. The coordinate s appearing as the argument of ζ could be formally identified as S^2_{II} or S^2_r coordinate so that generalized super-canonical conformal weights could be interpreted geometrically as inverses of $\zeta^{-1}(s)$ defined as a function in S^2_{II} or S^2_r.

In this case also the notion of number theoretic braids defined as sets of points for which $X^2_{M^4}$ projection intersects R_+ at same point could make sense for super-canonical conformal weights. This would require that the number for the branches of ζ^{-1} is same as the number of points of braid.

8.4.9 The relationship between λ and Higgs field

The generalized eigenvalue $\lambda(w)$ is only proportional to the vacuum expectation value of Higgs, not equal to it. Indeed, Higgs and gauge bosons as elementary particles correspond to wormhole contacts carrying fermion and antifermion at the two wormhole throats and must be distinguished from the space-time correlate of its vacuum expectation as something proportional to λ. In the fermionic case the vacuum expectation value of Higgs does not seem to be even possible since fermions do not correspond to wormhole contacts between two space-time sheets but possess only single wormhole throat (p-adic mass calculations are consistent with this). Gauge bosons can have Higgs expectation proportional to λ. The proportionality must be of form $\langle H\rangle \propto \lambda/p^{n/2}$ if gauge boson mass squared is of order $1/p^n$.

8.4.10 Possible objections related to the interpretation of Dirac determinant

Suppose that that Dirac determinant is defined as a product of determinants associated with various points z_k of number theoretical braids and that these determinants are defined as products of corresponding eigenvalues.

Since Dirac determinant is not real and is not invariant under isometries of CP_2 and of $\delta M^4_\pm$, it cannot give only the exponent of Kähler function which is real and $SU(3)\times SO(3,1)$ invariant. The natural guess is that Dirac determinant gives also the Chern-Simons exponential and possible phase factors depending on quantum numbers of parton.

1. The first manner to circumvent this objection is to restrict the consideration to maxima of Kähler function which select preferred light-like 3-surfaces X^3_l. The basic conjecture forced by the number theoretic universality and allowed by TGD based view about coupling constant evolution indeed is that perturbation theory at the level of configuration space can be restricted to the maxima of Kähler function and even more: the radiative corrections given by this perturbative series vanish being already coded by Kähler function having interpretation as analog of effective action.

2. There is also an alternative way out of the difficulty: define the Dirac determinant and zeta function using the minima of the modulus of the generalized Higgs as a function of coordinates of X^3_l so that continuous strands of braids are replaced by a discrete set of points in the generic case.

The fact that general Poincare transformations fail to be symmetries of Dirac determinant is not in conflict with Poincare invariance of Kähler action since preferred extremals of Kähler action are in question and must contain the fixed partonic 2-surfaces at $\delta M^4_\pm$ so that these symmetries are broken by boundary conditions which does not require that the variational principle selecting the preferred extremals breaks these symmetries.

One can exclude the possibility that the exponent of the stringy action defined by the area of X^2 emerges also from the Dirac determinant. The point is that Dirac determinant is invariant under the scalings of H metric whereas the area action is not.

The condition that the number of eigenvalues is finite is most naturally satisfied if generalized ζ coding information about the properties of partonic 2-surface and expressible as a rational function for which the inverse has a finite number of branches is in question.

8.4.11 How unique the construction of Higgs field is?

Is the construction of space-time correlate of Higgs as λ really unique? The replacement of H with its power H^r, $r > 0$, leaves the minima of H invariant as points of X^2 so that number theoretic braid is not affected. As a matter fact, the group of monotonically increasing maps real-analytic maps applied to H leaves number theoretic braids invariant.

The map $H \rightarrow H^r$ scales Kähler function to its r-multiple, which could be interpreted in terms of $1/r$- scaling of the Kähler coupling strength. Also super-canonical conformal weights identified as zeros of ζ are scaled as $h \rightarrow h/r$ and Chern-Simons charge k is replaced with k/r so that at least $r = 1/n$ might be allowed.

One can therefore ask whether the powers of H could define a hierarchy of quantum phases labelled by differen values of k and α_K. The interpretation as separate phases would conform with the idea that D in some sense has entire spectrum of generalized eigenvalues.

8.5 Quantization of the modified Dirac action

The quantization of the modified Dirac action involves a fusion of various number theoretical ideas. Stringy picture need not be correct with string being replaced number theoretic braids.

1. The first question is how M^4 and CP_2 braids relate. Since one assumes that the data associated with both braids are independent, it seems necessary to assume anti-commutativity between all points of X^2 belonging to some number theoretic braid.

2. There is no correlation between λ and eigenvalues associated with transverse degrees of freedom as in the case of d'Alember operator. Therefore an infinite number of eigen-modes of D for a given eigenvalue λ can be considered unless one poses some additional conditions. This would mean

that one could have anti-commutativity for different points of X^2 and anti-commutators of Ψ and conjugate at same point would be proportional to delta function. This would not conform with the stringy picture.

3. How could one obtain stringy anticommutations? The assumption that modes are holomorphic or antiholomorphic would guarantee this since formally only single coordinate variable would appear in Ψ. Anti-commutativity along string requires that in a given sector of configuration space isometries commute with the selection of quantization axes for the isometry algebra of the imbedding space. This might be justified by quantum classical correspondence. The unitarity for Yang-Baxter matrices and unitarity of the inner product for the radial modes r^{Δ}, $\Delta = 1/2 + iy$, is consistent with the stringy option where y would now label those points of R_+ which do not correspond to $z = 0$. String corresponds to the ζ-image of the critical line containing non-trivial zeros of zeta at the geodesic sphere of S^2_r.

4. One could ask whether number theoretic braids might have deeper meaning in terms of anticommutativity. This would be the case if the modes in transversal degrees of freedom reduce to a finite number and are actually labelled by λ. This could be achieved if there is no other dependence on transverse degrees of freedom than that coming through $\lambda(z)$. Anti-commutativity would hold true only at finite number of points and that anti-commutators would be finite in general. This outcome would be very nice.

5. An interesting question is whether the number theoretic braid could be also described by introducing a non-commutativity of the complex coordinate of X^2 provided by S^2_r or S^2_{II}. This should replace anti-commutativity in X^2 with anti-commutativity for different points of the number theoretic braid. The nice outcome would be the finiteness of anti-commutators at same point.

The following is an attempt to formulate this general vision in a more detail manner.

8.5.1 Fermionic anticommutation relations: non-stringy option

The fermionic anti-commutation relations must be consistent with the vacuum degeneracy and with the anti-commutation relations of configuration space gamma matrices defining the matrix elements of configuration space metric between complexified Hamiltonians.

1. The bosonic representation of configuration space Hamiltonians is naturally as Noether charges associated with Chern-Simons action:

$$H_A = \int d^2x \pi^0_k j^k_A ,$$

$$\pi^\alpha = \frac{\partial L_{C-S}}{\partial_\alpha h^k} . \quad (63)$$

π_k^0 denotes bosonic canonical momentum density. Note that also fermionic dynamics allows definition of Hamiltonians as fermionic charges) and this would give rise to fermionic representation of super-canonical algebra. Same applies to the super Kac-Moody algebra generators which super Kac-Moody generators realized as X^3-local isometries of the imbedding space.

2. Super Hamiltonians identifiable as contractions of configuration space gamma matrices with Killing vector fields of symplectic transformations in CH can be defined as matrix elements of $j_A^k\Gamma_k$ between $\overline{\nu}_R$ and Ψ:

$$J_A^K\Gamma_K \equiv \Gamma_A = H_{S,A} = \int d^2x\overline{\nu}_R j_A^k\Gamma_k\Psi . \quad (64)$$

$H_{S,A}^\dagger$ is obtained by Hermitian conjugation.

3. The anti-commutation relations read as

$$\{\overline{\Psi}(x), \Gamma_k\Psi(y)\} = \pi_k^0 J^{rs}\Sigma_{rs}\delta^2(x,y) . \quad (65)$$

Here J^{rs} denotes the degenerate Kähler form of $\delta M_+^4 \times CP_2$. What makes these anti-commutation relations non-stringy is that anti-commutator is proportional to 2-D delta function rather than 1-D delta function at 1-D sub-manifold of X^2 as in the case of conformal field theories. Hence one would have 3-D quantum field theory with one light-like direction.

4. The matrix elements of configuration space metric for the complexified Killing vector fields of symplectic transformations give the elements of configuration space Kähler form and metric as

$$\{\Gamma_A^\dagger, \Gamma_B\} = iG_{\overline{A},B} = J_{\overline{A},B} = \{\overline{H_A}, H_B\} = H_{[\overline{A},B]} . \quad (66)$$

8.5.2 Fermionic anti-commutation relations: stringy option

As already noticed, 2-dimensional delta function in the anti-commutation relations implies that spinor field is 2-D Euclidian free field rather than conformal field. The usual stringy picture would require anti-commutativity only along circle and nonlocal commutators outside this circle.

Also the original argument based on the observation that the points of CP_2 parameterize a large class of solutions of Yang-Baxter equation suggests the stringy option. The subset of commuting Yang-Baxter matrices corresponds to a geodesic sphere S^2 of CP_2 and the subset of unitary Yang-Baxter matrices to a geodesic circle of S^2 identifiable as real line plane compactified to S^2. Physical intuition strongly favors unitarity.

Stringy choice is consistent with the identification of the configuration space Hamiltonians as bosonic Noether charges only if Noether charges correspond to closed but in general not exact 2-forms and thus reduce to integrals of a 1-form over 1-dimensional manifold representing the discontinuity of the associated vector potential. That Noether charges would reduce to cohomology would conform with almost TQFT property. This is indeed the case under conditions which will be idenfied below.

1. The canonical momentum density associated with C-S action has the expression

$$\pi_k = \epsilon_{\alpha\beta 0}(\partial_\beta [A_\alpha A_k) - \partial_\alpha (A_\beta A_k] \ , \tag{67}$$

and is thus a closed two-form. Note that the discontinuity of the monopole like vector potential implies that the form in question is not exact.

2. Also the Hamiltonian densities

$$H_A = j^k_A \pi_k = J^{kl}\partial_l H_A \epsilon_{\alpha\beta 0} [\partial_\beta (A_\alpha A_k) - \partial_\alpha (A_\beta A_k] \tag{68}$$

should define closed forms

$$H_A = j^k_A \pi_k = \epsilon_{\alpha\beta 0} \left[\partial_\beta (A_\alpha A_k J^{kl}\partial_l H_A) - \partial_\alpha (A_\beta A_k \partial_l J^{kl} H_A\right] . \tag{69}$$

3. This is not the case in general since the derivatives coming from j^k_A give the term

$$\epsilon_{\alpha\beta 0} A_\alpha A_k J^{kl} D_r(\partial_l H_A)\partial_\beta h^r - A_\beta A_k J^{kl} D_r(\partial_l H_A)\partial_\alpha h^r \ . \tag{70}$$

which does not vanish unless the condition

$$A_k J^{kl} D_r(\partial_l H_A) = \partial_r \Phi \tag{71}$$

holds true.

The condition is equivalent with the vanishing of the Poisson bracket between Hamiltonian and components of the Kähler potential:

$$\partial_k H_A J^{kl} \partial_l A_r \quad = \quad 0 \ . \tag{72}$$

This poses a restriction on the group of isometries of configuration space. The restriction of Kähler potential to A_r is given by $(A_\theta, A_\phi) = (0, cos(\theta))$ and A_ϕ generates rotations in z-direction. Hence only the Hamiltonians commuting with Kähler gauge potential of $\delta M^4_\pm \times CP_2$ at X^2 would have vanishing color isospin and presumably also vanishing color hyper charge in the case of CP_2 and vanishing net spin in case of δM^4_+.

4. The discontinuity of Φ would result from the topological magnetic monopole character of the Kähler potential A_k in $\delta M^4_\pm \times CP_2$.

5. Quantum classical correspondence suggests that quantum measurement theory is realized at the level of the configuration space and induces a decomposition of the configuration space to a union of sub-configuration spaces corresponding to different choices of quantization axes of angular momentum and color quantum numbers. Hence the interpretation of configuration space isometries in terms of a maximal set of commuting observables would make sense. Of course, also the canonical transformations for which Hamiltonians do not reduce to 1-D integrals act as symmetries although they do not possess super counterparts. They play same role as Lorentz boosts whereas the super-symmetrizable part of the algebra is analogous to the little group of Lorentz group leaving momentum invariant. This means that complete reduction to string model type theory does not occur even at the level of quantum states.

Consider now the basic formulas for the stringy option.

1. Hamiltonians can be expressed as

$$H_A \quad = \quad \int dx A A_k J^{kl} \partial_l H_A \ . \tag{73}$$

where A denotes the projection of Kähler gauge potential to the 1-dimensional manifold in question.

2. The fermionic super-currents defining super-Hamiltonians and configuration space gamma matrices would be given by

$$J^K_A \Gamma_K \quad \equiv \quad \Gamma_A = H_{S,A} = \int dx \overline{\nu}_R j^k_A \Gamma_k \Psi \ . \tag{74}$$

$H^\dagger_{S,A}$ is obtained by Hermitian conjugation.

3. The anti-commutation relations would read as

$$\{\overline{\Psi}(x), \Gamma_k \Psi(y)\} = AA_k J^{kl} \partial_l H_A J^{rs} \Sigma_{rs} \delta(x,y) \quad . \tag{75}$$

The general formulas for the matrix elements of the configuration space metric and Kähler form are as for the non-stringy option.

8.5.3 String as the inverse image for image of critical line for zeros of zeta

Number theoretical argument suggests that 1-D dimensional delta function corresponds to the point set for which δM^4_+ projection corresponds to the line of non-trivial zeros for ζ: $z = \zeta(1/2+iy)$ that is intersection of X^2 with R_+. Thus stringy anti-commutation would be along R_+. In CP_2 the discrete set of points along which anticommutations would be given would be subset in S^2_{II}. Anti-commutativity on quantum critical set which corresponds to vacuum extremals would be indeed very natural.

In case of Riemann zeta one must consider also trivial zeros at $x = -2n$, $n = 1, 2....$ These would correspond to the integer powers of r^n for which the definition of inner product is problematic. Note however that for negative powers $-2n$ corresponding to zeros of ζ there are no problems if there is cutoff $r > r_0$.

The number theoretic counterpart of string would be most naturally a curve whose S^2_r projection belongs to the image of the critical line consisting of points $\zeta(1/2+iy)$. This image consist of the real axis of S^2 interpreted as compactified plane since ζ is real at the critical line. Note that in case of Riemann zeta also real axis is mapped to the real line so that it gives nothing new. Also this has a number theoretical justification since the basis $r^{1/2+iy}$, where r could correspond to the light-like coordinate of both $\delta M^4_\pm$ and partonic 3-surface, forms an orthogonal basis with respect to the inner product defined by the scaling invariant integration measure dx/x.

For number theoretical reasons which should be already clear, the values of y would be restricted to $y = \sum_k n_k y_k$ of imaginary parts of zeros of ζ. In the case of partonic 3-surface this would mean that eigenvalues of the modified Dirac operator would be of form $1/2 + i\sum_k n_k y_k$ and the number theoretical cutoff regularizing the Dirac determinant would emerge naturally. The important implication would be that not only q^{iy_k} but also y_k must be algebraic numbers. Note that the zeros of Riemann zeta at this line correspond to quantum criticality against phase transitions changing Planck constant meaning geometrically a leakage between different sectors of the imbedding space.

8.6 Number theoretic braids and global view about anti-commutations of induced spinor fields

The anti-commutations of induced spinor fields are reasonably well understood locally. The basic objects are 3-dimensional light-like 3-surfaces. These surfaces can be however seen as random light-like orbits of partonic 2-surfaces taking which would thus seem to take the role of fundamental dynamical objects. Conformal invariance in turn seems to make the 2-D partons 1-D objects and number theoretical braids in turn discretizes strings. And it also seems that the strands of number theoretic braid can in turn be discretized by considering the minima of Higgs potential in 3-D sense.

Somehow these apparently contradictory views should be unifiable in a more global view about the situation allowing to understand the reduction of effective dimension of the system as one goes to short scales. The notions of measurement resolution and number theoretic braid indeed provide the needed insights in this respect.

8.6.1 Anti-commutations of the induced spinor fields and number theoretical braids

The understanding of the number theoretic braids in terms of Higgs minima and maxima allows to gain a global view about anti-commutations. The coordinate patches inside which Higgs modulus is monotonically increasing function define a division of partonic 2-surfaces $X^2_t = X^3_l \cap \delta M^4_{\pm,t}$ to 2-D patches as a function of time coordinate of X^3_l as light-cone boundary is shifted in preferred time direction defined by the quantum critical sub-manifold $M^2 \times CP_2$. This induces similar division of the light-like 3-surfaces X^3_l to 3-D patches and there is a close analogy with the dynamics of ordinary 2-D landscape.

In both 2-D and 3-D case one can ask what happens at the common boundaries of the patches. Do the induced spinor fields associated with different patches anti-commute so that they would represent independent dynamical degrees of freedom? This seems to be a natural assumption both in 2-D and 3-D case and correspond to the idea that the basic objects are 2- *resp.* 3-dimensional in the resolution considered but this in a discretized sense due to finite measurement resolution, which is coded by the patch structure of X^3_l. A dimensional hierarchy results with the effective dimension of the basic objects increasing as the resolution scale increases when one proceeds from braids to the level of X^3_l.

If the induced spinor fields associated with different patches anti-commute, patches indeed define independent fermionic degrees of freedom at braid points and one has effective 2-dimensionality in discrete sense. In this picture the fundamental stringy curves for X^2_t correspond to the boundaries of 2-D patches and anti-commutation relations for the induced spinor fields can be formulated at these curves. Formally the conformal time evolution scaled down the boundaries of these patches. If anti-commutativity holds true at the boundaries of patches for spinor fields of neighboring patches, the patches would indeed represent independent degrees of freedom at stringy level.

The cutoff in transversal degrees of freedom for the induced spinor fields means cutoff $n \leq n_{max}$ for the conformal weight assignable to the holomorphic dependence of the induced spinor field on the complex coordinate. The dropping of higher conformal weights should imply the loss of the anti-commutativity of the induced spinor fields and its conjugate except at the points of the number theoretical braid. Thus the number theoretic braid should code for the value of n_{max}: the naive expectation is that for a given stringy curve the number of braid points equals to n_{max}.

8.6.2 The decomposition into 3-D patches and QFT description of particle reactions at the level of number theoretic braids

What is the physical meaning of the decomposition of 3-D light-like surface to patches? It would be very desirable to keep the picture in which number theoretic braid connects the incoming positive/negative energy state to the partonic 2-surfaces defining reaction vertices. This is not obvious if X_l^3 decomposes into causally independent patches. One can however argue that although each patch can define its own fermion state it has a vanishing net quantum numbers in zero energy ontology, and can be interpreted as an intermediate virtual state for the evolution of incoming/outgoing partonic state.

Another problem - actually only apparent problem - has been whether it is possible to have a generalization of the braid dynamics able to describe particle reactions in terms of the fusion and decay of braid strands. For some strange reason I had not realized that number theoretic braids naturally allow fusion and decay. Indeed, cusp catastrophe is a canonical representation for the fusion process: cusp region contains two minima (plus maximum between them) and the complement of cusp region single minimum. The crucial control parameter of cusp catastrophe corresponds to the time parameter of X_l^3. More concretely, two valleys with a mountain between them fuse to form a single valley as the two real roots of a polynomial become complex conjugate roots. The continuation of light-like surface to slicing of X^4 to light-like 3-surfaces would give the full cusp catastrophe.

In the catastrophe theoretic setting the time parameter of X_l^3 appears as a control variable on which the roots of the polynomial equation defining minimum of Higgs depend: the dependence would be given by a rational function with rational coefficients.

This picture means that particle reactions occur at several levels which brings in mind a kind of universal mimicry inspired by Universe as a Universal Computer hypothesis. Particle reactions in QFT sense correspond to the reactions for the number theoretic braids inside partons. This level seems to be the simplest one to describe mathematically. At parton level particle reactions correspond to generalized Feynman diagrams obtained by gluing partonic 3-surfaces along their ends at vertices. Particle reactions are realized also at the level of 4-D space-time surfaces. One might hope that this multiple realization could code the dynamics already at the simple level of single partonic 3-surface.

8.6.3 About 3-D minima of Higgs potential

The dominating contribution to the modulus of the Higgs field comes from $\delta M^4_\pm$ distance to the axis R_+ defining quantization axis. Hence in scales much larger than CP_2 size the geometric picture is quite simple. The orbit for the 2-D minimum of Higgs corresponds to a particle moving in the vicinity of R_+ and minimal distances from R_+ would certainly give a contribution to the Dirac determinant. Of course also the motion in CP_2 degrees of freedom can generate local minima and if this motion is very complex, one expects large number of minima with almost same modulus of eigenvalues coding a lot of information about X^3_l.

It would seem that only the most essential information about surface is coded: the knowledge of minima and maxima of height function indeed provides the most important general coordinate invariant information about landscape. In the rational category where X^3_l can be characterized by a finite set of rational numbers, this might be enough to deduce the representation of the surface.

What if the situation is stationary in the sense that the minimum value of Higgs remains constant for some time interval? Formally the Dirac determinant would become a continuous product having an infinite value. This can be avoided by assuming that the contribution of a continuous range with fixed value of Higgs minimum is given by the contribution of its initial point: this is natural if one thinks the situation information theoretically. Physical intuition suggests that the minima remain constant for the maxima of Kähler function so that the initial partonic 2-surface would determine the entire contribution to the Dirac determinant.

8.6.4 How generalized braid diagrams relate to the perturbation theory?

The association of generalized braid diagrams to incoming and outgoing partonic legs and possibly also vertices of the generalized Feynman diagrams forces to ask whether the generalized braid diagrams could give rise to a counterpart of perturbation theoretical formalism via the functional integral over configuration space degrees of freedom.

The question is how the functional integral over configuration space degrees of freedom relates to the generalized braid diagrams. The basic conjecture motivated also number theoretically is that radiative corrections in this sense sum up to zero for critical values of Kähler coupling strength and Kähler function codes radiative corrections to classical physics via the dependence of the scale of M^4 metric on Planck constant. Cancellation occurs only for critical values of Kähler coupling strength α_K: for general values of α_K cancellation would require separate vanishing of each term in the sum and does not occur.

This would mean following.

1. One would not have perturbation theory around a given maximum of Kähler function but as a sum over increasingly complex maxima of Kähler function. Radiative corrections in the sense of perturbative functional integral around a given maximum would vanish (so that the expansion in terms of braid topologies would not make sense around single maximum). Radiative corrections would not vanish in the sense of a sum over

3-topologies obtained by adding radiative corrections as zero energy states in shorter time scale.

2. Connes tensor product with a given measurement resolution would correspond to a restriction on the number of maxima of Kähler function labelled by the braid diagrams. For zero energy states in a given time scale the maxima of Kähler function could be assigned to braids of minimal complexity with braid vertices interpreted in terms of an addition of radiative corrections. Hence a connection with QFT type Feyman diagram expansion would be obtained and the Connes tensor product would have a practical computational realization.

3. The cutoff in the number of topologies (maxima of Kähler function contributing in a given resolution defining Connes tensor product) would be always finite in accordance with the algebraic universality.

4. The time scale resolution defined by the temporal distance between the tips of the causal diamond defined by the future and past light-cones applies to the addition of zero energy sub-states and one obtains a direct connection with p-adic length scale evolution of coupling constants since the time scales in question naturally come as negative powers of two. More precisely, p-adic primes near power of two are very natural since the coupling constant evolution comes in powers of two of fundamental 2-adic length scale.

There are still some questions. Radiative corrections around given 3-topology vanish. Could radiative corrections sum up to zero in an ideal measurement resolution also in 2-D sense so that the initial and final partonic 2-surfaces associated with a partonic 3-surface of minimal complexity would determine the outcome completely? Could the 3-surface of minimal complexity correspond to a trivial diagram so that free theory would result in accordance with asymptotic freedom as measurement resolution becomes ideal?

The answer to these questions seems to be 'No'. In the p-adic sense the ideal limit would correspond to the limit $p \rightarrow 0$ and since only $p \rightarrow 2$ is possible in the discrete length scale evolution defined by primes, the limit is not a free theory. This conforms with the view that CP_2 length scale defines the ultimate UV cutoff.

8.6.5 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

One can wonder how this picture relates to the earlier hypothesis that p-adic length coupling constant evolution is coded to the hypothesized $log(p)$ normalization of the eigenvalues of the modified Dirac operator D. There are objections against this normalization. $log(p)$ factors are not number theoretically favored and one could consider also other dependencies on p. Since the eigenvalue spectrum of D corresponds to the values of Higgs expectation at points of partonic

2-surface defining number theoretic braids, Higgs expectation would have $log(p)$ multiplicative dependence on p-adic length scale, which does not look attractive.

Is there really any need to assume this kind of normalization? Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5\ \mu$m (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3.

8.6.6 How quantum classical correspondence is realized at parton level?

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or a boundary term of YM action associated with a particle carrying gauge charges of the quantum state. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography. The difference between Chern-Simons action characterizing quantum state and the fundamental Chern-Simons type factor associated with the Kähler form would be that the latter emerges as the phase of the Dirac determinant.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

9 Super-symmetries at space-time and configuration space level

The first difference between TGD and standard conformal field theories and string models is that super-symmetry generators acting as configuration space gamma matrices acting as super generators carry either lepton or quark number. Only the anti-commutators of quark like generators expressible in terms of Hamiltonians H_A of $X^3_l \times CP_2$ can contribute to the super-symmetrization of the Poisson algebra and thus to CH metric via Poisson central extension, whereas leptonic generators, which are proportional to $j^{Ak}\Gamma_k$ can contribute to the super-symmetrization of the function algebra of CH. Quarks correspond to N-S type representations and kappa symmetry of string models whereas leptons correspond to Ramond type representations and ordinary super-symmetry.

Also Super Kac-Moody invariance allows lepton-quark dichotomy. What forces to assign leptons with Ramond representation is that covariantly constant neutrino must correspond to one conformal mode ($z^n, n = 0$). The p-adic mass calculations [6] carried for more than decade ago led to the same assignment on physical grounds: p-adic mass calculations also forced to include $SO(3,1)$ besides M^4 a tensor factor to super-conformal representations, which in recent context suggests that causal determinants $X_l^3 \times CP_2$, $X_l^3 \subset M^4$ an arbitrary light like 3-surface rather than just a translate of δM_+^4, must be allowed. Also now the lepton-quark, Ramond-NS and SUSY-kappa dichotomies correspond to one and same dichotomy so that the general structure looks quite satisfactory although it must be admitted that it is based on heuristic guess work.

Second deep difference is the appearance of the zeros of Riemann Zeta as conformal weights of the generating elements of the super-canonical algebra and the expected action of conformal algebra associated with 3-D CDS as a spectral flow in the space of super-canonical conformal weights inducing a mere gauge transformation infinitesimally and a braiding action in topological degrees of freedom.

In this section the relationship of Super Kac-Moody invariance to ordinary super-conformal symmetry and the interaction between Super-Kac Moody and super-canonical symmetries are discussed. For years the role of quaternions and octonions in TGD has been under an active speculation. These aspects are considered in [E2], where the number theoretic equivalent of spontaneous compactification is proposed. The conjecture states that space-time surfaces can be regarded either as 4-surfaces in $M^4 \times CP_2$ or as hyper-quaternionic 4-surfaces in the space $HO = M^8$ possessing hyper-octonionic structure (the attribute 'hyper' means that imaginary units are multiplied by $\sqrt{-1}$ in order to achieve number theoretic norm with Minkowskian signature).

9.1 Super-canonical and Super Kac-Moody symmetries

The proper understanding of super symmetries has turned out to be crucial for the understanding of quantum TGD and it seems that the mis-interpreted super-symmetries are one of the basic reasons for the difficulties of super string models too. At this moment one can fairly say that the construction of the configuration space spinor structure reduces to a purely group theoretical problem of constructing representations for the super generators of the super-canonical algebra of CP_2 localized with respect to $\delta M_\pm^4$ in terms of second quantized induced spinor fields.

9.1.1 Super canonical symmetries

One can imagine two kinds of causal determinants besides $\delta M_+^4 \times CP_2$. In principle all surfaces $X_l^3 \times CP_2$, where X_l^3 is a light like 3-surface of M^4, could act as effective causal determinants: the reason is that the creation of pairs of positive and negative energy space-time sheets is possible at these surfaces.

There are good hopes that the super-canonical and super-conformal symmetries associated with δX_l^3 allow to generalize the construction of the configuration space geometry performed at $\delta M_+^4 \times CP_2$. If X_l^3 can be restricted to be unions of future and past light cone boundaries, the generalization is more or less trivial: one just forms a union of configuration spaces associated with unions of translates of δM_+^4 and δM_-^4.

As explained in the previous chapter, one can understand how the causal determinants $X_l^3 \times CP_2$ emerge from the facts that space-time sheets with negative time orientation carry negative energy and that the most elegant theory results when the net quantum numbers and conserved classical quantities vanish for the entire Universe. Crossing symmetry allows consistency with elementary particle physics and the identification of gravitational 4-momentum as difference of conserved inertial (Poincare) 4-momenta for positive and negative energy matter provides consistency with macroscopic physics.

The emergence of these additional causal determinants means that super-canonical symmetries become microscopic, rather than only cosmological, symmetries commuting with Poincare transformations exactly for $M^4 \times CP_2$ and apart from small quantum gravitational effects for $M_+^4 \times CP_2$. Super-canonical symmetry differs in many respects from Kac-Moody symmetries of particle physics, which in fact correspond to the conformal invariance associated with the modified Dirac action and correspond to the product of Poincare, electro-weak and color groups. It seems that these symmetries are dually related.

9.1.2 Super Kac-Moody symmetries associated with the light like causal determinants

Also the light like 3-surfaces X_l^3 of H defining elementary particle horizons at which Minkowskian signature of the metric is changed to Euclidian and boundaries of space-time sheets can act as causal determinants, and thus contribute to the configuration space metric. In this case the symmetries correspond to the isometries of the imbedding space localized with respect to the complex coordinate of the 2-surface X^2 determining the light like 3-surface X_l^3 so that Kac-Moody type symmetry results. Also the condition $\sqrt(g_3) = 0$ for the determinant of the induced metric seems to define a conformal symmetry associated with the light like direction. This conforms with duality since also the 7-D causal determinants $X_l^3 \times CP_2$ allow both radial and transversal conformal symmetry.

Good candidate for the counterpart of this symmetry in the interior of space-time surface is hyper-quaternion conformal invariance [E2]. All that is needed for these symmetries to be equivalent that the spaces of super-gauge degrees of freedom defined by them are equivalent. Kac Moody generators and their super counterparts can be associated with the 3-D light like CDs.

If is enough to localize only the H-isometries with respect to X_l^3, the purely bosonic part of the Kac-Moody algebra corresponds to the isometry group $M^4 \times SO(3,1) \times SU(3)$. The physical interpretation of these symmetries is not so obvious as one might think. The point is that one can generalize the formulas characterizing the action of infinitesimal isometries on spinor fields of finite-dimensional Kähler manifold to the level of the configuration space. This gives rise to bosonic generators containing also a sigma-matrix term bilinear in fermionic oscillator operators. This representation is not equivalent with the purely fermionic representations provided by induced Dirac action. Thus one has two groups of local color charges and the challenge is to find a physical interpretation for them. The following arguments fix the identification.

1. The hint comes from the fact that $U(2)$ in the decomposition $CP_2 = SU(3)/U(2)$ corresponds in a well-defined sense electro-weak algebra identified as a holonomy algebra of the spinor connection. Hence one could argue that the $U(2)$ generators of either $SU(3)$ algebra might be identifiable as generators of local $U(2)_{ew}$ gauge transformations whereas non-diagonal generators would correspond to Higgs field. This interpretation would conform with the idea that Higgs field is a genuine scalar field rather than a composite of fermions.

2. Since X_l^3-local $SU(3)$ transformations represented by fermionic currents are characterized by central extension they would naturally correspond to the electro-weak gauge algebra and Higgs bosons. This is also consistent with the fact that both leptons and quarks define fermionic Kac Moody currents.

3. The fact that only quarks appear in the gamma matrices of the configuration space supports the view that action of the generators of X_l^3-local color transformations on configuration space spinor fields represents local color transformations. If the action of X_l^3-local $SU(3)$ transformations on configuration space spinor fields has trivial central extension term the identification as a representation of local color symmetries is possible.

The topological explanation of the family replication phenomenon is based on an assignment of 2-dimensional boundary to a 3-surface characterizing the elementary particle. The precise identification of this surface has remained open and one possibility is that the 2-surface X^2 defining the light light-like surface associated with an elementary particle horizon is in question. This assumption would conform with the notion of elementary particle vacuum functionals defined in the zero modes characterizing different conformal equivalences classes for X^2.

9.1.3 The relationship of the Super-Kac Moody symmetry to the standard super-conformal invariance

Super-Kac Moody symmetry can be regarded as $N = 4$ complex super-symmetry with complex H-spinor modes of H representing the 4 physical helicities of 8-component leptonic and quark like spinors acting as generators of complex dynamical super-symmetries. The super-symmetries generated by the covariantly constant right handed neutrino appear with *both* M^4 helicities: it however seems that covariantly constant neutrino does not generate any global super-symmetry in the sense of particle-sparticle mass degeneracy. Only righthanded neutrino spinor modes (apart from covariantly constant mode) appear in the expressions of configuration space gamma matrices forming a subalgebra of the full super-algebra.

$N = 2$ real super-conformal algebra is generated by the energy momentum tensor $T(z)$, $U(1)$ current $J(z)$, and super generators $G^{\pm}(z)$ carrying $U(1)$ charge. Now $U(1)$ current would correspond to right-handed neutrino number and super generators would involve contraction of covariantly constant neutrino spinor with second quantized induced spinor field. The further facts that $N = 2$ algebra is associated naturally with Kähler geometry, that the partition functions associated with $N = 2$ super-conformal representations are modular invariant, and that $N = 2$ algebra defines so called chiral ring defining a topological quantum field theory [40], lend a further support for the belief that $N = 2$ super-conformal algebra acts in super-canonical degrees of freedom.

The values of c and conformal weights for $N = 2$ super-conformal field theories are given by

$$\begin{aligned} c &= \frac{3k}{k+2} \ , \\ \Delta_{l,m}(NS) &= \frac{l(l+2)-m^2}{4(k+2)} \ , \ \ l = 0,1,...,k \ , \\ q_m &= \frac{m}{k+2} \ , \ \ m = -l,-l+2,....,l-2,l \ . \end{aligned} \tag{76}$$

q_m is the fractional value of the $U(1)$ charge, which would now correspond to a fractional fermion number. For $k = 1$ one would have $q = 0, 1/3, -1/3$, which brings in mind anyons. $\Delta_{l=0,m=0} = 0$ state would correspond to a massless state with a vanishing fermion number. Note that $SU(2)_k$ Wess-Zumino model has the same value of c but different conformal weights. More information about conformal algebras can be found from the appendix of [40].

For Ramond representation $L_0 - c/24$ or equivalently G_0 must annihilate the massless states. This occurs for $\Delta = c/24$ giving the condition $k = 2\left[l(l+2) - m^2\right]$ (note that k must be even and that $(k, l, m) = (4, 1, 1)$ is the simplest non-trivial solution to the condition). Note the appearance of a fractional vacuum fermion number $q_{vac} = \pm c/12 = \pm k/4(k + 2)$. I have proposed that NS and Ramond algebras could combine to a larger algebra containing also lepto-quark type generators but this not necessary.

The conformal algebra defined as a direct sum of Ramond and NS $N = 4$ complex sub-algebras associated with quarks and leptons might further extend to a larger algebra if lepto-quark generators acting effectively as half odd-integer Virasoro generators can be allowed. The algebra would contain spin and electro-weak spin as fermionic indices. Poincare and color Kac-Moody generators would act as symplectically extended isometry generators on configuration space Hamiltonians expressible in terms of Hamiltonians of $X^3_l \times CP_2$. Electro-weak and color Kac-Moody currents have conformal weight $h = 1$ whereas T and G have conformal weights $h = 2$ and $h = 3/2$.

The experience with $N = 4$ complex super-conformal invariance suggests that the extended algebra requires the inclusion of also second quantized induced spinor fields with $h = 1/2$ and their super-partners with $h = 0$ and realized as fermion-antifermion bilinears. Since G and Ψ are labelled by 2×4 spinor indices, super-partners would correspond to $2 \times (3+1) = 8$ massless electro-weak gauge boson states with polarization included. Their inclusion would make the theory highly predictive since induced spinor and electro-weak fields are the fundamental fields in TGD.

9.1.4 How could conformal symmetries of light like 3-D CDs act on super-canonical degrees of freedom?

An important challenge is to understand the action of super-conformal symmetries associated with the light like 3-D CDs on super-canonical degrees of freedom. The breakthrough in this respect via the algebraic formulation for the vision about vanishing loop corrections of ordinary Feynman diagrams in terms of equivalence of generalized Feynman diagrams with loops with tree diagrams [C7]. The formulation involves Yang-Baxter equations, braid groups, Hopf algebras, and so called ribbon categories and led to the following vision. The original formulation to be discussed in this sub-subsection is very heuristic and a more quantitative formulation follows in the next subsection.

1. Quantum classical correspondence suggests that the complex conformal weights of super-canonical algebra generators have space-time counterparts. The proposal is that the weights are mapped to the points of the homologically non-trivial geodesic sphere S^2 of CP_2 corresponds to the super-canonical conformal weights, and corresponds to a discrete set of points at the space-time surface. These points would also label mutually commutating R-matrices. The map is completely analogous to the map of momenta of quantum particles to the points of celestial sphere. These points would belong to a "time=constant" section of 2-dimensional "space-time", presumably circle, defining physical states of a two-dimensional conformal field theory for which the scaling operator L_0 takes the role of Hamiltonian.

2. One could thus regard super-generators as conformal fields in space-time or complex plane having super-canonical conformal weights as punctures. The action of super-conformal algebra and braid group on these points realizing monodromies of conformal field theories [40] would induce by a pull-back a braid group action on the super-canonical conformal weights of configuration space gamma matrices (super generators) and corresponding isometry generators.

At the first sight the explicit realization of super-canonical and Kac Moody generators seems however to be in conflict with this vision. The interaction of the conformal algebra of X_l^3 on super-canonical algebra is a pure gauge interaction since the definition of super canonical generators is not changed by the action of conformal transformations of X_l^3. This is however consistent with the assumption that the action defined by the quantum-classical correspondence is also a pure gauge interaction locally. The braiding action would be analogous to the holonomies encountered in the case of non-Abelian gauge fields with a vanishing curvature in spaces possessing non-trivial first homotopy group.

Quantum classical correspondence would allow to map abstract configuration space level to space-time level.

1. The complex argument z of Kac Moody and Virasoro algebra generators $T(z) = \sum T_n z^n$ would be discretized so that it would have values on the set of supercanonical conformal weights corresponding to the space t in the Cartan decomposition $g = t + h$ of the tangent space of the configuration space. These points could be interpreted as punctures of the complex plane restricted to the lines $Re(z) = \pm 1/2$ and positive real axis if zeros of Riemann zeta define the conformal weights.

2. The vacuum expectation values of the enveloping algebra of the super-canonical algebra would reduce to n-point functions of a super-conformal quantum field theory in the complex plane containing infinite number of punctures defined by the super-canonical conformal weights, for which primary fields correspond to the representations of $SO(3)\times SU(3)$. These representations would combine to form infinite-dimensional representations of super-canonical algebra. The presence of the gigantic super-canonical symmetries raises the hope that quantum TGD could be solvable to a very high degree.

3. The Super Virasoro algebra and Super Kac Moody algebra associated with 3-D light like CDs would act as symmetries of this theory and the S-matrix of TGD would involve the n-point functions of this field theory. By 7–3 duality this indeed makes sense. The situation would reduce to that encountered in WZW theory in the sense that one would have space-like 3-surfaces X^3 containing two-dimensional closed surfaces carrying representations of Super Kac-Moody algebra.

This picture also justifies the earlier proposal that configuration space Clifford algebra defined by the gamma matrices acting as super generators defines an infinite-dimensional von Neumann algebra possessing hierarchies of type II_1 factors [41] having a close connection with the non-trivial representations of braid group and quantum groups. The sequence of non-trivial zeros of Riemann Zeta along the line $Re(s) = 1/2$ in the plane of conformal weights could be regarded an an infinite braid behind the von Neumann algebra [41]. Contrary to the expectations, also trivial zeros seem to be important. The finite braids defined by subsets of zeros, and also superpositions of non-trivial zeros of form $1/2+\sum_i y_i$, could be seen as a hierarchy of completely integrable 1-dimensional spin chains leading to quantum groups and braid groups [39, 40] naturally.

It seems that not only Riemann's zeta but also polyzetas [42, 43, 44, 45] could play a fundamental role in TGD Universe. The super-canonical conformal weights of interacting particles, in particular of those forming bound states, are expected to have "off mass shell" values. An attractive hypothesis is that they correspond to zeros of Riemann's polyzetas. Interaction would allow quite concretely the realization of braiding operations dynamically. The physical justification for the hypothesis would be quantum criticality. Indeed, it has been found that the loop corrections of quantum field theory are expressible in terms of polyzetas [46]. If the arguments of polyzetas correspond to conformal weights of particles of many-particle bound state, loop corrections vanish when the super-canonical conformal weights correspond to the zeros of polyzetas including zeta.

9.2 The relationship between super-canonical and Super Kac-Moody algebras, Equivalence Principle, and justification of p-adic thermodynamics

The relationship between super-canonical algebra (SC) acting at light-cone boundary and Super Kac-Moody algebra (SKM) acting on light-like 3-surfaces has remained somewhat enigmatic due to the lack of physical insights. This is not the only problem. The question to precisely what extent Equivalence Principle (EP) remains true in TGD framework and what might be the precise mathematical realization of EP is waiting for an answer. Also the justification of p-adic thermodynamics for the scaling generator L_0 of Virasoro algebra -in obvious conflict with the basic wisdom that this generator should annihilate physical states- is lacking. It seems that these three problems could have a common solution.

Before going to describe the proposed solution, some background is necessary. The latest proposal for $SC-SKM$ relationship relies on non-standard and therefore somewhat questionable assumptions.

1. SKM Virasoro algebra (SKMV) and SC Virasoro algebra (SCV) (anti)commute for physical states.

2. SC algebra generates states of negative conformal weight annihilated by SCV generators L_n, $n<0$, and serving as ground states from which SKM generators create states with non-negative conformal weight.

This picture could make sense for elementary particles. On other hand, the recent model for hadrons [F4] assumes that SC degrees of freedom contribute about 70 per cent to the mass of hadron but at space-time sheet different from those assignable to quarks. The contribution of SC degrees of freedom to the thermal average of the conformal weight would be positive. A contradiction results unless one assumes that there exists also SCV ground states with positive conformal weight annihilated by SCV elements L_n, $n<0$, but also this seems implausible.

9.2.1 New vision about the relationship between SCV and $SKMV$

Consider now the new vision about the relationship between SCV and $SKMV$.

1. The isometries of H assignable with SKM are also symplectic transformations [B3] (note that I have used the term canonical instead of symplectic previously). Hence might consider the possibility that SKM could be identified as a subalgebra of SC. If this makes sense, a generalization of the coset construction obtained by replacing finite-dimensional Lie group with infinite-dimensional symplectic group suggests itself. The differences of SCV and $SKMV$ elements would annihilate physical states and (anti)commute with $SKMV$. Also the generators O_n, $n > 0$, for both algebras would annihilate the physical states so that the differences of the elements would annihilate automatically physical states for $n > 0$.

2. The super-generator G_0 contains the Dirac operator D of H. If the action of SCV and $SKMV$ Dirac operators on physical states are identical then cm of degrees of freedom disappear from the differences $G_0(SCV) - G_0(SKMV)$ and $L_0(SCV) - L_0(SKMV)$. One could interpret the identical action of the Dirac operators as the long sought-for precise realization of Equivalence Principle (EP) in TGD framework. EP would state that the total inertial four-momentum and color quantum numbers assignable to SC (imbedding space level) are equal to the gravitational four-momentum and color quantum numbers assignable to SKM (space-time level). Note that since super-canonical transformations correspond to the isometries of the "world of classical worlds" the assignment of the attribute "inertial" to them is natural.

3. The analog of coset construction applies also to SKM and SC algebras which means that physical states can be thought of as being created by an operator of SKM carrying the conformal weight and by a genuine SC operator with vanishing conformal weight. Therefore the situation does not reduce to that encountered in super-string models.

4. The reader can recognize $SC - SKM$ as a precise formulation for $7-3$ duality discussed in the section *About dualities and conformal symmetries in TGD framework* stating that 3-D light-like causal determinants and 7-D causal determinants $\delta M^4_\pm \times CP_2$ are equivalent.

9.2.2 Consistency with p-adic thermodynamics

The consistency with p-adic thermodynamics provides a strong reality test and has been already used as a constraint in attempts to understand the super-conformal symmetries in partonic level.

1. In physical states the p-adic thermal expectation value of the SKM and SC conformal weights would be non-vanishing and identical and mass squared could be identified to the expectation value of SKM scaling generator L_0. There would be no need to give up Super Virasoro conditions for $SCV - SKMV$.

2. There is consistency with p-adic mass calculations for hadrons [F4] since the non-perturbative SC contributions and perturbative SKM contributions to the mass correspond to space-time sheets labeled by different p-adic primes. The earlier statement that SC is responsible for the dominating non-perturbative contributions to the hadron mass transforms to a statement reflecting $SC-SKM$ duality. The perturbative quark contributions to hadron masses can be calculated most conveniently by using p-adic thermodynamics for SKM whereas non-perturbative contributions to hadron masses can be calculated most conveniently by using p-adic thermodynamics for SC. Also the proposal that the exotic analogs of baryons resulting when baryon looses its valence quarks [F5] remains intact in this framework.

3. The results of p-adic mass calculations depend crucially on the number N of tensor factors contributing to the Super-Virasoro algebra. The required number is $N=5$ and during years I have proposed several explanations for this number. It seems that holonomic contributions that is electro-weak and spin contributions must be regarded as contributions separate from those coming from isometries. SKM algebras in electro-weak degrees and spin degrees of of freedom, would give 2+1=3 tensor factors corresponding to $U(2)_{ew}\times SU(2)$. $SU(3)$ and $SO(3)$ (or $SO(2)\subset SO(3)$ leaving the intersection of light-like ray with S^2 invariant) would give 2 additional tensor factors. Altogether one would indeed have 5 tensor factors.

There are some further questions which pop up in mind immediately.

1. Why mass squared corresponds to the thermal expectation value of the net conformal weight? This option is forced among other things by Lorentz invariance but it is not possible to provide a really satisfactory answer to this question yet. In the coset construction there is no reason to require that the mass squared equals to the integer value conformal weight for SKM algebra. This allows the possibility that mass squared has same value for states with different values of SKM conformal weights appearing in the thermal state and equals to the average of the conformal weight.

   The coefficient of proportionality can be however deduced from the observation that the mass squared values for CP_2 Dirac operator correspond to definite values of conformal weight in p-adic mass calculations. It is indeed possible to assign to partonic 2-surface X^2 CP_2 partial waves correlating strongly with the net electro-weak quantum numbers of the parton so that the assignment of ground state conformal weight to CP_2 partial waves makes sense. In the case of M^4 degrees of freedom it is not possible to talk about momentum eigen states since translations take parton out of δH_+ so that momentum must be assigned with the tip of the light-cone containing the particle.

2. The additivity of conformal weight means additivity of mass squared at parton level and this has been indeed used in p-adic mass calculations.

This implies the conditions

$$(\sum_i p_i)^2 = \sum_i m_i^2 \quad (77)$$

The assumption $p_i^2 = m_i^2$ makes sense only for massless partons moving collinearly. In the QCD based model of hadrons only longitudinal momenta and transverse momentum squared are used as labels of parton states, which together with the presence of preferred plane M^2 would suggest that one has

$$\begin{aligned} p_{i,||}^2 &= m_i^2 \ , \\ -\sum_i p_{i,\perp}^2 + 2\sum_{i,j} p_i \cdot p_j &= 0 \ . \end{aligned} \quad (78)$$

The masses would be reduced in bound states: $m_i^2 \rightarrow m_i^2 - (p_T^2)_i$. This could explain why massive quarks can behave as nearly massless quarks inside hadrons.

3. Single particle super-canonical conformal weights can have also imaginary part, call it y. The question is what complex mass squared means physically. Complex conformal weights have been assigned with an inherent time orientation distinguishing positive energy particle from negative energy antiparticle (in particular, phase conjugate photons from ordinary photons). This suggests an interpretation of y in terms of a decay width. p-Adic thermodynamics suggest that y vanishes for states with vanishing conformal weight (mass square

4. and that the measured value of y is a p-adic thermal average with non-vanishing contributions from states with mass of order CP_2 mass. This makes sense if y_k are algebraic or perhaps even rational numbers.

   For instance, if a massless state characterized by p-adic prime p has p-adic thermal average $y = psy_k$, where s is the denominator of rational valued $y_k = r/s$, the lowest order contribution to the decay width is proportional to $1/p$ by the basic rules of p-adic mass calculations and the decay rate is of same order of magnitude as mass. If the p-adic thermal average of y is of form $p^n y_k$ for massless state then a decay width of order $\Gamma \sim p^{-(n-1)/2}m$ results. For electron n should be rather large. This argument generalizes trivially to the case in which massless state has vanishing value of y.

9.2.3 Can SKM be lifted to a sub-algebra of SC?

A picture introducing only a generalization of coset construction as a new element, realizing mathematically Equivalence Principle, and justifying p-adic thermodynamics is highly attractive but there is a problem. SKM is defined at light-like 3-surfaces X^3 whereas SC acts at light-cone boundary $\delta H_\pm = \delta M^4_\pm \times CP_2$. One should be able to lift SKM to imbedding space level somehow. Also SC should be lifted to entire H. This problem was the reason why I gave up the idea about coset construction and $SC-SKM$ duality as it appeared for the first time.

A possible solution of the lifting problem comes from the observation making possible a more rigorous formulation of $HO - H$ duality stating that one can regard space-time surfaces either as surfaces in hyper-octonionic space $HO = M^8$ or in $H = M^4 \times CP_2$ [C1, E2]. Consider first the formulation of $HO - H$ duality.

1. Associativity also in the number theoretical sense becomes the fundamental dynamical principle if $HO-H$ duality holds true [E2]. For a space-time surface $X^4 \subset HO = M^8$ associativity is satisfied at space-time level if the tangent space at each point of X^4 is some hyper-quaternionic sub-space $HQ = M^4 \subset M^8$. Also partonic 2-surfaces at the boundaries of causal diamonds formed by pairs of future and past directed light-cones defining the basic imbedding space correlate of zero energy state in zero energy ontology and light-like 3-surfaces are assumed to belong to $HQ = M^4 \subset HO$.

2. $HO - H$ duality requires something more. If the tangent spaces contain the same preferred commutative and thus hyper-complex plane $HC = M^2$, the tangent spaces of X^4 are parameterized by the points s of CP_2 and $X^4 \subset HO$ can be mapped to $X^4 \subset M^4 \times CP_2$ by assigning to a point of X^4 regarded as point (m, e) of $M^4_0 \times E^4 = M^8$ the point (m, s). Note that one must also fix a preferred global hyper-quaternionic subspace $M^4_0 \subset M^8$ containing M^2 to be not confused with the local tangent planes M^4.

3. The preferred plane M^2 can be interpreted as the plane of non-physical polarizations so that the interpretation as a number theoretic analog of gauge conditions posed in both quantum field theories and string models is possible.

4. An open question is whether the resulting surface in H is a preferred extremal of Kähler action. This is possible since the tangent spaces at light-like partonic 3-surfaces are fixed to contain M^2 so that the boundary values of the normal derivatives of H coordinates are fixed and field equations fix in the ideal case X^4 uniquely and one obtains space-time surface as the analog of Bohr orbit.

5. The light-like "Higgs term" proportional to $O = \gamma_k t^k$ appearing in the generalized eigenvalue equation for the modified Dirac operator is an essential element of TGD based description of Higgs mechanism. This term can cause complications unless t is a covariantly constant light-like vector. Covariant constancy is achieved if t is constant light-like vector in M^2. The interpretation as a space-time correlate for the light-like 4-momentum assignable to the parton might be considered.

6. Associativity requires that the hyper-octonionic arguments of N-point functions in HO description are restricted to a hyperquaternionic plane $HQ = M^4 \subset HO$ required also by the $HO - H$ correspondence. The intersection $M^4 \cap int(X^4)$ consists of a discrete set of points in the generic case. Partonic 3-surfaces are assumed to be associative and belong to M^4. The set of commutative points at the partonic 2-surface X^2 is discrete in the generic case whereas the intersection $X^3 \cap M^2$ consists of 1-D curves so that the notion of number theoretical braid crucial for the p-adicization of the theory as almost topological QFT is uniquely defined.

7. The preferred plane $M^2 \subset M^4 \subset HO$ can be assigned also to the definition of N-point functions in HO picture. It is not clear whether it must be same as the preferred planes assigned to the partonic 2-surfaces. If not, the interpretation would be that it corresponds to a plane containing the over all cm four-momentum whereas partonic planes M_i^2 would contain the partonic four-momenta. M^2 is expected to change at wormhole contacts having Euclidian signature of the induced metric representing horizons and connecting space-time sheets with Minkowskian signature of the induced metric.

The presence of globally defined plane M^2 and the flexibility provided by the hyper-complex conformal invariance raise the hopes of achieving the lifting of SC and SKM to H. At the light-cone boundary the light-like radial coordinate can be lifted to a hyper-complex coordinate defining coordinate for M^2. At X^3 one can fix the light-like coordinate varying along the braid strands can be lifted to some hyper-complex coordinate of M^2 defined in the interior of X^4. The total four-momenta and color quantum numbers assignable to the SC and SKM degrees of freedom are naturally identical since they can be identified as the four-momentum of the partonic 2-surface $X^2 \subset X^3 \cap \delta M^4_\pm \times CP_2$. Equivalence Principle would emerge as an identity.

9.2.4 Questions about conformal weights

One can pose several non-trivial questions about conformal weights.

1. The negative SKM conformal weights of ground states for elementary particles [F3] remain to be understood in this framework. In the case of light-cone boundary the natural value for ground state conformal weight of a scalar field is $-1/2$ since this implies a complete analogy with a plane wave with respect to the radial light-like coordinate r_M with inner product defined by a scale invariant integration measure dr_M/r_M. If the coset construction works same should hold true for SKM degrees of freedom for a proper choice of the light-like radial coordinate. There are thus good hopes that negative ground state conformal weights could be understood.

2. Further questions relate to the imaginary parts of ground state conformal weights, which can be vanishing in principle. Do the ground state

conformal weights correspond to the zeros of some zeta function- most naturally the zeta function defined by generalized eigenvalues of the modified Dirac operator and satisfying Riemann hypothesis so that ground state conformal weight would have real part -1/2? Do SC and SKM have same spectrum of complex conformal weights as the coset construction suggests? Does the imaginary part of the conformal weight bring in a new degree of freedom having interpretation in terms of space-time correlate for the arrow of time with the generalization of the phase conjugation of laser physics representing the reversal of the arrow of geometric time?

3. The opposite light-cone boundaries of the causal diamond bring in mind the hemispheres of S^2 in ordinary conformal theory. In ordinary conformal theory positive/negative powers of z correspond to these hemispheres. Could it be that the radial conformal weights are of opposite sign and of same magnitude for the positive and negative energy parts of zero energy state?

9.2.5 Further questions

There are still several open questions.

1. Is it possible to define hyper-quaternionic variants of the superconformal algebras in both H and HO or perhaps only in HO. A positive answer to this question would conform with the conjecture that the geometry of "world of classical worlds" allows Hyper-Kähler property in either or both pictures [B3].

2. How this picture relates to what is known about the extremals of field equations [D1] characterized by generalized Hamilton-Jacobi structure bringing in mind the selection of preferred M^2?

3. Is this picture consistent with the views about Equivalence Principle and its possible breaking based on the identification of gravitational four-momentum in terms of Einstein tensor is interesting [D3]?

9.3 Brief summary of super-conformal symmetries in partonic picture

The notion of conformal super-symmetry is very rich and involves several non-trivial aspects, and as the following discussions shows, one could assign the attribute super-conformal to several symmetries. In the following I try to sum up what I see as important. What is new is that it is now possible to tie everything to the fundamental description in terms of the parton level action principle and provide a rigorous justification and precise realization for the claimed super-conformal symmetries.

9.3.1 Super-canonical symmetries

Super-canonical symmetries correspond to the isometries of the configuration space CH (the world of classical worlds) and are induced from the corresponding symmetries of $\delta H_\pm \equiv \delta M^4_\pm \times CP_2$. The explicit representations have been constructed for both 2-D and stringy options. The most stringent option having strong support from various considerations is that single particle conformal weights are of form $1/2 + i\sum_k n_k y_k$, where $s_k = 1/2 + iy_k$ is zero of Riemann zeta. The construction of many particle conformally bound states for poly-zetas leads to the same spectrum for bound states and predicts that only 2- and 3-parton bound states are irreducible. On the other hand, conformal weights are additive for the (anti)commutators of (super)Hamiltonians and gives thus all weights of form $s = n + i\sum_k n_k y_k$.

The interpretation of this picture is not obvious.

1. The first interpretation would be that also other conformal weights are possible but that the commutator and anti-commutator algebras of super-canonical algebra containing conformal weights $Re(s) = k/2$, $k > 1$, represent gauge degrees of freedom. The sub-Virasoro algebra generated by L_n, $n > 0$, would generate these conformal weights which would suggest that L_n, $n > 0$, but not L_0, must annihilate the physical states. The problem is that this makes p-adic thermodynamics impossible.

2. p-Adic mass calculations would suggest that Super Kac-Moody Virasoro (SKMV) generators L_n, $n > 0$, do not correspond to pure gauge degrees of freedom, and a more general interpretation would be that all these conformal weights are possible and represent genuine physical degrees of freedom. The extension of the algebra using the standard assumption $L_{-n} = L_n^\dagger$ would bring in also the conformal weights $Re(s) = -k/2$, $k \geq 1$. p-adic mass calculations would encourage to think that it is super-canonical (SC) generators L_{-n}, $n > 0$, which annihilate tachyonic ground states and stabilize them against tachyonic p-adic thermodynamics. The physical ground state with a vanishing conformal weight would be constructed from this tachyonic ground state and p-adic thermodynamics for $SKMV$ generators L_n, $n > 0$, would apply to it.

3. In the discrete variant of theory required by number theoretic universality all stringy sub-manifolds of X^2 corresponding to the inverse images of $z = \zeta(n/2 + i\sum_k n_k y_k) \in S^2 \subset CP_2$ would be realized so that one would have probability amplitude in the discrete set of these number theoretic strings. $SKMV$ generators L_n and G_r would excite $n > 0$ "shells" in this structure whereas SC generators would generate $n < 0$ shells.

4. Also the trivial zeros $s_n = -2n$, $n > 0$, of Riemann Zeta could correspond to physically interesting conformal weights for the super-canonical algebra (at least). In the region $r \geq r_0$ the function r^{-2n} approaches zero and these powers are square integrable in this region. The orthogonality with other

states could be achieved by arranging things suitably in other degrees of freedom [B2]. Since ζ is real also along real line, the set of even integers $\sum_k n_k s_k$, $n_k \in Z$ is mapped by ζ to the same real line of $S^2 \subset CP_2$ as non-trivial zeros of ζ. p-Adic mass calculations would suggest that states with conformal weight $s_{min} = -2n_{max}$ (at least thes

5. could represent null states annihilated by L_{-n}, $n > 0$.

9.3.2 Bosonic super Kac-Moody algebra

The generators of bosonic super Kac-Moody algebra leave the light-likeness condition $\sqrt{g_3} = 0$ invariant. This gives the condition

$$\delta g_{\alpha\beta} Cof(g^{\alpha\beta}) = 0 \ , \tag{79}$$

Here Cof refers to matrix cofactor of $g_{\alpha\beta}$ and summation over indices is understood. The conditions can be satisfied if the symmetries act as combinations of infinitesimal diffeomorphisms $x^\mu \to x^\mu + \xi^\mu$ of X^3 and of infinitesimal conformal symmetries of the induced metric

$$\delta g_{\alpha\beta} = \lambda(x) g_{\alpha\beta} + \partial_\mu g_{\alpha\beta}\xi^\mu + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta\xi^\mu \ . \tag{80}$$

1. Ansatz as an X^3-local conformal transformation of imbedding space

Write δh^k as a super-position of X^3-local infinitesimal diffeomorphisms of the imbedding space generated by vector fields $J^A = j^{A,k}\partial_k$:

$$\delta h^k = c_A(x) j^{A,k} \ . \tag{81}$$

This gives

$$\begin{aligned} c_A(x) \quad & \left[D_k j_l^A + D_l j_k^A\right] \partial_\alpha h^k \partial_\beta h^l + 2\partial_\alpha c_A h_{kl} j^{A,k} \partial_\beta h^l \\ = \quad & \lambda(x) g_{\alpha\beta} + \partial_\mu g_{\alpha\beta}\xi^\mu + g_{\mu\beta}\partial_\alpha\xi^\mu + g_{\alpha\mu}\partial_\beta\xi^\mu \ . \end{aligned} \tag{82}$$

If an X^3-local variant of a conformal transformation of the imbedding space is in question, the first term is proportional to the metric since one has

$$D_k j_l^A + D_l j_k^A = 2h_{kl} \ . \tag{83}$$

The transformations in question includes conformal transformations of $H_\pm$ and isometries of the imbedding space H.

The contribution of the second term must correspond to an infinitesimal diffeomorphism of X^3 reducible to infinitesimal conformal transformation ψ^μ:

$$2\partial_\alpha c_A h_{kl} j^{A,k} \partial_\beta h^l = \xi^\mu \partial_\mu g_{\alpha\beta} + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta \xi^\mu \quad . \tag{84}$$

2. A rough analysis of the conditions

One could consider a strategy of fixing c_A and solving solving ξ^μ from the differential equations. In order to simplify the situation one could assume that $g_{ir} = g_{rr} = 0$. The possibility to cast the metric in this form is plausible since generic 3-manifold allows coordinates in which the metric is diagonal.

1. The equation for g_{rr} gives

$$\partial_r c_A h_{kl} j^{Ak} \partial_r h^k \quad = \quad 0 \quad . \tag{85}$$

The radial derivative of the transformation is orthogonal to X^3. No condition on ξ^α results. If c_A has common multiplicative dependence on $c_A = f(r)d_A$ by a one obtains

$$d_A h_{kl} j^{Ak} \partial_r h^k \quad = \quad 0 \quad . \tag{86}$$

so that J^A is orthogonal to the light-like tangent vector $\partial_r h^k$ X^3 which is the counterpart for the condition that Kac-Moody algebra acts in the transversal degrees of freedom only. The condition also states that the components g_{ri} is not changed in the infinitesimal transformation.

It is possible to choose $f(r)$ freely so that one can perform the choice $f(r) = r^n$ and the notion of radial conformal weight makes sense. The dependence of c_A on transversal coordinates is constrained by the transversality condition only. In particular, a common scale factor having free dependence on the transversal coordinates is possible meaning that X^3-local conformal transformations of H are in question.

2. The equation for g_{ri} gives

$$\partial_r \xi^i \quad = \quad \partial_r c_A h_{kl} j^{Ak} h^{ij} \partial_j h^k \quad . \tag{87}$$

The equation states that g_{ri} are not affected by the symmetry. The radial dependence of ξ^i is fixed by this differential equation. No condition on ξ^r results. These conditions imply that the local gauge transformations are dynamical with the light-like radial coordinate r playing the role of the time variable. One should be able to fix the transformation more or less arbitrarily at the partonic 2-surface X^2.

3. The three independent equations for g_{ij} give

$$\xi^\alpha \partial_\alpha g_{ij} + g_{kj}\partial_i \xi^k + g_{ki}\partial_j \xi^k = \partial_i c_A h_{kl} j^{Ak} \partial_j h^l \ . \qquad (88)$$

These are 3 differential equations for 3 functions ξ^α on 2 independent variables x^i with r appearing as a parameter. Note however that the derivatives of ξ^r do not appear in the equation. At least formally equations are not over-determined so that solutions should exist for arbitrary choices of c_A as functions of X^3 coordinates satisfying the orthogonality conditions. If this is the case, the Kac-Moody algebra can be regarded as a local algebra in X^3 subject to the orthogonality constraint.

This algebra contains as a subalgebra the analog of Kac-Moody algebra for which all c_A except the one associated with time translation and fixed by the orthogonality condition depends on the radial coordinate r only. The larger algebra decomposes into a direct sum of representations of this algebra.

3. Commutators of infinitesimal symmetries

The commutators of infinitesimal symmetries need not be what one might expect since the vector fields ξ^μ are functionals c_A and of the induced metric and also c_A depends on induced metric via the orthogonality condition. What this means that $j^{A,k}$ in principle acts also to ϕ_B in the commutator $[c_A J^A, c_B J^B]$.

$$\left[c_A J^A, c_B J^B\right] = c_A c_B J^{[A,B]} + J^A \circ c_B J^B - J^B \circ c_A J^A \ , \qquad (89)$$

where $\circ$ is a short hand notation for the change of c_B induced by the effect of the conformal transformation J^A on the induced metric.

Luckily, the conditions in the case $g_{rr} = g_{ir} = 0$ state that the components g_{rr} and g_{ir} of the induced metric are unchanged in the transformation so that the condition for c_A resulting from g_{rr} component of the metric is not affected. Also the conditions coming from $g_{ir} = 0$ remain unchanged. Therefore the commutation relations of local algebra apart from constraint from transversality result.

The commutator algebra of infinitesimal symmetries should also close in some sense. The orthogonality to the light-like tangent vector creates here a problem since the commutator does not obviously satisfy this condition automatically. The problem can be solved by following the recipes of non-covariant quantization of string model.

1. Make a choice of gauge by choosing time translation P^0 in a preferred M^4 coordinate frame to be the preferred generator $J^{A_0} \equiv P^0$, whose coefficient $\Phi_{A_0} \equiv \Psi(P^0)$ is solved from the orthogonality condition. This assumption is analogous with the assumption that time coordinate is non-dynamical in the quantization of strings. The natural basis for the algebra is obtained by allowing only a single generator J^A besides P^0 and putting $d_A = 1$.

2. This prescription must be consistent with the well-defined radial conformal weight for the $J^A \neq P^0$ in the sense that the proportionality of d_A to r^n for $J^A \neq P^0$ must be consistent with commutators. SU(3) part of the algebra is of course not a problem. From the Lorentz vector property of P^k it is clear that the commutators resulting in a repeated commutation have well-defined radial conformal weights only if one restricts $SO(3,1)$ to $SO(3)$ commuting with P^0. Also D could be allowed without losing well-defined radial conformal weights but the argument below excludes it. This picture conforms with the earlier identification of the Kac-Moody algebra.

   Conformal algebra contains besides Poincare algebra and the dilation $D = m^k\partial_{m^k}$ the mutually commuting generators $K^k = (m^r m_r \partial_{m^k} - 2m^k m^l \partial_{m^l})/2$. The commutators involving added generators are

$$\begin{aligned} \left[D,K^k\right] &= -K^k \ , & \left[D,P^k\right] &= P^k \ , \\ \left[K^k,K^l\right] &= 0 \ , & \left[K^k,P^l\right] &= m^{kl}D - M^{kl} \ . \end{aligned} \tag{90}$$

   From the last commutation relation it is clear that the inclusion of K^k would mean loss of well-defined radial conformal weights.

3. The coefficient dm^0/dr of $\Psi(P^0)$ in the equation

$$\Psi(P^0)\frac{dm^0}{dr} = -J^{Ak}h_{kl}\partial_r h^l$$

   is always non-vanishing due to the light-likeness of r. Since P^0 commutes with generators of $SO(3)$ (but not with D so that it is excluded!), one can *define* the commutator of two generators as a commutator of the remaining part and identify $\Psi(P^0)$ from the condition above.

4. Of course, also the more general transformations act as Kac-Moody type symmetries but the interpretation would be that the sub-algebra plays the same role as $SO(3)$ in the case of Lorentz group: that is gives rise to generalized spin degrees of freedom whereas the entire algebra divided by this sub-algebra would define the coset space playing the role of orbital degrees of freedom. In fact, also the Kac-Moody type symmetries for which c_A depends on the transversal coordinates of X^3 would correspond to orbital degrees of freedom. The presence of these orbital degrees of freedom arranging super Kac-Moody representations into infinite multiplets labelled by function basis for X^2 means that the number of degrees of freedom is much larger than in string models.

5. It is possible to replace the preferred time coordinate m^0 with a preferred light-like coordinate. There are good reasons to believe that orbifold singularity for phases of matter involving non-standard value of Planck constant corresponds to a preferred light-ray going through the tip of $\delta M^4_\pm$.

Thus it would be natural to assume that the preferred M^4 coordinate varies along this light ray or its dual. The Kac-Moody group $SO(3) \times E^3$ respecting the radial conformal weights would reduce to $SO(2) \times E^2$ as in string models. E^2 would act in tangent plane of $S^2_{\pm}$ along this ray defining also $SO(2)$ rotation axis.

4. Hamiltonians

The action of these transformations on Chern-Simons action is well-defined and one can deduce the conserved quantities having identification as configuration space Hamiltonians. Hamiltonians also correspond to closed 2-forms. The condition that the Hamiltonian reduces to a dual of closed 2-form is satisfied because X^2-local conformal transformations of $M^4_{\pm} \times CP_2$ are in question (X^2-locality does not imply any additional conditions).

5. Action on spinors

One can imagine two interpretations for the action of generalized Kac-Moody transformations on spinors.

1. Both SO(3) and SU(3) rotations have a standard action as spin rotation and electro-weak rotation allowing to define the action of the Kac-Moody algebra J^A on spinors. This action is not consistent with the generalized eigenvalue equation unless one restricts it to X^2 at $\delta H_{\pm}$.
2. Since Kac-Moody generator performs a local spinor rotation and increases the conformal weight by n units, the simplest possibility is that the action of transformation adds to Ψ_λ with $\lambda = 1/2 + i\sum_k n_k y_k$, a term with eigenvalue $\lambda + n$ and having $J^A \Psi_\lambda$ as initial values at X^2. This would make natural the interpretation as a gauge transformation apart from the effects caused by the possible central extension term.

6. How central extension term could emerge?

The central extension term of Kac-Moody algebra could correspond to a symplectic extension which can emerge from the freedom to add a constant term to Hamiltonians as in the case of super-canonical algebra. The expression of the Hamiltonians as closed forms could allow to understand how the central extension term emerges.

In principle one can construct a representation for the action of Kac-Moody algebra on fermions a representations as a fermionic bilinear and the central extension of Kac-Moody algebra could emerge in this construction just as it appears in Sugawara construction.

9.3.3 Fermionic Kac-Moody algebra in spin and electro-weak degrees of freedom

The action of spin rotations and electro-weak rotations can be identified in terms of the group $SU(2) \times SU(2) \times U(1)$ associated inherently with $N =$

4 super-conformal symmetry. The action on zero modes and eigen modes Ψ is straightforward to write as multiplication on the initial values at X^2 and assuming that λ in the generalized eigenvalue equation is replaced by $\lambda + n$.

Fermionic super-generators correspond naturally to zero modes and eigen modes of the modified Dirac operator labelled by the radial conformal weights $\lambda = 1/2 + i\sum_k n_k yk$ and by the quantum numbers labelling the dependence on transversal degrees of freedom. The real part of the conformal weight would corresponds for $D\Psi = 0$ to ground state conformal weight $h = 0$ (Ramond) and to $h = 1/2$ for $\lambda \neq 0$ (N-S). That also bosonic super-canonical Hamiltonians can have half odd integer conformal weight is however in conflict with the intuition that half-odd integer conformal weights correspond to states with odd fermion number.

For Ramond representations the lines $\zeta(Re(s) = n) \subset S^2$, $n \geq 0$, would represent the conformal weights at space-time level and for N-S representations the lines would correspond to $\zeta(Re(s) = n + 1/2) \subset S^2$. If also trivial zeros are possible they would correspond to the lines $\zeta(Re(s) = n - 2k) \subset S^2$, $k = 1, 2, ...$

9.3.4 Radial Super Virasoro algebras

The radial Super Virasoro transformations act on both $\delta H_\pm$ and partonic 3-surface X^3 and are consistent with the freedom to choose the basis of $H_\pm$ Hamiltonians and the eigenmode basis of the modified Dirac operator by a re-scaling the light-like vector (t^k or more plausibly, its dual n^k) appearing in the definition of the generalized eigenvalue equation.

In the partonic sector a possible interpretation is as local diffeomorphisms of X^3. These transformations do not however leave X^3 invariant as a whole, which brings in some delicacies. In the case of $\delta H_\pm$ the tip of the future light-cone remains invariant only for $n \geq 0$ and $r = \infty$ only for $n \leq 0$. These facts could explain why only the generators L_n, $n < 0$ (or $n < 0$ depending on whether positive or negative energy component of zero energy state is in question) annihilate the ground states.

One can assign to the Virasoro algebra of $H_\pm$ Hamiltonians as Noether charges defined by current $\Pi^0_k j^{Ak}$ which reduces to a dual of a closed 2-form in the case of $H_\pm$ because its symplectic form annihilates j^{Ak}. The transformations associated with X^3 correspond to a unique shift of X^2 in the light-like direction by $\delta h^k = r^n \partial_r h^k$ so that the Hamiltonian is well-defined and reduces to a value of a closed 2-form so that the stringy picture emerges.

The corresponding fermionic super Hamiltonians $G_r = \overline{\nu} r^n \Gamma_r \Psi$ anti-commute to these as is easy to see by noticing that the light-like radial gamma matrices Γ_r appear in the combination $\Gamma_r \gamma^0 \Gamma_r = \gamma_0$ in the anti-commutator so that it does not vanish. One can consider also more general fermionic generators obtained by replacing right-handed neutrino spinor with an arbitrary solution of $D\Psi = 0$ which is eigen spinor of $J^{kl}\Sigma_{kl}$ appearing in the fermionic anti-commutation relations. This would give rise to a full $N = 4$ super-conformal symmetry of Ramond type but having infinite degeneracy due to the dependence on transversal coordinates of X^3. If one allows also the solutions of $D\Psi = \lambda\Psi$ one obtains counterparts of N-S type representations with a similar degeneracy.

It must be emphasized that four-momentum does not appear neither in the representations of Super Virasoro generators as it does in string models and this is consistent with the Lorentz invariant identification of mass squared as vacuum expectation value of the net conformal weight. Also the problems with tachyons are avoided. Four-momentum could creep in if one had Sugawara type representation of Super Virasoro generators in terms of Kac-Moody generators which indeed contain also translation generators now. Note also that the stringy conformal weight would be associated with partonic 2-surface, whereas radial conformal weight is associated with its light-like orbit. Furthermore, the origin of the radial super-conformal symmetries is light-likeness rather than stringy character. It is not clear whether it is useful to assign the usual conformal weights with the conformal fields at X^2 and whether the stringy anti-commutation relations for Ψ force this kind of assignment.

9.3.5 Gauge super-symmetries associated with the generalized eigenvalue equation for D

Zero modes which are annihilated by the operator $T = t^k\gamma_k$ or $N = n^k\gamma_k$. t^k (n^k) is the light-like appearing in the generalized eigenvalue equation for the modified Dirac operator. t^k is parallel to X^3 and n^k, which corresponds to the more plausible option, is obtained by changing the direction of the spatial part of t^k in the preferred M^4 coordinate frame associated with the space-time sheet (the rest system or number theoretically determined M^4 time). n^k defines inwards directed tangent vector to the space-time sheet containing X^3. The zero modes of the modified Dirac operator annihilated by T (N) act as super gauge symmetries for the generalized eigen modes of the generalized Dirac operator. They do not depend on r and thus have a vanishing conformal weight.

The freedom to choose the scaling of t^k (n^k) rather freely gives rise to a further symmetry which does not affect the eigenvalue spectrum but modifies the eigen modes. This symmetry is definitely a pure gauge symmetry.

9.3.6 What about ordinary conformal symmetries?

Ordinary conformal symmetries acting on the complex coordinate of X^2 have not yet been discussed. These symmetries involve the dependence on the induced metric through the moduli of characterizing the conformal structure of X^2. Stringy picture would suggest in the case of a spherical topology that the zero modes and eigen modes of Ψ are proportional to z^n at X^2. Only $n \geq 0$ mode would be non-singular at the northern hemisphere and $n \leq 0$ at the southern hemisphere and the eigen modes are non-normalizable.

One cannot glue these modes together at equator unless one assumes the behavior z^n, $n \geq 0$, on the northern hemisphere and $\overline{z}^{-n}$, $n \geq 0$, on the southern hemisphere. The identification $\Psi_+(z) = \Psi_-^\dagger(z)$ ($z \rightarrow \overline{z}$ in Hermitian conjugation) at equator would state that "positive energy" particle at the northern hemisphere corresponds to a negative energy antiparticle at the southern hemisphere. The assumption that energy momentum generators $T_+(z)$ and $T_-(z)$ are related in the same manner at equator gives $L_n = L^\dagger_{-n}$ as required. Second candidate for the basis are spherical harmonics which are eigenstates of $L_0 - \overline{L_0}$ defining angular momentum operator L_z but they do not possess well defined conformal weights.

The radial time evolution for the Kac-Moody generators does not commute with L_0 whereas well-defined radial conformal weights are possible. This would support the view that the conformal weight associated with X^2 degrees of freedom does not contribute to the mass squared. If this picture is correct, L_0 would label different SKM representations and play a role similar to that in conformal field theories for critical systems.

9.3.7 How to interpret the overall sign of conformal weight?

The overall sign of conformal weight can be changed by replacing r with $1/r$ and the region $r > r_0$ with $r < r_0$ of $\delta H_\pm$ or of partonic 3-surface. The earlier idea that the conformal weights associated with the super-conformal algebras assignable to $\delta H_\pm$ and to light-like partonic 3-surfaces have opposite signs would allow to construct representations of super-canonical algebra by constructing a tachyonic ground state using super-canonical generators and its excitations using super Super-Kac Moody generators as in super string models.

There is however an objection against this idea. The partons at $\delta H_\pm$ would have a finite distance from the tip of the light cone at all points where they correspond to non-vacuum extremals, so that the phase transitions changing the value of Planck constant should always occur via vacuum extremals. This would not allow the leakage of Kähler magnetic flux between different sectors of imbedding space. The cautious conclusion is that at least in the super-canonical sector both $r > r_0$ and $r < r_0$ sectors related by the conformal transformation $r \to 1/r$ must be allowed and correspond to positive and negative values for the radial super-conformal weights.

In zero energy ontology particle reactions correspond to zero energy states which at space-time level carry positive energy particles at the end of world in geometric past and negative energy particles at the end of world in the geometric future. Also conformal weights are of opposite sign so that vanishing of the net conformal weights holds true only for zero energy states in accordance with the spirit of p-adic mass calculations. If the states of geometric past correspond to positive (negative) super Kac-Moody (super-canonical) conformal weights, the scattering could be regarded as a process leading from the region $r > r_0$ at δM^4_+0 to the region $r < r_0$ at δM^4_-. At partonic level the incoming partons would correspond to the region $r < r_0$ and outgoing partons to the region $r > r_0$, which conforms with the idea that the final state can partons can be arbitrary far in the geometric future.

In certain sense this picture would reproduce big ban-big crush picture at the level of super-canonical algebra. $r < r_0$ means that partons can be arbitrarily near to the tip of δM^4_- representing the final singularity whereas $r > r_0$ for δM^4_+ would be the counterpart for big bang.

9.3.8 Absolute extremum property for Kähler action implies dynamical Kac-Moody and super conformal symmetries

The identification of the criterion selecting the preferred extremal of Kähler action defining space-time surface as a counterpart of Bohr orbit has been a long standing challenge. The first guess was that an absolute minimum is in question. The number theoretic theoretic picture, in particular $HO-H$ duality [E2] resolves the problem by assigning to each point of X^4 a preferred plane M^2, which also fixes the boundary conditions for the field equations at light-like partonic 3-surfaces. The still open questions are whether the H images of hyper-quaternionic 4-surfaces of $HO = M^8$ are indeed extremals of Kähler action and whether these preferred extremals satisfy absolute extremum property. Be as it may, the following argument suggests that absolute extremum property gives rise to additional symmetries.

The extremal property for Kähler action with respect to variations of time derivatives of initial values keeping h^k fixed at X^3 implies the existence of an infinite number of conserved charges assignable to the small deformations of the extremum and to H isometries. Also infinite number of local conserved super currents assignable to second variations and to covariantly constant right handed neutrino are implied. The corresponding conserved charges vanish so that the interpretation as dynamical gauge symmetries is appropriate. This result provides strong support that the local extremal property is indeed consistent with the almost-topological QFT property at parton level.

The starting point are field equations for the second variations. If the action contain only derivatives of field variables one obtains for the small deformations δh^k of a given extremal

$$\begin{aligned} \partial_\alpha J_k^\alpha &= 0 \ , \\ J_k^\alpha &= \frac{\partial^2 L}{\partial h_\alpha^k \partial h_\beta^l} \delta h_\beta^l \ , \end{aligned} \tag{91}$$

where h_α^k denotes the partial derivative $\partial_\alpha h^k$. A simple example is the action for massless scalar field in which case conservation law reduces to the conservation of the current defined by the gradient of the scalar field. The addition of mass term spoils this conservation law.

If the action is general coordinate invariant, the field equations read as

$$D_\alpha J^{\alpha,k} = 0 \tag{92}$$

where D_α is now covariant derivative and index raising is achieved using the metric of the imbedding space.

The field equations for the second variation state the vanishing of a covariant divergence and one obtains conserved currents by the contraction this equation with covariantly constant Killing vector fields j_A^k of M^4 translations which means that second variations define the analog of a local gauge algebra in M^4 degrees of freedom.

$$\begin{aligned} \partial_\alpha J_n^{A,\alpha} &= 0 \ , \\ J_n^{A,\alpha} &= J_n^{\alpha,k} j_k^A \ . \end{aligned} \tag{93}$$

Conservation for Killing vector fields reduces to the contraction of a symmetric tensor with $D_k j_l$ which vanishes. The reason is that action depends on induced metric and Kähler form only.

Also covariantly constant right handed neutrino spinors Ψ_R define a collection of conserved super currents associated with small deformations at extremum

$$\begin{aligned} J_n^\alpha &= J_n^{\alpha,k} \gamma_k \Psi_R \ , \\ & . \end{aligned} \tag{94}$$

Second variation gives also a total divergence term which gives contributions at two 3-dimensional ends of the space-time sheet as the difference

$$\begin{aligned} Q_n(X_f^3) - Q_n(X^3) &= 0 \ , \\ Q_n(Y^3) &= \int_{Y^3} d^3x J_n \ , \ J_n = J^{tk} h_{kl} \delta h_n^l \ . \end{aligned} \tag{95}$$

The contribution of the fixed end X^3 vanishes. For the extremum with respect to the variations of the time derivatives $\partial_t h^k$ at X^3 the total variation must vanish. This implies that the charges Q_n defined by second variations are identically vanishing

$$Q_n(X_f^3) = \int_{X_f^3} J_n = 0 \ . \tag{96}$$

Since the second end can be chosen arbitrarily, one obtains an infinite number of conditions analogous to the Virasoro conditions. The analogs of unbroken loop group symmetry for H isometries and unbroken local super symmetry generated by right handed neutrino result. Thus extremal property is a necessary condition for the realization of the gauge symmetries present at partonic level also at the level of the space-time surface. The breaking of super-symmetries could perhaps be understood in terms of the breaking of these symmetries for light-like partonic 3-surfaces which are not extremals of Chern-Simons action.

9.4 Large $N = 4$ SCA is the natural option

The arguments below support the view that "large" $N = 4$ SCA is the natural algebra in TGD framework.

9.4.1 How $N = 4$ super-conformal invariance emerges from the parton level formulation of quantum TGD?

The discovery of the formulation of TGD as a $N = 4$ almost topological super-conformal QFT with light-like partonic 3-surfaces identified as basic dynamical objects led to the final understanding of super-conformal symmetries and their breaking. $N = 4$ super-conformal algebra corresponds to the maximal algebra with $SU(2)\times U(2)$ Kac-Moody algebra as inherent fermionic Kac-Moody algebra having interpretation in terms of rotations and electro-weak gauge group.

9.4.2 Large $N = 4$ SCA algebra

Large $N = 4$ super-conformal symmetry with $SU(2)_+ \times SU(2)_- \times U(1)$ inherent Kac-Moody symmetry seems to define the fundamental partonic super-conformal symmetry in TGD framework. In the case of SKM algebra the groups would act on induced spinors with $SU(2)_+$ representing spin rotations and $SU(2)_- \times U(1) = U(2)_{ew}$ electro-weak rotations. In super-canonical sector the action would be geometric: $SU(2)_+$ would act as rotations on light-cone boundary and $U(2)$ as color rotations leaving invariant a preferred CP_2 point.

A concise discussion of this symmetry with explicit expressions of commutation and anticommutation relations can be found in [73]. The representations of SCA are characterized by three central extension parameters for Kac-Moody algebras but only two of them are independent and given by

$$\begin{aligned} k_\pm &\equiv k(SU(2)_\pm) \ , \\ k_1 &\equiv k(U(1)) = k_+ + k_- \ . \end{aligned} \tag{97}$$

The central extension parameter c is given as

$$c = \frac{6k_+k_-}{k_+ + k_-} \ . \tag{98}$$

and is rational valued as required.

A much studied $N = 4$ SCA corresponds to the special case

$$\begin{aligned} k_- &= 1 \ , \ k_+ = k+1 \ , \ k_1 = k+2 \ , \\ c &= \frac{6(k+1)}{k+2} \ . \end{aligned} \tag{99}$$

$c = 0$ would correspond to $k_+ = 0, k_- = 1, k_1 = 1$. Central extension would be trivial in rotational degrees of freedom but non-trivial in $U(2)_{ew}$. For $k_+ > 0$ one has $k_1 = k_+ + k_- \neq k_+$. A possible interpretation is in terms of electro-weak symmetry breaking with $k_+ > 0$ signalling for the massivation of electro-weak gauge bosons.

An interpretation consistent with the general vision about the quantization of Planck constants is that k_+ and k_- relate directly to the integers n_a and n_b characterizing the values of $M^4_\pm$ and CP_2 Planck constants via the formulas $n_a = k_+ + 2$ and $n_b = k_- + 2$. This would require $k_\pm \geq 1$ for G_i a finite subgroup of $SU(2)$ ("anyonic" phases). In stringy phases with $G_i = SU(2)$ for $i = a$ or $i = b$ or for both, k_i could also vanish so that also $n_i = 2$ corresponding to A_2 ADE diagram and $SU(2)$ Kac-Moody algebra becomes possible. In the super-canonical sector $k_+ = 0$ would mean massless gluons and $k_- = k_1$ that $U(2) \subset SU(3)$ and possibly entire $SU(3)$ represents an unbroken symmetry.

9.4.3 About breaking of large $N = 4$ SCA

Partonic formulation predicts that large $N = 4$ SCA is a broken symmetry, and the first guess is that breaking could be thought to occur via several steps. First a "small" $N = 4$ SCA with Kac-Moody group $SU(2) \times U(1)$ would result. The next step would lead to $N = 2$ SCA and the final step to $N = 0$ SCA. Several symmetry breaking scenarios are possible.

1. $SU(2) \times U(1)$ could correspond to electro-weak gauge group such that rotational degrees of freedom are frozen dynamically by the massivation of the corresponding excitations. This interpretation could apply in stringy phase: for cosmic strings rotational excitations are indeed hyper-massive.

2. The interpretation of $SU(2)$ as spin rotation group and $U(1)$ as electromagnetic gauge group conforms with the general vision about electroweak symmetry breaking in non-stringy phase. The interpretation certainly makes sense for covariantly constant right handed neutrinos for which spin direction is free.

The next step in the symmetry breaking sequence would be $N = 2$ SCA with $U(1) \subset SU(2) \times U(2)$ sub-algebra. The interpretation could be as electro-weak symmetry breaking in the stringy sector (cosmic strings) so that $U(1)$ would correspond to em charge or possibly weak isospin.

9.4.4 Relationship to super-strings and M-theory

The (4,4) signature characterizing $N = 4$ SCA topological field theory is not a problem since in TGD framework the target space becomes a fictive concept defined by the Cartan algebra. Both $M^4 \times CP_2$ decomposition of the imbedding space and space-time dimension are crucial for the $2+2+2+2$ structure of the Cartan algebra, which together with the notions of the configuration space and generalized coset representation formed from super Kac-Moody and super-canonical algebras guarantees $N = 4$ super-conformal invariance.

Including the 2 gauge degrees of freedom associated with M^2 factor of $M^4 = M^2 \times E^2$ the critical dimension becomes $D = 10$ and and including the radial degree of light-cone boundary the critical dimension becomes $D = 11$ of M-theory. Hence the fictive target space associated with the vertex operator construction corresponds to a flat background of super-string theory and flat background of M-theory with one light-like direction. From TGD point view the difficulties of these approaches are due to the un-necessary assumption that the fictive target space defined by the Cartan algebra corresponds to the physical imbedding space. The flatness of the fictive target space forces to introduce the notion of spontaneous compactification and dynamical imbedding space and this in turn leads to the notion of landscape.

9.4.5 Questions

A priori one can consider 3 different options concerning the identification of quarks and leptons.

1. Could also quarks define $N = 4$ superconformal symmetry?

One can ask, whether the construction could be extended by allowing H-spinors of opposite chirality to have leptonic quantum numbers so that free quarks would have integer charge. The construction does not work. The direct sum of $N = 4$ SCAs can be realized but $N = 8$ algebra would require SO(7) rotations mixing states with different fermion numbers: for $N = 4$ SCA this is not needed. Furthermore, only $N < 4$ super-conformal algebras allow an associative realization and $N = 8$ non-associative realization discovered first by Englert exists only at the limit when Kac-Moody central extension parameter k becomes infinite (this corresponds to a critical phase formally and $q = 1$ Jones inclusion). This is not enough for the purposes of TGD and number theoretic vision strongly supports "small" $N = 4$ SCA.

2. Integer charged leptons and fractionally charged quarks?

Second option would be leptons and fractionally charged quarks with $N = 4$ SCA in leptonic sector. It is indeed possible to realize both quark and lepton spinors as super generators of super affinized quaternion algebras (a generalization of super-Kac Moody algebras) so that the fundamental spectrum generating algebra is obtained. Quarks with their natural charges can appear only in $n = 3, k = 1$ phase together with fractionally charged leptons. Leptons in this phase would have strong interactions with quarks. The penetration of lepton into hadron would give rise to this kind of situation. Leptons can indeed move in triality 1 states since 3-fold covering of CP_2 points by M^4 points means that 3 full rotations for the phase angle of CP_2 complex coordinate corresponds to single 2π rotation for M^4 point.

Hadron like states would correspond to the lowest possible Jones inclusion characterized by n=3 and the subgroup A_2 (Z_3) of SU(2). The work with quantization of Planck constant had already earlier led to the realization that ADE Dynkin diagrams assignable to Jones inclusions indeed correspond to gauge groups [C8]: in particular, A_2 corresponds to color group SU(3). Infinite hierarchy of hadron like states with $n = 3, 4, 5...$ quarks or leptons is predicted corresponding to the hierarchy of Jones inclusions, and I have already earlier proposed that this hierarchy should be crucial for the understanding of living matter [M3]. For states containing quarks n would be multiple of 3.

One can understand color confinement of quarks as absolute if one accepts the generalization of the notion of imbedding space forced by the quantization of Planck constant. Ordinary gauge bosons come in two varieties depending on whether their couplings are H-vectorial or H-axial. Strong interactions inside hadrons could be also interpreted as H-axial electro-weak interactions which have become strong (presumably because corresponding gauge bosons are massless) as is clear from the fact that arbitrary high n-point functions are non-vanishing in the phases with $q \neq 1$. Already earlier the so called HO-H duality inspired by the number theoretical vision [E2] led to the same proposal but for ordinary electro-weak interactions which can be also imagined in the scenario in which only leptons are fundamental fermions.

3. Quarks as fractionally charged leptons?

For the third option only leptons would appear as free fermions. The dramatic prediction would be that quarks would be fractionally charged leptons. It is however not clear whether proton can decay to positron plus something (recall the original erratic interpretation of positron as proton by Dirac!): lepton number fractionization meaning that baryon consists of three positrons with fermion number 1/3 might allow this. If not, then only the interactions mediated by the exchanges of gauge bosons (vanishing lepton number is essential) between worlds corresponding to different Jones inclusions are possible and proton would be stable.

There are however also objections. In particular, the resulting states are not identical with color partial waves assignable to quarks and the nice predictions of p-adic mass calculations for quark and hadron masses might be lost. Hence the cautious conclusion is that the original scenario with integer charged quarks predicting confinement automatically is the correct one.

10 About the construction of S-matrix

During years I have proposed a long list of nice looking ideas concerning the construction of S-matrix. After the progress in understanding the role of hyper-finite factors of type II_1 it become clear that the basic problems have been more at the conceptual level rather than calculational. Thus the key questions seem to be following ones.

What does one actually mean with S-matrix? How does S-matrix differ from the U-matrix associated with the quantum jump? What could S-matrix with a finite measurement resolution mean? What is the precise mathematical characterization of a physical state when the measurement resolution is finite? How does the fuzziness due to a finite measurement resolution affect the definition of transition probabilities defined by S-matrix?

The proper formulation of the notion of measurement resolution leads to a rather dramatic modification of the standard mathematical picture. S-matrix could be fractal and more or less the same for $\mathcal{M}$ and its sub-factors. Transition probabilities would be defined by "quantum S-matrix" with non-commuting $\mathcal{N}$ valued elements in non-commutative fuzzy "quantum quantum state space" with $\mathcal{N}$ valued coefficients generated by $\mathcal{M}/\mathcal{N}$, where Jones inclusion $\mathcal{N} \subset \mathcal{M}$ defines the measurement resolution. Transition probabilities would be eigenvalues of the transition probabilities, which would be commuting Hermitian operators in $\mathcal{N}$.

Classical TGD forces to question even the basic ontology and strongly suggests the notion of zero energy ontology in which physical states possess vanishing net quantum numbers and are creatable from vacuum: S-matrix would represent entanglement coefficients between positive and negative energy parts of the state. U-matrix would characterize transition amplitudes between zero energy states and could have elements between states belonging to different number fields. In particular, it could characterize transitions in which intention transforms to action.

At the more technical level the requirement of number theoretical universality leads to a rather concrete picture about the general form of S-matrix based on the notion of number theoretic braid. This notion emerges also from the non-commutativity implied by the finite measurement resolution characterized in terms of Jones inclusions.

The improved understanding of super-conformal symmetries during last year provides powerful additional constraints and suggest a modification of stringy picture replacing number theoretic strings with number theoretic braids.

10.1 About the general conceptual framework behind quantum TGD

Let us first list the basic conceptual framework in which I try to concretize the ideas about S-matrix.

10.1.1 $N = 4$ super-conformal invariance and light-like 3-surfaces as fundamental dynamical objects

Super-conformal symmetries generalized from string model context to TGD framework are symmetries of S-matrix and of its generalization to M-matrix. This is very powerful constraint but useless unless one has precisely defined ontology translated to a rigorous mathematical framework. The zero energy ontology of TGD is now rather well understood but differs dramatically from that of standard quantum field theories. Second deep difference is that path integral formalism is given up and the goal is to construct S-matrix as a generalization of braiding S-matrices with reaction vertices replaced with the replication of number theoretic braids associated with partonic 2-surfaces taking the role of vertices.

The path leading to the understanding of super-conformal invariance in TGD framework was long but the final outcome is briefly described. The are two kinds of super-conformal symmetries.

1. The first super-conformal invariance is associated with light-cone bound-

ary and is due to its metric 2-dimensionality putting 4-D Minkowski space in a unique position. The canonical transformations of $\delta H_{\pm} = \delta M^4_{\pm} \times CP_2$ are identified as isometries of the configuration space. The super-generators of super-canonical algebra correspond to the gamma matrices of configuration space.

2. Light-like partonic 3-surfaces X^3 are the basic dynamical objects and light-likeness is respected by the 3-D variant of Kac-Moody algebra of conformal transformations of imbedding space made local with respect to X^3. Ordinary 1-D Kac-Moody algebra with complex coordinate z replaced with a light-like radial coordinate r takes a special role and super Kac-Moody symmetry is associated with this. The conformal symmetries associated with X^2 are counterpart of stringy conformal symmetries but have a role analogous to the conformal symmetries of critical statistical systems.

3. By the generalized coset construction the differences of SKMV and SCV generators annihilate physical states: the interpretation is in terms of Equivalence Principle. This also justifies the assuption that mass squared is p-adic thermal expectation value of Super Kac-Moody conformal weight. SKM algebra creates tachyonic ground states with various conformal weights as null states annihilated by L_n, $n > 0$ to which p-adic thermodynamics in SKMV degrees of freedom applies.

The light-likeness property allows the fermionic counterpart of the Chern-Simons action for the induced Kähler gauge potential as the only possible action principle. The resulting almost topological conformal field theory has maximal $N = 4$ super-conformal symmetry with the inherent gauge group $SU(2) \times U(2)$ identified in terms of rotations and electro-weak symmetries acting on imbedding space spinors.

Fermionic dynamics is determined by the modified Dirac action fixed uniquely by the requirement of super-conformal symmetry. The generalized eigen modes and the generalized eigen-values λ of the modified Dirac operator D are expected to play a fundamental role in quantum TGD. In particular, Dirac determinant and the zeta function assignable to the generalized eigenvalues are expected to be important. The following interprtation is perhaps the most plausible interpretation.

1. TGD based description of Higgs mechanism encourages to consider the interpretation of the generalized eigenvalue λ as an analog of a complex square root of conformal weight (mass squared rather than mass corresponds to conformal weight). For this option $|\lambda|^2$ would correspond to Higgs expectation value [A9].

2. If the zeta function defined by the generalized eigenvalues satisfies Riemann hypothesis, its zeros $\Delta = 1/2 + iy_k$ would have interpretation as negatives of conformal weights identifiable as both super-canonical and Super Kac-Moody ground state conformal weights.

10.1.2 S-matrix as a functor

Almost topological QFT property of quantum allows to identify S-matrix as a functor from the category of generalized Feynman cobordisms to the category of operators mapping the Hilbert space of positive energy states to that for negative energy states: these Hilbert spaces are assignable to partonic 2-surfaces. Feynman cobordism is the generalized Feynman diagram having light-like 3-surfaces as lines glued together along their ends defining vertices as 2-surfaces. This picture differs dramatically from that of string models. There is a functional integral over the small deformations of Feynman cobordisms corresponding to maxima of Kähler function. Functor property generalizes the unitary condition and allows also thermal S-matrices which seem to be unavoidable since imbedding space degrees of freedom give rise to a factor of type I with $Tr(id) = \infty$.

10.1.3 S-matrix in zero energy ontology

Zero energy ontology allows to construct unitary S-matrix in fermionic degrees of freedom as unitary entanglement coefficients between positive and negative energy parts of zero energy state. The basic properties of hyper-finite factor II_1 are absolutely crucial. The inclusion of bosonic degrees of freedom lead to a replacement of HFF of type II_1 with HFF of type $II_\infty = II_1 \otimes I_\infty$. However, normalizability of the states allows only a projection of S-matrix to a finite-dimensional subspace of incoming or outgoing states. Hence the S-matrix is effectively restricted to $II_1 \otimes I_n = II_1$ factor so that at the level of physical states HFF of type II_1 results. This is absolutely crucial for the unitary of the S-matrix since it makes possible to have $Tr(SS^\dagger) = Tr(Id) = 1$. If factor of type I is present as a tensor factor, thermal S-matrix is the only possibility and later arguments in favor of the idea that thermodynamics is unavoidable part of quantum theory in zero energy ontology will be developed.

One can worry whether unitarity condition is consistent with the idea that fermionic degrees of freedom should allow to represent Boolean functions in terms of time-like entanglement. That unitary time evolution is able to represent this kind of functions in the case of quantum computers suggests that unitarity is not too strong a restriction. The basic question is whether only a "cognitive" representation of physical S-matrix in terms of time like entanglement or a genuine physical S-matrix is in question. It seems that the latter option is the only possible one so that physical systems would represent the laws of physics.

10.1.4 U-matrix

Besides S-matrix there is also U-matrix defining the unitary process associated with the quantum jump. S- *resp.* U-matrix characterizes quantum state *resp.* quantum jump so that they cannot be one and same thing.

1. There are good arguments supporting the view that U-matrix is almost trivial, and the real importance of U-matrix seems to be related to the to the description of intentional action identified as a transition between

p-adic and real zero energy states and to the possibility to perceive states rather than only changes as quantum jumps leaving the state almost unchanged.

2. State function reduction corresponds to a projection sub-factor in TGD inspired quantum measurement theory whereas U process in some sense corresponds its reversal. Therefore U matrix might correspond to unitary isomorphism mapping factor to a larger factor containing it.

3. State function reduction must be consistent with the unitarity of S-matrix defining time-like entanglement. Since state function reduction means essentially multiplication by a projector to a sub-space it seems that state function reduction for both incoming and outgoing states are possible and would naturally correspond to projections to sub-factors of corresponding HFFs of type II_1.

10.1.5 Unitarity of S-matrix is not necessary in zero energy ontology

U-matrix is necessarily unitary. There are good reasons to believe that this condition combined with Lorentz invariance makes it almost trivial. In the case of S-matrix unitarity is not absolutely necessary.

The restriction of the time-like entanglement coefficients to a unitary S-matrix would conform with the idea that light-like partonic 2-surfaces represent a dynamical evolution at quantum level so that zero energy states must be orthogonal both with respect to positive and negative energy parts of the states. On the other hand, the light-like 3-surface can be chosen arbitrarily and its choice indeed affects S-matrix. Hence the theory cannot fully reduce to a 2-dimensional theory. The interpretation is that light-like 3-surfaces are in 1-1 correspondence with the ground states of super-conformal representations identifiable as light particles.

There are several arguments supporting the view that S-matrix need not be unitary. The simplest observation is that imbedding space degrees of freedom naturally give rise to a factor of type I so that only thermal S-matrix defines a normalizable zero energy state. S-matrix as functor from the category of Feynman cobordisms to the category operators defining entanglement coefficients implies that S-matrix in fermionic degrees of freedom for a product of cobordisms is product of the S-matrices for cobordisms. This implies that in fermionic degrees of freedom S-matrix is thermal S-matrix with time parameter replaced with complex time parameter whose imaginary part corresponds to inverse temperature. Also an argument based on the existence of universal thermal S-matrix with a complex time parameter for hyper-finite factors of type III_1 supports the view that unitarity is not necessary. A further argument is based on the finding that in dimensions $D < 4$ unitary S-matrix exists only if cobordism is trivial so that topology change would not be possible. This raises the fascinating possibility that thermodynamics - in particular p-adic thermodynamics - is an unavoidable and inherent property of quantum TGD.

10.1.6 Does Connes tensor product fix the allowed M-matrices?

Hyperfinite factors of type II_1 and the inclusion $\mathcal{N} \subset \mathcal{M}$ inclusions have been proposed to define quantum measurement theory with a finite measurement resolution characterized by $\mathcal{N}$ and with complex rays of state space replaced with $\mathcal{N}$ rays. What this really means is far from clear.

1. Naively one expects that matrices whose elements are elements of $\mathcal{N}$ give a representation for M. Now however unit operator has unit trace and one cannot visualize the situation in terms of matrices in case of $\mathcal{M}$ and $\mathcal{N}$.

2. The state space with $\mathcal{N}$ resolution would be formally $\mathcal{M}/\mathcal{N}$ consisting of $\mathcal{N}$ rays. For $\mathcal{M}/\mathcal{N}$ one has finite-D matrices with non-commuting elements of $\mathcal{N}$. In this case quantum matrix elements should be multiplets of selected elements of $\mathcal{N}$, **not all** possible elements of $\mathcal{N}$. One cannot therefore think in terms of the tensor product of $\mathcal{N}$ with $\mathcal{M}/\mathcal{N}$ regarded as a finite-D matrix algebra.

3. What does this mean? Obviously one must pose a condition implying that $\mathcal{N}$ action commutes with matrix action just like C: this poses conditions on the matrices that one can allow. Connes tensor product [71] does just this. Note I have proposed already earlier the reduction of interactions to Connes tensor product (see the section *"Could Connes tensor product...."* later in this chapter) but without reference to zero energy ontology as a fundamental manner to define measurement resolution with respect time and assuming unitarity.

The starting point is the Jones inclusion sequence

$$\mathcal{N} \subset \mathcal{M} \subset \mathcal{M} \otimes_N \mathcal{M} ...$$

Here $\mathcal{M} \otimes_N \mathcal{M}$ is Connes tensor product which can be seen as elements of the ordinary tensor product commuting with $\mathcal{N}$ action so that $\mathcal{N}$ indeed acts like complex numbers in $\mathcal{M}$. $\mathcal{M}/\mathcal{N}$ is in this picture represented with $\mathcal{M}$ in which operators defined by Connes tensor products of elements of $\mathcal{M}$. The replacement $\mathcal{M} \rightarrow \mathcal{M}/\mathcal{N}$ corresponds to the replacement of the tensor product of elements of $\mathcal{M}$ defining matrices with Connes tensor product.

One can try to generalize this picture to zero energy ontology.

1. $\mathcal{M} \otimes_{\mathcal{N}} \mathcal{M}$ would be generalized by $\mathcal{M}_+ \otimes_{\mathcal{N}} \mathcal{M}_-$. Here $\mathcal{M}_+$ would create positive energy states and $\mathcal{M}_-$ negative energy states and $\mathcal{N}$ would create zero energy states in some shorter time scale resolution: this would be the precise meaning of finite measurement resolution.

2. Connes entanglement with respect to $\mathcal{N}$ would define a non-trivial and unique recipe for constructing M-matrices as a generalization of S-matrices expressible as products of square root of density matrix and unitary S-matrix but it is not how clear how many M-matrices this allows. In

any case M-matrices would depend on the triplet $(\mathcal{N}, \mathcal{M}_+, \mathcal{M}_-)$ and this would correspond to p-adic length scale evolution giving replacing coupling constant evolution in TGD framework. Thermodynamics would enter the fundamental quantum theory via the square root of density matrix.

3. Zero energy ontology is a key element of this picture and the most compelling argument for zero energy ontology is the possibility of describing coherent states of Cooper pairs without giving up fermion number, charge, etc. conservation and automatic emerges of length scale dependent notion of quantum numbers (quantum numbers identified as those associated with positive energy factor).

To sum up, interactions would be an outcome of a finite measurement resolution and at the never-achievable limit of infinite measurement resolution the theory would be free: this would be the counterpart of asymptotic freedom.

10.1.7 Quantum classical correspondence

Quantum classical correspondence states that there is a correspondence between quantum fluctuating degrees of freedom associated with partonic 2-surfaces and classical dynamics. The weakest form of this principle is that the ground states of partonic super-conformal representations (massless states which generate light masses observed in laboratory) correspond to the interior dynamics of space-time sheets containing the partonic 2-surfaces. At the space-time level there would be 1-1 correspondence with the maxima of Kähler function giving rise to the analog of spin glass energy landscape.

One could protest by saying that excited states of super-conformal representations have no space-time correlate in this picture. Quantum states are replaced with states in which the projection of S-matrix to a finite-dimensional space in bosonic degrees of freedom appears as time-like entanglement coefficients so that quantum classical correspondence is obtained in strict sense after all. These states states are formally analogous which raises the question whether an actual relationship exists. For HFFs of type III unitary time evolution and thermal equilibrium are indeed closely related aspects of states [48]. $I_\infty \rightarrow I_n$ cutoff in the bosonic degrees of freedom would naturally have the discretization represented by number theoretic braids as a space-time correlate.

The effective elimination of the degrees of freedom associated with the space-time interior implied by the 1-1 correlation would allow to forget 4-D space-time degrees of freedom more or less completely as far as calculation of S-matrix is considered and everything would reduce to Fock space level as it does in quantum field theories. The functional integral around the maximum of Kähler function would select a set of preferred light-like partonic 3-surfaces. Quantum criticality suggests that the functional integral can be carried out exactly.

10.1.8 How TGD differs from string models

An important detail which deserves to be mentioned separately is one crucial deviation from string model picture: the stringy decays of partonic 2-surfaces or 3-surfaces are space-time correlates for the propagation of particle via several different routes rather than genuine particle decay. Note that partonic 2-surfaces can have arbitrarily large size and the outer boundary of any physical system represents the basic example of this kind of surface. Particle reactions correspond to branchings of light-like partonic 2-surfaces so that incoming and outgoing partons are glued together along their ends. This picture makes sense because quantum TGD reduces to almost topological conformal QFT at parton level (only light-likeness brings in the notion of metric).

Quantum classical correspondence allows to interpret light-like partonic 3-surface either as a time evolution of a highly non-deterministic 2-D system or as a 3-D system. This state-dynamics duality was discovered already in [E9], where it was realized that topological quantum computation has interpretation either as a program (state) or running of program (dynamics). Complete reduction to 2-D dynamics is not possible since the light-like 3-surfaces associated with maxima of Kähler action define spin glass energy landscape such that each maximum corresponds to its own S-matrix.

In this picture particle reactions correspond classically to branchings of partonic 2-surfaces generalizing the branchings for lines in Feynman diagrams. The stringy vertices for decays of surfaces correspond in TGD framework to the classical space-time correlate for a particle travelling along different paths and the particle creation and annihilation is a generalization of what occurs in Feynman diagrams with vertices replaced with 2-dimensional partonic surfaces along which light-like partonic 3-surfaces meet.

10.1.9 Physics as a generalized number theory vision

TGD as a generalized number theory vision gives powerful constraints. New view about space-time involves p-adic space-time sheets as space-time correlates for cognitive representations in fermionic case and for intentions in the bosonic case. This leads to the notion of number theoretic braid belonging to the algebraic intersection of real and p-adic partonic surfaces obeying same algebraic equations.

The implication is that the data characterizing S-matrix elements should come from discrete algebraic points of number theoretic braids. The Galois groups for braids occupying regions of partonic 2-surface emerge as a new element and relate closely to the representations of braid groups in HFFs of type II_1. Number theoretic universality leads to the condition that S-matrix elements are algebraic numbers in the extension of rational defined by the extension of p-adic numbers involved.

10.1.10 The role of hyper-finite factors of type II_1

The Clifford algebra of configuration space ("world of classical worlds") spinors is very naturally a hyper-finite factor of type II_1. During the last few years I have gradually learned something about the magnificent mathematical beauty of these objects.

1. TGD inspired quantum measurement theory with measurement resolution characterized in terms of Jones inclusion and based on HFFs of type II_1 brings in non-commutative quantum physics and leads to powerful general predictions [C8, H2]. The basic idea is that complex rays of the state space are replaced with $\mathcal{N}$ rays for Jones inclusion $\mathcal{N} \subset \mathcal{M}$. $\mathcal{N}$ defines the measurement resolution in the sense that the group G leaving elements of $\mathcal{N}$ invariant characterizes the measured quantum numbers.

2. Hyper-finite factors have the property that they are isomorphic with their tensor powers. This makes possible the construction of vertices as unitary isomorphisms between tensor products of HFFs of type II_1 associated with incoming and outgoing states. The core part of S-matrix boils down to a unitary isomorphism between tensor products of hyper-finite factors of type II_1 associated with incoming *resp.* outgoing partonic 3-surfaces whose ends meet at the partonic 2-surface representing reaction vertex.

3. The study of Jones inclusions leads to the idea that Planck constant is dynamical and quantized. The predicted hierarchy of Planck constants involving a generalization of imbedding space concept and an explanation of dark matter as macroscopic quantum phases [A9]. Here the special mathematical role of Jones inclusions with index $r \leq 4$ is crucial.

4. The properties of HFFs inspire also the idea that TGD based physics should able to mimic any imaginable quantum physical system defined by gauge theory or conformal field theory involving Kac-Moody symmetry. Thus the ultimate physics would be kind of analog for Turing machine. The prediction inspired by TGD based explanation of McKay correspondence [77] is that TGD Universe is indeed able to simulate gauge and Kac-Moody dynamics of a very large subset of ADE type groups. In fact, also much more general prediction that simulation should be possible for any compact Lie group emerges.

5. HFFs of type II_1 lead also to deep connections with number theory [77] and number theoretical braids can be interpreted in terms of representations of Galois groups assignable with partonic 2-surfaces in terms of HFFs of type II_1. Particle decay represents a replication of number theoretical braids and this together with p-adic fractality and hierarchy of Planck constants suggests strongly direct connections with genetic code and DNA.

10.1.11 Could TGD emerge from a local version of infinite-dimensional Clifford algebra?

A crucial step in the progress was the realization that TGD emerges from the mere idea that a local version of hyper-finite factor of type II_1 represented as an infinite-dimensional Clifford algebra must exist (as analog of say local gauge groups). This implies a connection with the classical number fields. Quantum version of complexified octonions defining the coordinate with respect to which one localizes is unique by its non-associativity allowing to uniquely separate the powers of octonionic coordinate from the associative infinite-dimensional Clifford algebra elements appearing as Taylor coefficients in the expansion of Clifford algebra valued field.

Associativity condition implies the classical and quantum dynamics of TGD. Space-time surfaces are hyper-quaternionic of co-hyper-quatenionic sub-manifolds of hyper-octonionic imbedding space HO. Also the interpretation as a four-surface in $H = M^4 \times CP_2$ emerges and implies $HO - H$ duality. What is also nice that Minkowski spaces correspond to the spectra for the eigenvalues of maximal set of commuting quantum coordinates of suitably defined quantum spaces. Thus Minkowski signature has quantal explanation.

10.2 S-matrix as a functor in TQFTs

John Baez's [78] discusses in a physicist friendly manner the possible application of category theory to physics. The lessons obtained from the construction of topological quantum field theories (TQFTs) suggest that category theoretical thinking might be very useful in attempts to construct theories of quantum gravitation.

The point is that the Hilbert spaces associated with the initial and final state n-1-manifold of n-cobordism indeed form in a natural manner category. Morphisms of Hilb in turn are unitary or possibly more general maps between Hilbert spaces. TQFT itself is a functor assigning to a cobordism the counterpart of S-matrix between the Hilbert spaces associated with the initial and final n-1-manifold. The surprising result is that for $n \leq 4$ the S-matrix can be unitary S-matrix only if the cobordism is trivial. This should lead even string theorist to raise some worried questions.

In the hope of feeding some category theoretic thinking into my spine, I briefly summarize some of the category theoretical ideas discussed in the article and relate it to the TGD vision, and after that discuss the worried questions from TGD perspective. That space-time makes sense only relative to imbedding space would conform with category theoretic thinking.

10.2.1 The *-category of Hilbert spaces

Baez considers first the category of Hilbert spaces. Intuitively the definition of this category looks obvious: take linear spaces as objects in category Set, introduce inner product as additional structure and identify morphisms as maps preserving this inner product. In finite-D case the category with inner product is however identical to the linear category so that the inner product does not seem to be absolutely essential. Baez argues that in infinite-D case the morphisms need not be restricted to unitary transformations: one can consider also bounded linear operators as morphisms since they play key role in quantum theory (consider only observables as Hermitian operators). For hyper-finite factors of type II_1 inclusions define very important morphisms which are not unitary transformations but very similar to them. This challenges the belief about the fundamental role of unitarity and raises the question about how to weaken the unitarity condition without losing everything.

The existence of the inner product is essential only for the metric topology of the Hilbert space. Can one do without inner product as an inherent property of state space and reduce it to a morphism? One can indeed express inner product in terms of morphisms from complex numbers to Hilbert space and their conjugates. For any state Ψ of Hilbert space there is a unique morphisms T_Ψ from C to Hilbert space satisfying $T_\Psi(1) = \Psi$. If one assumes that these morphisms have conjugates T^*_Ψ mapping Hilbert space to C, inner products can be defined as morphisms $T^*_\Phi T_\Psi$. The Hermitian conjugates of operators can be defined with respect to this inner product so that one obtains *-category. Reader has probably realized that T_Ψ and its conjugate correspond to ket and bra in Dirac's formalism.

Note that in TGD framework based on hyper-finite factors of type II_1 (HFFs) the inclusions of complex rays might be replaced with inclusions of HFFs with included factor representing the finite measurement resolution. Note also the analogy of inner product with the representation of space-times as 4-surfaces of the imbedding space in TGD.

10.2.2 The monoidal *-category of Hilbert spaces and its counterpart at the level of nCob

One can give the category of Hilbert spaces a structure of monoid by introducing explicitly the tensor products of Hilbert spaces. The interpretation is obvious for physicist. Baez describes the details of this identification, which are far from trivial and in the theory of quantum groups very interesting things happen. A non-commutative quantum version of the tensor product implying braiding is possible and associativity condition leads to the celebrated Yang-Baxter equations: inclusions of HFFs lead to quantum groups too.

At the level of nCob the counterpart of the tensor product is disjoint union of n-1-manifolds. This unavoidably creates the feeling of cosmic loneliness. Am I really a disjoint 3-surface in emptiness which is not vacuum even in the geometric sense? Cannot be true!

This horrifying sensation disappears if n-1-manifolds are n-1-surfaces in some higher-dimensional imbedding space so that there would be at least something between them. I can emit a little baby manifold moving somewhere perhaps being received by some-one somewhere and I can receive radiation from some-one at some distance and in some direction as small baby manifolds making gentle tosses on my face!

This consoling feeling could be seen as one of the deep justifications for identifying fundamental objects as light-like partonic 3-surfaces in TGD framework. Their ends correspond to 2-D partonic surfaces at the boundaries of future or past directed light-cones (states of positive and negative energy respectively) and are indeed disjoint but not in the desperately existential sense as 3-geometries of General Relativity.

This disjointness has also positive aspect in TGD framework. One can identify the color degrees of freedom of partons as those associated with CP_2 degrees of freedom. For instance, SU(3) analogs for rotational states of rigid body become possible. 4-D space-time surfaces as preferred extremals of Kähler action connect the partonic 3-surfaces and bring in classical representation of correlations and thus of interactions. The representation as sub-manifolds makes it also possible to speak about positions of these sub-Universes and about distances between them. The habitants of TGD Universe are maximally free but not completely alone.

10.2.3 TQFT as a functor

The category theoretic formulation of TQFT relies on a very elegant and general idea. Quantum transition has as a space-time correlate an n-dimensional surface having initial final states as its n-1-dimensional ends. One assigns Hilbert spaces of states to the ends and S-matrix would be a unitary morphism between the ends. This is expressed in terms of the category theoretic language by introducing the category nCob with objects identified as n-1-manifolds and morphisms as cobordisms and *-category Hilb consisting of Hilbert spaces with inner product and morphisms which are bounded linear operators which do not however preserve the unitarity. Note that the morphisms of nCob cannot anymore be identified as maps between n-1-manifolds interpreted as sets with additional structure so that in this case category theory is more powerful than set theory.

TQFT is identified as a functor nCob → Hilb assigning to n-1-manifolds Hilbert spaces, and to cobordisms unitary S-matrices in the category Hilb. This looks nice but the surprise is that for $n \leq 4$ unitary S-matrix exists only if the cobordism is trivial so that topology changing transitions are not possible unless one gives up unitarity.

This raises several worried questions.

1. Does this result mean that in TQFT sense unitary S-matrix for topology changing transitions from a state containing n_i closed strings to a state containing $n_f \neq n_i$ strings does not exist? Could the situation be same also for more general non-topological stringy S-matrices? Could the non-converging perturbation series for S-matrix with finite individual terms matrix fail to no non-perturbative counterpart? Could it be that M-theory is doomed to remain a dream with no hope of being fulfilled?

2. Should one give up the unitarity condition and require that the theory predicts only the relative probabilities of transitions rather than absolute rates? What the proper generalization of the S-matrix could be?

3. What is the relevance of this result for quantum TGD?

10.3 S-matrix as a functor in quantum TGD

The result about the non-existence of unitary S-matrix for topology changing cobordisms allows new insights about the meaning of the departures of TGD from string models.

10.3.1 Cobordism cannot give interesting selection rules

When I started to work with TGD for more than 28 years ago, one of the first ideas was that one could identify the selection rules of quantum transitions as topological selection rules for cobordisms. Within week or two came the great disappointment: there were practically no selection rules. Could one revive this naive idea? Could the existence of unitary S-matrix force the topological selection rules after all? I am skeptic. If I have understood correctly the discussion of what happens in 4-D case [79] only the exotic diffeo-structures modify the situation in 4-D case.

10.3.2 Light-like 3-surfaces allow cobordism

In the physically interesting GRT like situation one would expect the cobordism to be mediated by a space-time surface possessing Lorentz signature. This brings in metric and temporal distance. This means complications since one must leave the pure TQFT context. Also the classical dynamics of quantum gravitation brings in strong selection rules related to the dynamics in metric degrees of freedom so that TQFT approach is not expected to be useful from the point of view of quantum gravity and certainly not the limit of a realistic theory of quantum gravitation.

In TGD framework situation is different. 4-D space-time sheets can have Euclidian signature of the induced metric so that Lorentz signature does not pose conditions. The counterparts of cobordisms correspond at fundamental level to light-like 3-surfaces, which are arbitrarily except for the light-likeness condition (the effective 2-dimensionality implies generalized conformal invariance and analogy with 3-D black-holes since 3-D vacuum Einstein equations are satisfied). Field equations defined by the Chern-Simons action imply that CP_2 projection is at most 2-D but this condition holds true only for the extremals and one has functional integral over all light-like 3-surfaces. The temporal distance between points along light-like 3-surface vanishes. The constraints from light-likeness bring in metric degrees of freedom but in a very gentle manner and just to make the theory physically interesting.

10.3.3 Feynmann cobordism as opposed to ordinary cobordism

In string model context the discouraging results from TQFT hold true in the category of nCob, which corresponds to trouser diagrams for closed strings or for their open string counterparts. In TGD framework these diagrams are replaced with a direct generalization of Feynman diagrams for which 3-D light-like partonic 3-surfaces meet along their 2-D ends at the vertices. In honor of Feynman one could perhaps speak of Feynman cobordisms. These surfaces are singular as 3-manifolds but vertices are nice 2-manifolds. I contrast to this, in string models diagrams are nice 2-manifolds but vertices are singular as 1-manifolds (say eye-glass type configurations for closed strings).

This picture gains a strong support for the interpretation of fermions as light-like throats associated with connected sums of CP_2 type extremals with space-time sheets with Minkowski signature and of bosons as pairs of light-like wormhole throats associated with CP_2 type extremal connecting two space-time sheets with Minkowski signature of induced metric. The space-time sheets have opposite time orientations so that also zero energy ontology emerges unavoidably. There is also consistency TGD based explanation of the family replication phenomenon in terms of genus of light-like partonic 2-surfaces.

One can wonder what the 4-D space-time sheets associated with the generalized Feynman diagrams could look like? One can try to gain some idea about this by trying to assign 2-D surfaces to ordinary Feynman diagrams having a subset of lines as boundaries. In the case of $2\rightarrow 2$ reaction open string is pinched to a point at vertex. $1\rightarrow 2$ vertex, and quite generally, vertices with odd number of lines, are impossible. The reason is that 1-D manifolds of finite size can have either 0 or 2 ends whereas in higher-D the number of boundary components is arbitrary. What one expects to happen in TGD context is that wormhole throats which are at distance characterized by CP_2 fuse together in the vertex so that some kind of pinches appear also now.

10.3.4 Zero energy ontology

Zero energy ontology gives rise to a second profound distinction between TGD and standard QFT. Physical states are identified as states with vanishing net quantum numbers, in particular energy. Everything is creatable from vacuum - and one could add- by intentional action so that zero energy ontology is profoundly Eastern. Positive *resp.* negative energy parts of states can be identified as states associated with 2-D partonic surfaces at the boundaries of future *resp.* past directed light-cones, whose tips correspond to the arguments of n-point functions. Each incoming/outgoing particle would define a mini-cosmology corresponding to not so big bang/crunch. If the time scale of perception is much shorter than time interval between positive and zero energy states, the ontology looks like the Western positive energy ontology. Bras and kets correspond naturally to the positive and negative energy states and phase conjugation for laser photons making them indeed something which seems to travel in opposite time direction is counterpart for bra-ket duality.

10.3.5 Finite temperature S-matrix defines genuine quantum state in zero energy ontology

In TGD framework one encounters two S-matrix like operators.

1. There is U-matrix between zero energy states. This is expected to be rather trivial but very important from the point of view of description of intentional actions as transitions transforming p-adic partonic 3-surfaces to their real counterparts.

2. The S-matrix like operator describing what happens in laboratory corresponds to the time-like entanglement coefficients between positive and negative energy parts of the state. Measurement of reaction rates would be a measurement of observables reducing time like entanglement and very much analogous to an ordinary quantum measurement reducing space-like entanglement. There is a finite measurement resolution described by inclusion of HFFs and this means that situation reduces effectively to a finite-dimensional one.

p-Adic thermodynamics strengthened with p-adic length scale hypothesis predicts particle masses with an amazing success. At first the thermodynamical approach seems to be in contradiction with the idea that elementary particles are quantal objects. Unitarity is however *not* necessary if one accepts that only relative probabilities for reductions to pairs of initial and final states interpreted as particle reactions can be measured.

The beneficial implications of unitarity are not lost if one replaces QFT with thermal QFT. Category theoretically this would mean that the time-like entanglement matrix associated with the product of cobordisms is a product of these matrices for the factors. The time parameter in S-matrix would be replaced with a complex time parameter with the imaginary part identified as inverse temperature. Hence the interpretation in terms of time evolution is not lost.

In the theory of hyper-finite factors of type III_1 the partition function for thermal equilibrium states and S-matrix can be neatly fused to a thermal S-matrix for zero energy states and one could introduce p-adic thermodynamics at the level of quantum states. It seems that this picture applies to HFFs by restriction. Therefore the loss of unitarity S-matrix might after all turn to a victory by more or less forcing both zero energy ontology and p-adic thermodynamics.

10.3.6 Time-like entanglement coefficients as a square root of density matrix?

All quantum states do not correspond to thermal states and one can wonder what might be the most general identification of the quantum state in zero energy ontology. Density matrix formalism defines a very general formulation of quantum theory. Since the quantum states in zero energy ontology are analogous to operators, the idea that time-like entanglement coefficients in some sense define a square root of density matrix is rather natural. This would give the defining conditions

$$\begin{aligned} \rho^{+} &= SS^{\dagger} \ , \rho^{-} = S^{\dagger}S \ , \\ Tr(\rho^{\pm}) &= 1 \ . \end{aligned} \tag{100}$$

$\rho^{\pm}$ would define density matrix for positive/negative energy states. In the case HFFs of type II_1 one obtains unitary S-matrix and also the analogs of pure quantum states are possible for factors of type I. The numbers $p^{+}_{m,n} = |S^2_{m,n}|/\rho^{+}_{m,m}$ and $p^{-}_{m,n} = |S^2_{n,m}|/\rho^{-}_{m,m}$ give the counterparts of the usual scattering probabilities.

A physically well-motivated hypothesis would be that S has expression $S = \sqrt{\rho}S_0$ such that S_0 is a universal unitary S-matrix, and $\sqrt{\rho}$ is square root of a state dependent density matrix. Note that in general S is not diagonalizable in the algebraic extension involved so that it is not possible to reduce the scattering to a mere phase change by a suitable choice of state basis. Clearly, S-matrix can be seen as matrix valued generalization of Schrö;dinger amplitude. Note that the "indices" of the S-matrices correspond to configuration space spinors (fermions and their bound states giving rise to gauge bosons and gravitons) and to configuration space degrees of freedom (world of classical worlds). For hyperfinite factor of II_1 it is not strictly speaking possible to speak about indices since the matrix elements are traces of the S-matrix multiplied by projection operators to infinite-dimensional subspaces from right and left.

The functor property of S-matrices implies that they form a multiplicative structure analogous but not identical to groupoid [82]. Recall that groupoid has associative product and there exist always right and left inverses and identity in the sense that ff^{-1} and $f^{-1}f$ are always defined but not identical and one has $fgg^{-1} = f$ and $f^{-1}fg = g$.

The reason for the groupoid like property is that S-matrix is a map between state spaces associated with initial and final sets of partonic surfaces and these state spaces are different so that inverse must be replaced with right and left inverse. The defining conditions for groupoid are replaced with more general ones. Also now associativity holds but the role of inverse is taken by hermitian conjugate. Thus one has the conditions $fgg^{\dagger} = f\rho_{g,+}$ and $f^{\dagger}fg = \rho_{f,-}g$, and the conditions $ff^{\dagger} = \rho_{+}$ and $f^{\dagger}f = \rho_{-}$ are satisfied. Here $\rho_{\pm}$ is density matrix associated with positive/negative energy parts of zero energy state. If the inverses of the density matrices exist, groupoid axioms hold true since $f_L^{-1} = f^{\dagger}\rho^{-1}_{f,+}$ satisfies $ff_L^{-1} = Id_{+}$ and $f_R^{-1} = \rho^{-1}_{f,-}f^{\dagger}$ satisfies $f_R^{-1}f = Id_{-}$.

There are good reasons to believe that also tensor product of its appropriate generalization to the analog of co-product makes sense with non-triviality characterizing the interaction between the systems of the tensor product. If so, the S-matrices would form very beautiful mathematical structure bringing in mind the corresponding structures for 2-tangles and N-tangles. Knowing how incredibly powerful the group like structures have been in physics one has good reasons to hope that groupoid like structure might help to deduce a lot of information about the quantum dynamics of TGD.

A word about nomenclature is in order. S has strong associations to unitarity and it might be appropriate to replace S with some other letter. The interpretation of S-matrix as a generalized Schrödinger amplitude would suggest Ψ-matrix. Since the interaction with Kea's M-theory blog at http://keamonad.blogspot.com/ (M denotes Monad or Motif in this context) was led ot the realization of the connection with density matrix, also M-matrix might be considered. S-matrix as a functor from the category of Feynman cobordisms in turn suggests C or F. Or could just Matrix denoted by M in formulas be enough? Certainly it would inspire feeling of awe!

10.4 Finite measurement resolution: from S-matrix to quantum S-matrix

The finite resolution of quantum measurement leads in TGD framework naturally to the notion of quantum S-matrix for which elements have values in sub-factor of HFF rather than being complex numbers. It is still possible to satisfy generalized unitarity condition but one can also consider the possibility that only probabilities are conserved.

10.4.1 Quantum S-matrix

The description of finite measurement resolution in terms of Jones inclusion $\mathcal{N} \subset \mathcal{M}$ seems to boil down to a simple rule. Replace ordinary quantum mechanics in complex number field C with that in $\mathcal{N}$. This means that the notions of unitarity, hermiticity, Hilbert space ray, etc.. are replaced with their $\mathcal{N}$ counterparts.

The full S-matrix in $\mathcal{M}$ should be reducible to a finite-dimensional quantum S-matrix in the state space generated by quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ which can be regarded as a finite-dimensional matrix algebra with non-commuting $\mathcal{N}$-valued matrix elements. This suggests that full S-matrix can be expressed as S-matrix with $\mathcal{N}$-valued elements satisfying $\mathcal{N}$-unitarity conditions.

Physical intuition also suggests that the transition probabilities defined by quantum S-matrix must be commuting hermitian $\mathcal{N}$-valued operators inside every row and column. The traces of these operators give $\mathcal{N}$-averaged transition probabilities. The eigenvalue spectrum of these Hermitian gives more detailed information about details below experimental resolution. $\mathcal{N}$-hermicity and commutativity pose powerful additional restrictions on the S-matrix.

Quantum S-matrix defines $\mathcal{N}$-valued entanglement coefficients between quantum states with $\mathcal{N}$-valued coefficients. How this affects the situation? The non-commutativity of quantum spinors has a natural interpretation in terms of fuzzy state function reduction meaning that quantum spinor corresponds effectively to a statistical ensemble which cannot correspond to pure state. Does this mean that predictions for transition probabilities must be averaged over the ensemble defined by "quantum quantum states"?

10.4.2 Quantum fluctuations and Jones inclusions

Jones inclusions $\mathcal{N} \subset \mathcal{M}$ provide also a first principle description of quantum fluctuations since quantum fluctuations are by definition quantum dynamics below measurement resolution. This gives hopes for articulating precisely what the important phrase "long range quantum fluctuations around quantum criticality" really means mathematically.

1. Phase transitions involve a change of symmetry. One might hope that the change of the symmetry group $G_a \times G_b$ could universally code this aspect of phase transitions. This need not always mean a change of Planck constant but it means always a leakage between sectors of imbedding space. At quantum criticality 3-surfaces would have regions belonging to at least two sectors of H.

2. The long range of quantum fluctuations would naturally relate to a partial or total leakage of the 3-surface to a sector of imbedding space with larger Planck constant meaning zooming up of various quantal lengths.

3. For S-matrix in $\mathcal{M}/\mathcal{N}$ quantum criticality would mean a special kind of eigen state for the transition probability operator defined by the S-matrix. The properties of the number theoretic braids contributing to the S-matrix should characterize this state. The strands of the critical braids would correspond to fixed points for $G_a \times G_b$ or its subgroup.

4. Accepting number theoretical vision, quantum criticality would mean that super-canonical conformal weights and/or generalized eigenvalues of the modified Dirac operator correspond to zeros of Riemann ζ so that the points of the number theoretic braids would be mapped to fixed points of G_a and G_b at geodesic spheres of $\delta M^4_+ = S^2 \times R_+$ and CP_2. Also weaker critical points which are fixed points of only subgroup of G_a or G_b can be considered.

10.5 Number theoretic constraints on S-matrix

Number theoretical universality leads to the hypothesis that S-matrix elements must be algebraic numbers [C2]. This is achieved naturally if the definition of S-matrix elements involves only the data associated with the number theoretic braid. This leads naturally to a connection with braiding S-matrices also in the case of real-to-real transitions. Also the concept of number theoretic string emerges. This picture becomes highly predictive if one accepts number theoretic universality of Riemann Zeta [C2] to be discussed at the end of the article.

The partonic vertices appearing in S-matrix elements should be expressible in terms of N-point functions of almost topological $N = 4$ super-conformal field theory but with the p-adically questionable N-fold integrals over string replaced with sums over the strands of a braid: spin chain type string discretization could be in question [C2]. Propagators, that is correlations between partonic 2-surfaces, would be due to the interior dynamics of space-time sheets which means a deviation from super string theory. Another function of interior degrees of freedom is to provide zero modes of metric of WCW identifiable as classical degrees of freedom of quantum measurement theory entangling with quantal degrees of freedom at partonic 3-surfaces.

10.6 Does Connes tensor product fix the allowed M-matrices?

Hyperfinite factors of type II_1 and the inclusion $\mathcal{N} \subset \mathcal{M}$ inclusions have been proposed to define quantum measurement theory with a finite measurement resolution characterized by $\mathcal{N}$ and with complex rays of state space replaced with $\mathcal{N}$ rays. What this really means is far from clear.

1. Naively one expects that matrices whose elements are elements of $\mathcal{N}$ give a representation for M. Now however unit operator has unit trace and one cannot visualize the situation in terms of matrices in case of $\mathcal{M}$ and $\mathcal{N}$.
2. The state space with $\mathcal{N}$ resolution would be formally $\mathcal{M}/\mathcal{N}$ consisting of $\mathcal{N}$ rays. For $\mathcal{M}/\mathcal{N}$ one has finite-D matrices with non-commuting elements of $\mathcal{N}$. In this case quantum matrix elements should be multiplets of selected elements of $\mathcal{N}$, **not all** possible elements of $\mathcal{N}$. One cannot therefore think in terms of the tensor product of $\mathcal{N}$ with $\mathcal{M}/\mathcal{N}$ regarded as a finite-D matrix algebra.
3. What does this mean? Obviously one must pose a condition implying that $\mathcal{N}$ action commutes with matrix action just like C: this poses conditions on the matrices that one can allow. Connes tensor product [71] does just this. Note I have proposed already earlier the reduction of interactions to Connes tensor product (see the section *"Could Connes tensor product...."* later in this chapter) but without reference to zero energy ontology as a fundamental manner to define measurement resolution with respect time and assuming unitarity.

10.6.1 The argument demonstrating almost uniqueness of M-matrix

The starting point is the Jones inclusion sequence

$$\mathcal{N} \subset \mathcal{M} \subset \mathcal{M} \otimes_N \mathcal{M}...$$

Here $\mathcal{M} \otimes_N \mathcal{M}$ is Connes tensor product which can be seen as elements of the ordinary tensor product commuting with $\mathcal{N}$ action so that $\mathcal{N}$ indeed acts like complex numbers in $\mathcal{M}$. $\mathcal{M}/\mathcal{N}$ is in this picture represented with $\mathcal{M}$ in which operators defined by Connes tensor products of elements of $\mathcal{M}$. The replacement $\mathcal{M} \rightarrow \mathcal{M}/\mathcal{N}$ corresponds to the replacement of the tensor product of elements of $\mathcal{M}$ defining matrices with Connes tensor product.

One can try to generalize this picture to zero energy ontology.

1. $\mathcal{M} \otimes_{\mathcal{N}} \mathcal{M}$ would be generalized by $\mathcal{M}_+ \otimes_{\mathcal{N}} \mathcal{M}_-$. Here $\mathcal{M}_+$ would create positive energy states and $\mathcal{M}_-$ negative energy states and $\mathcal{N}$ would create

zero energy states in some shorter time scale resolution: this would be the precise meaning of finite measurement resolution.

2. Connes entanglement with respect to $\mathcal{N}$ would define a non-trivial and unique recipe for constructing M-matrices as a generalization of S-matrices expressible as products of square root of density matrix and unitary S-matrix but it is not how clear how many M-matrices this allows. In any case M-matrices would depend on the triplet $(\mathcal{N}, \mathcal{M}_+, \mathcal{M}_-)$ and this would correspond to p-adic length scale evolution giving replacing coupling constant evolution in TGD framework. Thermodynamics would enter the fundamental quantum theory via the square root of density matrix.

3. The defining condition for the variant of the Connes tensor product proposed here has the following equivalent forms

$$MN = N^*M \ , \quad N = M^{-1}N^*M \ , \quad N^* = MNM^{-1} \ . \tag{101}$$

   If M_1 and M_2 are two M-matrices satisfying the conditions then the matrix $M_{12} = M_1M_2^{-1}$ satisfies the following equivalent conditions

$$N = M_{12}NM_{12}^{-1} \ , \quad [N, M_{12}] = 0 \ . \tag{102}$$

   Jones inclusions with $\mathcal{M} : \mathcal{N} \leq 4$ are irreducible which means that the operators commuting with $\mathcal{N}$ consist of complex multiples of identity. Hence one must have $M_{12} = 1$ so that M-matrix is unique in this case. For $\mathcal{M} : \mathcal{N} > 4$ the complex dimension of commutator algebra of $\mathcal{N}$ is 2 so that M-matrix depends should depend on single complex parameter. The dimension of the commutator algebra associated with the inclusion gives the number of parameters appearing in the M-matrix in the general case.

   When the commutator has complex dimension $d > 1$, the representation of $\mathcal{N}$ in $\mathcal{M}$ is reducible: the matrix analogy is the representation of elements of $\mathcal{N}$ as direct sums of d representation matrices. M-matrix is a direct sum of form $M = a_1M_1 \oplus a_2M_2 \oplus ...$, where M_i are unique. The condition $\sum_i ; |a_i|^2 = 1$ is satisfied and *-commutativity holds in each summand separately.

   There are several questions. Could M_i define unique universal unitary S-matrices in their own blocks? Could the direct sum define a counterpart of a statistical ensemble? Could irreducible inclusions correspond to pure states and reducible inclusions to mixed states? Could different values of energy in thermodynamics and of the scaling generator L_0 in p-adic thermodynamics define direct summands of the inclusion? The values of conserved quantum numbers for the positive energy part of the state indeed naturally define this kind of direct direct summands.

It must be of course noticed that reducibility and thermodynamics emerge naturally also in another sense since a direct sum of HFFs of type II_1 is what one expects. The radial conformal weights associated light-cone boundary and X_l^3 would indeed naturally label the factors in the direct sum.

4. Zero energy ontology is a key element of this picture and the most compelling argument for zero energy ontology is the possibility of describing coherent states of Cooper pairs without giving up fermion number, charge, etc. conservation and automatic emerges of length scale dependent notion of quantum numbers (quantum numbers identified as those associated with positive energy factor).

To sum up, interactions would be an outcome of a finite measurement resolution and at the never-achievable limit of infinite measurement resolution the theory would be free: this would be the counterpart of asymptotic freedom.

10.6.2 How to define the inclusion of $\mathcal{N}$ physically?

The overall picture looks beautiful but it is not clear how one could define the inclusion $\mathcal{N} \subset \mathcal{M}$ precisely. One must distinguish between two cases corresponding to the unitary U-matrix representing unitary process associated with the quantum jump and defined between zero energy states and M-matrix defining the time-like entanglement between positive and negative energy states.

1. In the case of U-matrix both $\mathcal{N}$ and $\mathcal{M}$ corresponds to zero energy states. The time scale of the zero energy state created by $\mathcal{N}$ should be shorter than that for the state defined naturally as the temporal distance t_{+-} between the tips of the light-cones $M^4_\pm$ associated with the state and defining diamond like structure.

2. In the case of M-matrix one has zero energy subalgebra of algebra creating positive or negative energy states in time scale t_{+-}. In this case the time scale for zero energy states is smaller than $t_{+-}/2$. The defining conditions for the Connes tensor product are analogous to crossing symmetry but with the restriction that the crossed operators create zero energy states.

Quantum classical correspondence requires a precise formulation for the action of $\mathcal{N}$ at space-time level and this is a valuable guideline in attempts to understand what is involved. Consider now the definition of the action of $\mathcal{N}$ in the case of M-matrix.

1. In standard QFT picture the action of the element of $\mathcal{N}$ multiplies the positive or negative energy parts of the state with an operator creating a zero energy state.

2. At the space-time level one can assign positive/negative energy states to the incoming/outcoing 3-D lines of generalized Feynman diagrams (recall

that in vertices the 3-D light-like surfaces meet along their ends). At the parton level the addition of a zero energy state would be simply addition of a collection of light-like partonic 3-surfaces describing a zero energy state in a time scale shorter than that associated with incoming/outgoing positive/negative energy space-time sheet. The points of the discretized number theoretic braid would naturally contain the insertions of the second quantized induced spinor field in the description of M-matrix element in terms of N-point function.

3. At first look this operation looks completely trivial but this is not the case. The point is that the 3-D lines of zero energy diagram and those of the original positive/negative energy diagram must be assigned to *single connected* 4-D space-time surface. Note that even the minima of the λ are not same as for the original positive energy state and free zero energy state since the minimization is affected by the constraint that the resulting space-time sheet is connected.

4. What happens if one allows several disconnected space-time sheets in the initial state? Could/should one assign the zero energy state to a particular incoming space-time sheet? If so, what space-time sheet of the final state should one attach the *-conjugate of this zero energy state? Or should one allow a non-unique assignment and interpret the result in terms of different phases? If one generalizes the connectedness condition to the connectedness of the entire space-time surface characterizing zero energy state one would bet rid of the question but can still wonder how unique the assignment of the 4-D space-time surface to a given collection of light-like 3-surfaces is.

10.6.3 How to define Hermitian conjugation physically?

Second problem relates to the realization of Hermitian conjugation $\mathcal{N} \rightarrow \mathcal{N}^*$ at the space-time level. Intuitively it seems clear that the conjugation must involve M^4 time reflection with respect to some origin of M^4 time mapping partonic 3-surfaces to their time mirror images and performing T-operation for induced spinor fields acting at the points of discretized number theoretic braids.

Suppose that incoming and outgoing states correspond to light-cones M^4_+ and M^4_- with tips at points $m^0 = 0$ and $m^0 = t_{+-}$. This does not require that the preferred sub-manifolds M^2 and S^2_{II} are same for positive/negative energy states and inserted zero energy states. In this case the point $(m^0 = t_{+-}/2, m^k = 0)$ would be the natural reflection point and the operation mapping the action of $\mathcal{N}$ to the action of $\mathcal{N}^*$ would be unique.

Can one allow several light cones in the initial and final states or should one restrict M-matrix to single diamond like structure defined by the two light-cones? The most reasonable option seems to be an assignment of a diamond shape pair of light-cones to each zero energy component of the state. The temporal distance t_{+-} between the tips of the light-cones would assign a precise time scale assigned to the zero state. The zero energy states inserted to a state characterized by a time scale t_{+-} would correspond to time scales $t < t_{+-}/2$ so that a hierarchy in powers of 2 would emerge naturally. Note that the choice of quantization axes (manifolds M^2 and S^2_{II}) could be different at different levels of hierarchy.

This picture would apply naturally also in the case of U-matrix and make the cutoff hierarchy discrete in accordance with p-adic length scale hypothesis bringing in also quantization of the time scales t_{+-}. In the case of U-matrix $\mathcal{N}$ would contain besides the zero energy algebra of M-matrix also the subalgebra for which the positive and negative energy parts reside at different sides of the center of the diamond.

10.6.4 How to generalize the notion of observable?

The almost-uniqueness of M-matrix seems too good to be true and in this kind of situation it is best to try to find an argument killing the hypothesis. The first test is whether the ordinary quantum measurement theory with Hermitian operators identified as observables generalizes.

The basic implication is that M should commute with Hermitian operators of $\mathcal{N}$ assuming that they exist in some sense. All Hermitian elements of $\mathcal{N}$ could be regarded not only as observables but also as conserved charges defining symmetries of M which would be thus maximal. The geometric counterpart for this would be the fact that configuration space is a union of symmetric spaces having maximal isometry group. Super-conformal symmetries of M-matrix would be in question.

The task is to define what Hermiticity means in this kind of situation. The super-positions $N + N^*$ and products N^*N defined in an appropriate sense should Hermitian operators. One can define what the products MN^* and NM mean. There are also two Hermitian conjugations involved: $\mathcal{M}$ conjugation and $\mathcal{N}$ conjugation.

1. Consider first Hermitian conjugation in $\mathcal{M}$. The operators of $\mathcal{N}$ creating zero energy states on the positive energy side and $\mathcal{N}^*$ acting on the negative energy side are not Hermitian in the hermitian conjugation of $\mathcal{M}$. If one defines $MN^* \equiv N^*M$ and $NM \equiv MN$, the operators $N + N^*$ and N^*N indeed commute with M by the basic condition. One could label the states created by $\mathcal{M}$ by eigenvalues of a maximally commuting sub-algebra of $\mathcal{N}$. Clearly, the operators acting on positive and negative energy state spaces should be interpreted in terms of a polarization $\mathcal{N} = \mathcal{N}_+ + \mathcal{N}_-$ such that $\mathcal{N}_{+/-}$ acts on positive/negative energy states.

2. In the Hermitian conjugation of $\mathcal{N}$ which does not move the operator from positive energy state to negative energy state there certainly exist Hermitian operators and they correspond to zero energy states invariant under exchange of the incoming and outgoing states but in time scale $t_{+-}/2$. These operators are not Hermitian in $\mathcal{M}$. The commutativity of M with these operators follows also from the basic conditions.

10.6.5 Fractal hierarchy of state function reductions

In accordance with fractality, the conditions for the Connes tensor product at a given time scale imply the conditions at shorter time scales. On the other hand, in shorter time scales the inclusion would be deeper and would give rise to a larger reducibility of the representation of $\mathcal{N}$ in $\mathcal{M}$. Formally, as $\mathcal{N}$ approaches to a trivial algebra, one would have a square root of density matrix and trivial S-matrix in accordance with the idea about asymptotic freedom.

M-matrix would give rise to a matrix of probabilities via the expression $P(P_+ \to P_-) = Tr[P_+ M^\dagger P_- M]$, where P_+ and P_- are projectors to positive and negative energy energy $\mathcal{N}$-rays. The projectors give rise to the averaging over the initial and final states inside $\mathcal{N}$ ray. The reduction could continue step by step to shorter length scales so that one would obtain a sequence of inclusions. If the U-process of the next quantum jump can return the M-matrix associated with $\mathcal{M}$ or some larger HFF, U process would be kind of reversal for state function reduction.

Analytic thinking proceeding from vision to details; human life cycle proceeding from dreams and wild actions to the age when most decisions relate to the routine daily activities; the progress of science from macroscopic to microscopic scales; even biological decay processes: all these have an intriguing resemblance to the fractal state function reduction process proceeding to to shorter and shorter time scales. Since this means increasing thermality of M-matrix, U process as a reversal of state function reduction might break the second law of thermodynamics.

The conservative option would be that only the transformation of intentions to action by U process giving rise to new zero energy states can bring in something new and is responsible for evolution. The non-conservative option is that the biological death is the U-process of the next quantum jump leading to a new life cycle. Breathing would become a universal metaphor for what happens in quantum Universe. The 4-D body would be lived again and again.

10.6.6 How generalized braid diagrams relate to the perturbation theory?

Many steps of progress have occurred in the understanding of TGD lately.

1. In a given measurement resolution characterized by the inclusion of HFFs of type II_1 Connes tensor product defines an almost universal M-matrix apart from the non-uniqueness due to the facts that one has a direct sum of hyper-finite factors of type II_1 (sum over conformal weights at least) and the fact that the included algebra defining the measurement resolution can be represented in a reducible manner. The S-matrices associated with irreducible factors would be unique in a given measurement resolution and the non-uniqueness would make possible non-trivial density matrices and thermodynamics.

2. As explained in [C1], Higgs vacuum expectation is proportional to the

generalized position dependent eigenvalue of the modified Dirac operator and its minima define naturally number theoretical braids as orbits for the minima of the universal Higgs potential: fusion and decay of braid strands emerge naturally. Thus the speculation [C7] about a generalization of braid diagrams to Feynman diagram likes objects, which I already began to think to be too crazy to be true, finds a very natural realization.

In [C1] I explained how generalized braid diagrams emerge naturally as orbits of the minima of Higgs defined as a generalized eigenvalue of the modified Dirac operator. The association of generalized braid diagrams to incoming and outgoing 3-D partonic legs and possibly also vertices of the generalized Feynman diagrams forces to ask whether the generalized braid diagrams could give rise to a counterpart of perturbation theoretical formalism via the functional integral over configuration space degrees of freedom.

The association of generalized braid diagrams to incoming and outgoing partonic legs and possibly also vertices of the generalized Feynman diagrams forces to ask whether the generalized braid diagrams could give rise to a counterpart of perturbation theoretical formalism via the functional integral over configuration space degrees of freedom.

The question is how the functional integral over configuration space degrees of freedom relates to the generalized braid diagrams. The basic conjecture motivated also number theoretically is that radiative corrections in this sense sum up to zero for critical values of Kähler coupling strength and Kähler function codes radiative corrections to classical physics via the dependence of the scale of M^4 metric on Planck constant. Cancellation occurs only for critical values of Kähler coupling strength α_K: for general values of α_K cancellation would require separate vanishing of each term in the sum and does not occur.

This would mean following.

1. One would not have perturbation theory around a given maximum of Kähler function but as a sum over increasingly complex maxima of Kähler function. Radiative corrections in the sense of perturbative functional integral around a given maximum would vanish (so that the expansion in terms of braid topologies would not make sense around single maximum). Radiative corrections would not vanish in the sense of a sum over 3-topologies obtained by adding radiative corrections as zero energy states in shorter time scale.

2. Connes tensor product with a given measurement resolution would correspond to a restriction on the number of maxima of Kähler function labelled by the braid diagrams. For zero energy states in a given time scale the maxima of Kähler function could be assigned to braids of minimal complexity with braid vertices interpreted in terms of an addition of radiative corrections. Hence a connection with QFT type Feyman diagram expansion would be obtained and the Connes tensor product would have a practical computational realization.

3. The cutoff in the number of topologies (maxima of Kähler function contributing in a given resolution defining Connes tensor product) would be always finite in accordance with the algebraic universality.

4. The time scale resolution defined by the temporal distance between the tips of the causal diamond defined by the future and past light-cones applies to the addition of zero energy sub-states and one obtains a direct connection with p-adic length scale evolution of coupling constants since the time scales in question naturally come as negative powers of two. More precisely, p-adic p-adic primes near power of two are very natural since the coupling constant evolution comes in powers of two of fundamental 2-adic length scale.

There are still some questions. Radiative corrections around given 3-topology vanish. Could radiative corrections sum up to zero in an ideal measurement resolution also in 2-D sense so that the initial and final partonic 2-surfaces associated with a partonic 3-surface of minimal complexity would determine the outcome completely? Could the 3-surface of minimal complexity correspond to a trivial diagram so that free theory would result in accordance with asymptotic freedom as measurement resolution becomes ideal?

The answer to these questions seems to be 'No'. In the p-adic sense the ideal limit would correspond to the limit $p \rightarrow 0$ and since only $p \rightarrow 2$ is possible in the discrete length scale evolution defined by primes, the limit is not a free theory. This conforms with the view that CP_2 length scale defines the ultimate UV cutoff.

10.6.7 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

One can wonder how this picture relates to the earlier hypothesis that p-adic length coupling constant evolution is coded to the hypothesized $log(p)$ normalization of the eigenvalues of the modified Dirac operator D. There are objections against this normalization. $log(p)$ factors are not number theoretically favored and one could consider also other dependencies on p. Since the eigenvalue spectrum of D corresponds to the values of Higgs expectation at points of partonic 2-surface defining number theoretic braids, Higgs expectation would have $log(p)$ multiplicative dependence on p-adic length scale, which does not look attractive.

Is there really any need to assume this kind of normalization? Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic

thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5$ μm (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3.

10.6.8 How quantum classical correspondence is realized at parton level?

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or a boundary term of YM action associated with a particle carrying gauge charges of the quantum state. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography. The difference between Chern-Simons action characterizing quantum state and the fundamental Chern-Simons type factor associated with the Kähler form would be that the latter emerges as the phase of the Dirac determinant.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

10.6.9 Some further comments about Connes tensor product

Below some further comments related to Connes tensor product.

1. M-matrix as an anti-unitary operator

The proposed form of Connes tensor product cannot be correct if M is a linear operator. The point is that if the conditions hold true for operator N and M then they cannot hold true for iN and M. One could restrict Connes tensor product to only Hermitian operators of $\mathcal{N}$ so that M-matrix would have $\mathcal{N}$ as symmetries. Another equivalent way to cope with the difficulty is to assume that M is anti-unitary operator. This assumption is natural since negative energy states are identified as hermitian conjugates of positive energy states so that entanglement matrix is interpreted as matrix multiplication plus conjugation acting on (say) negative energy states.

In both cases the interpretation is that the Hermitian operators of $\mathcal{N}$ act as symmetries of M-matrix. Quite generally, the interpretation would be in terms of symmetries of $U(n)$ or its subgroup. This conforms with the earlier view that finite measurement resolution allows to have any compact group as the group of dynamical symmetries. This might also relate to the connection between between Jones inclusions and Dynkin diagrams of ADE groups.

2. Connes tensor product in finite-D case

In the finite-dimensional case Connes tensor product of Hilbert spaces has a rather simple interpretation. If the matrix algebra N of $n \times n$ matrices acts on V from right, V can be regarded as a space formed by $m \times n$ matrices for some value of m. If N acts from left on W, W can be regarded as space of $n \times r$

matrices.

1. In the first representation the Connes tensor product of spaces V and W consists of $m \times r$ matrices and Connes tensor product is represented as the product VW of matrices as $(VW)_{mr}e^{mr}$. In this representation the information about N disappears completely as the interpretation in terms of measurement resolution suggests. The sum over intermediate states defined by N brings in mind path integral.

2. An alternative and more physical representation is as a state

$$\sum_n V_{mn}W_{nr}e^{mn} \otimes e^{nr}$$

in the tensor product $V \otimes W$.

3. One can also consider two spaces V and W in which N acts from right and define Connes tensor product for $A^\dagger \otimes_N B$ or its tensor product counterpart. This case corresponds to the modification of the Connes tensor product of positive and negative energy states. Since Hermitian conjugation is involved, matrix product does not define the Connes tensor product now. For $m = r$ case entanglement coefficients should define a unitary matrix commuting with the action of the Hermitian matrices of N and interpretation would be in terms of symmetry. HFF property would encourage to think that this representation has an analog in the case of HFFs of type II_1.

4. Also type I_n factors are possible and for them Connes tensor product makes sense if one can assign the inclusion of finite-D matrix algebras to a measurement resolution.

3. Connes tensor product in positive/negative energy sector

Ordinary linear Connes tensor product makes sense also in positive/negative energy sector and also now it makes sense to speak about measurement resolution. Hence one can ask whether Connes tensor product should be posed as a constraint on space-like entanglement. The interpretation could be in terms of the formation of bound states. The reducibility of HFFs and inclusions means that the tensor product is not uniquely fixed and ordinary entanglement could correspond to this kind of entanglement. Also the counterpart of p-adic coupling constant evolution would makes sense. The interpretation of Connes tensor product would be as the variance of the states with respect to some subgroup of $U(n)$ associated with the measurement resolution: the analog of color confinement would be in question.

10.7 Summary about the construction of S-matrix

It is perhaps wise to summarize briefly the vision about S-matrix.

1. S-matrix defines entanglement between positive and negative energy parts of zero energy states. This kind of S-matrix need not be unitary unlike the U-matrix associated with unitary process forming part of quantum jump. There are several good arguments suggesting that that S-matrix cannot be unitary but can be regarded as thermal S-matrix so that thermodynamics would become an essential part of quantum theory. In TGD framework path integral formalism is given up although functional integral over the 3-surfaces is present.

2. Almost topological QFT property of quantum allows to identify S-matrix as a functor from the category of generalized Feynman cobordisms to the category of operators mapping the Hilbert space of positive energy states to that for negative energy states: these Hilbert spaces are assignable to partonic 2-surfaces. It is difficult to overestimate the importance of this result bringing category theory absolutely essential part of quantum TGD. One can assign to S-matrix a complex parameter whose real part has interpretation as interaction time and imaginary part as the inverse temperature. S-matrices and thus also quantum states in zero energy ontology possess a semigroup like structure and in the product time and inverse temperature are additive. This suggests that the cosmological evolution of temperature as $T \propto 1/t$ could be understood at the level of fundamental quantum theory. The most general identification of the time like entanglement coefficients would be as a "square root" of density matrix thus satisfying the condition $\rho = SS^{\dagger}$, $Tr(\rho) = 1$.

3. S-matrix should be constructible as a generalization of braiding S-matrix in such a manner that the number theoretic braids assignable to light-like partonic 3-surfaces glued along their ends at 2-dimensional partonic 2-surfaces representing reaction vertices replicate in the vertex [C7].

4. The construction of braiding S-matrices assignable to the incoming and outgoing partonic 2-surfaces is not a problem [C7]. The problem is to express mathematically what happens in the vertex. Here the observation that the tensor product of HFFs of type II is HFF of type II is the key observation. Many-parton vertex can be identified as a unitary isomorphism between the tensor product of incoming *resp.* outgoing HFFs. A reduction to HFF of type II_1 occurs because only a finite-dimensional projection of S-matrix in bosonic degrees of freedom defines a normalizable state. In the case of factor of type II_∞ only thermal S-matrix is possible without finite-dimensional projection and thermodynamics would thus emerge as an essential part of quantum theory.

5. HFFs of type III could also appear at the level of field operators used to create states but at the level of quantum states everything reduces to HFFs of type II_1 and their tensor products giving these factors back. If braiding automorphisms reduce to the purely intrinsic unitary automorphisms of HFFs of type III then for certain values of the time parameter

of automorphism having interpretation as a scaling parameter these automorphisms are trivial. These time scales could correspond to p-adic time scales so that p-adic length scale hypothesis would emerge at the fundamental level. In this kind of situation the braiding S-matrices associated with the incoming and outgoing partons could be trivial so that everything would reduce to this unitary isomorphism: a counterpart for the elimination of external legs from Feynman diagram in QFT.

6. One might hope that all complications related to what happens for *space-like* 3-surfaces could be eliminated by quantum classical correspondence stating that space-time view about particle reaction is only a space-time correlate for what happens in quantum fluctuating degrees of freedom associated with partonic 2-surfaces. This turns out to be the case only in non-perturbative phase. The reason is that the arguments of n-point function appear as continuous moduli of Kähler function. In non-perturbative phases the dependence of the maximum of Kähler function on the arguments of n-point function cannot be regarded as negligible and Kähler function becomes the key to the understanding of these effects including formation of bound states and color confinement.

7. In this picture light-like 3-surface would take the dual role as a correlate for both state and time evolution of state and this dual role allows to understand why the restriction of time like entanglement to that described by S-matrix must be made. For fixed values of moduli each reaction would correspond to a minimal braid diagram involving exchanges of partons being in one-one correspondence with a maximum of Kähler function. By quantum criticality and the requirement of ideal quantum-classical correspondence only one such diagram would contribute for given values of moduli.

8. A completely unexpected prediction deserving a special emphasis is that number theoretic braids replicate in vertices. This is of course the braid counterpart for the introduction of annihilation and creation of particles in the transition from free QFT to an interacting one. This means classical replication of the number theoretic information carried by them. This allows to interpret one of the TGD inspired models of genetic code [L4] in terms of number theoretic braids representing at deeper level the information carried by DNA. This picture provides also further support for the proposal that DNA acts as topological quantum computer utilizing braids associated with partonic light-like 3-surfaces (which can have arbitrary siz

9. [E9]. In the reverse direction one must conclude that even elementary particles could be information processing and communicating entities in TGD Universe.

11 General vision about coupling constant evolution

Zero energy ontology, the construction of M-matrix as time like entanglement coefficients defining Connes tensor product characterizing finite measurement resolution in terms of inclusion of hyper-finite factors of type II_1, the realization that symplectic invariance of N-point functions provides a detailed mechanism eliminating UV divergences, and the understanding of the relationship between super-canoniocal and super Kac-Moody symmetries: these are the pieces of the puzzle whose combination making possible a rather concrete vision about coupling constant evolution in TGD Universe and even a rudimentary form of generalize Feynman rules.

11.1 General ideas about coupling constant evolution

11.1.1 Zero energy ontology

In zero energy ontology one replaces positive energy states with zero energy states with positive and negative energy parts of the state at the boundaries of future and past direct light-cones forming a causal diamond. All conserved quantum numbers of the positive and negative energy states are of opposite sign so that these states can be created from vacuum. "Any physical state is creatable from vacuum" becomes thus a basic principle of quantum TGD and together with the notion of quantum jump resolves several philosophical problems (What was the initial state of universe?, What are the values of conserved quantities for Universe, Is theory building completely useless if only single solution of field equations is realized?).

At the level of elementary particle physics positive and negative energy parts of zero energy state are interpreted as initial and final states of a particle reaction so that quantum states become physical events. Equivalence Principle would hold true in the sense that the classical gravitational four-momentum of the vacuum extremal whose small deformations appear as the argument of configuration space spinor field is equal to the positive energy of the positive energy part of the zero energy quantum state. Equivalence Principle is expected to hold true for elementary particles and their composites but not for the quantum states defined around non-vacuum extremals.

11.1.2 Does the finiteness of measurement resolution dictate the laws of physics?

The hypothesis that the mere finiteness of measurement resolution could determine the laws of quantum physics [C2] completely belongs to the category of not at all obvious first principles. The basic observation is that the Clifford algebra spanned by the gamma matrices of the "world of classical worlds" represents a von Neumann algebra [63] known as hyperfinite factor of type II_1

(HFF) [A9, C8, C2]. HFF [65, 48] is an algebraic fractal having infinite hierarchy of included subalgebras isomorphic to the algebra itself [58]. The structure of HFF is closely related to several notions of modern theoretical physics such as integrable statistical physical systems [49], anyons [95], quantum groups and conformal field theories[71, 40], and knots and topological quantum field theories [51, 52].

Zero energy ontology is second key element. In zero energy ontology these inclusions allow an interpretation in terms of a finite measurement resolution: in the standard positive energy ontology this interpretation is not possible. Inclusion hierarchy defines in a natural manner the notion of coupling constant evolution and p-adic length scale hypothesis follows as a prediction. In this framework the extremely heavy machinery of renormalized quantum field theory involving the elimination of infinities is replaced by a precisely defined mathematical framework. More concretely, the included algebra creates states which are equivalent in the measurement resolution used. Zero energy states are associated with causal diamond formed by a pair of future and past directed light-cones having positive and negative energy parts of state at their boundaries. Zero energy state can be modified in a time scale shorter than the time scale of the zero energy state itself.

On can imagine two kinds of measurement resolutions. The element of the included algebra can leave the quantum numbers of the positive and negative energy parts of the state invariant, which means that the action of subalgebra leaves M-matrix invariant. The action of the included algebra can also modify the quantum numbers of the positive and negative energy parts of the state such that the zero energy property is respected. In this case the Hermitian operators subalgebra must commute with M-matrix.

The temporal distance between the tips of light-cones corresponds to the secondary p-adic time scale $T_{p,2} = \sqrt{p}T_p$ by a simple argument based on the observation that light-like randomness of light-like 3-surface is analogous to Brownian motion. This gives the relationship $T_p = L_p^2/Rc$, where R is CP_2 size. The action of the included algebra corresponds to an addition of zero energy parts to either positive or negative energy part of the state and is like addition of quantum fluctuation below the time scale of the measurement resolution. The natural hierarchy of time scales is obtained as $T_n = 2^{-n}T$ since these insertions must belong to either upper or lower half of the causal diamond. This implies that preferred p-adic primes are near powers of 2. For electron the time scale in question is .1 seconds defining the fundamental biorhythm of 10 Hz.

M-matrix representing a generalization of S-matrix and expressible as a product of a positive square root of the density matrix and unitary S-matrix would define the dynamics of quantum theory [C2]. The notion of thermodynamical state would cease to be a theoretical fiction and in a well-defined sense quantum theory could be regarded as a square root of thermodynamics. M-matrix is identifiable in terms of Connes tensor product [48] and therefore exists and is almost unique. Connes tensor product implies that the Hermitian elements of the included algebra commute with M-matrix and hence act like infinitesimal symmetries. A connection with integrable quantum field theories is suggestive. The remaining challenge is the calculation of M-matrix and the needed machinery might already exist.

The tension is present also now. The connection with visions should come from the discretization in terms of number theoretic braids providing space-time correlate for the finite measurement resolution and making p-adicization in terms of number theoretic braids possible. Number theoretic braids give a connection with the construction of configuration space geometry in terms of Dirac determinant and with TGD as almost TQFT and with conformal field theory approach. The mathematics for the inclusions of hyper-finite factors of type II_1 is also closely related to that for conformal field theories including quantum groups relating closely to Connes tensor product and non-commutativity.

11.1.3 How do p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

Zero energy ontology in which zero energy states have as imbedding space correlates causal diamonds for which the distance between the tips of future and past directed light-cones are power of 2 multiples of fundamental time scale implies in a natural manner coupling constant evolution.

Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond

to $L(169) \simeq 5\ \mu$m (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3.

11.2 Both symplectic and conformal field theories are needed in TGD framework

Before one can say anything quantitative about coupling constant evolution, one must have a formulation for its TGD counterpart and thus also a more detailed formulation for how to calculate M-matrix elements. There is also the question about infinities. By very general arguments infinities of quantum field theories are predicted to cancel in TGD Universe - basically by the non-locality of Kähler function as a functional of 3-surface and by the general properties of the vacuum functional identified as the exponent of Kähler function. The precise mechanism leading to the cancellation of infinities of local quantum field theories has remained unspecified. Only the realization that the symplectic invariance of quantum TGD provides a mechanism regulating the short distance behavior of N-point functions changed the situation in this respect. This also leads to concrete view about the generalized Feynman diagrams giving M-matrix elements and rather close resemblance with ordinary Feynman diagrammatics.

11.2.1 Symplectic invariance

Symplectic (or canonical as I have called them) symmetries of $\delta M^4_+ \times CP_2$ (light-cone boundary briefly) act as isometries of the "world of classical worlds". One can see these symmetries as analogs of Kac-Moody type symmetries with symplectic transformations of $S^2 \times CP_2$, where S^2 is $r_M = constant$ sphere of lightcone boundary, made local with respect to the light-like radial coordinate r_M taking the role of complex coordinate. Thus finite-dimensional Lie group G is replaced with infinite-dimensional group of symplectic transformations. This inspires the question whether a symplectic analog of conformal field theory at $\delta M^4_+ \times CP_2$ could be relevant for the construction of n-point functions in quantum TGD and what general properties these n-point functions would have. This section appears already in the previous chapter about symmetries of quantum TGD [C1] but because the results of the section provide the first concrete construction recipe of M-matrix in zero energy ontology, it is included also in this chapter.

11.2.2 Symplectic QFT at sphere

Actually the notion of symplectic QFT emerged as I tried to understand the properties of cosmic microwave background which comes from the sphere of last scattering which corresponds roughly to the age of 5×10^5 years [D8]. In this situation vacuum extremals of Kähler action around almost unique critical Robertson-Walker cosmology imbeddable in $M^4 \times S^2$, where there is homologically trivial geodesic sphere of CP_2. Vacuum extremal property is satisfied for any space-time surface which is surface in $M^4 \times Y^2$, Y^2 a Lagrangian sub-manifold of CP_2 with vanishing induced Kähler form. Symplectic transformations of CP_2 and general coordinate transformations of M^4 are dynamical symmetries of the vacuum extremals so that the idea of symplectic QFT emerges natural. Therefore I shall consider first symplectic QFT at the sphere S^2 of last scattering with temperature fluctution $\Delta T/T$ proportional to the fluctuation of the metric component g_{aa} in Robertson-Walker coordinates.

1. In quantum TGD the symplectic transformation of the light-cone boundary would induce action in the "world of classical worlds" (light-like 3-surfaces). In the recent situation it is convenient to regard perturbations of CP_2 coordinates as fields at the sphere of last scattering (call it S^2) so that symplectic transformations of CP_2 would act in the field space whereas those of S^2 would act in the coordinate space just like conformal transformations. The deformation of the metric would be a symplectic field in S^2. The symplectic dimension would be induced by the tensor properties of R-W metric in R-W coordinates: every S^2 coordinate index would correspond to one unit of symplectic dimension. The symplectic invariance in CP_2 degrees of freedom is guaranteed if the integration measure over the vacuum deformations is symplectic invariant. This symmetry does not play any role in the sequel.

2. For a symplectic scalar field $n \geq 3$-point functions with a vanishing anomalous dimension would be functions of the symplectic invariants defined by the areas of geodesic polygons defined by subsets of the arguments as points of S^2. Since n-polygon can be constructed from 3-polygons these invariants can be expressed as sums of the areas of 3-polygons expressible in terms of symplectic form. n-point functions would be constant if arguments are along geodesic circle since the areas of all sub-polygons would vanish in this case. The decomposition of n-polygon to 3-polygons brings in mind the decomposition of the n-point function of conformal field theory to products of 2-point functions by using the fusion algebra of conformal fields (very symbolically $\Phi_k\Phi_l = c_{kl}^m\Phi_m$). This intuition seems to be correct.

3. Fusion rules stating the associativity of the products of fields at different points should generalize. In the recent case it is natural to assume a non-local form of fusion rules given in the case of symplectic scalars by the equation

$$\Phi_k(s_1)\Phi_l(s_2) = \int c_{kl}^m f(A(s_1, s_2, s_3))\Phi_m(s)d\mu_s \quad . \tag{103}$$

Here the coefficients c_{kl}^m are constants and $A(s_1, s_2, s_3)$ is the area of the geodesic triangle of S^2 defined by the sympletic measure and integration is over S^2 with symplectically invariant measure $d\mu_s$ defined by symplectic form of S^2. Fusion rules pose powerful conditions on n-point functions and one can hope that the coefficients are fixed completely.

4. The application of fusion rules gives at the last step an expectation value of 1-point function of the product of the fields involves unit operator term $\int c_{kl} f(A(s_1, s_2, s)) I d d\mu_s$ so that one has

$$\langle \Phi_k(s_1)\Phi_l(s_2)\rangle = \int c_{kl} f(A(s_1, s_2, s)) d\mu_s \ . \tag{104}$$

Hence 2-point function is average of a 3-point function over the third argument. The absence of non-trivial symplectic invariants for 1-point function means that $n = 1$- an are constant, most naturally vanishing, unless some kind of spontaneous symmetry breaking occurs. Since the function $f(A(s_1, s_2, s_3))$ is arbitrary, 2-point correlation function can have both signs. 2-point correlation function is invariant under rotations and reflections.

11.2.3 Symplectic QFT with spontaneous breaking of rotational and reflection symmetries

CMB data suggest breaking of rotational and reflection symmetries of S^2. A possible mechanism of spontaneous symmetry breaking is based on the observation that in TGD framework the hierarchy of Planck constants assigns to each sector of the generalized imbedding space a preferred quantization axes. The selection of the quantization axis is coded also to the geometry of "world of classical worlds", and to the quantum fluctuations of the metric in particular. Clearly, symplectic QFT with spontaneous symmetry breaking would provide the sought-for really deep reason for the quantization of Planck constant in the proposed manner.

1. The coding of angular momentum quantization axis to the generalized imbedding space geometry allows to select South and North poles as preferred points of S^2. To the three arguments s_1, s_2, s_3 of the 3-point function one can assign two squares with the added point being either North or South pole. The difference

$$\Delta A(s_1, s_2, s_3) \quad \equiv \quad A(s_1, s_2, s_3, N) - A(s_1, s_2, s_3, S) \tag{105}$$

of the corresponding areas defines a simple symplectic invariant breaking the reflection symmetry with respect to the equatorial plane. Note that ΔA vanishes if arguments lie along a geodesic line or if any two arguments co-incide. Quite generally, symplectic QFT differs from conformal QFT in that correlation functions do not possess singularities.

2. The reduction to 2-point correlation function gives a consistency conditions on the 3-point functions

$$\langle(\Phi_k(s_1)\Phi_l(s_2))\Phi_m(s_3)\rangle = c_{kl}^r \int f(\Delta A(s_1,s_2,s))\langle\Phi_r(s)\Phi_m(s_3)\rangle d\mu_s$$
$$= \tag{106}$$
$$c_{kl}^r c_{rm} \int f(\Delta A(s_1,s_2,s)) f(\Delta A(s,s_3,t)) d\mu_s d\mu_t \ . \tag{107}$$

   Associativity requires that this expression equals to $\langle\Phi_k(s_1)(\Phi_l(s_2)\Phi_m(s_3))\rangle$ and this gives additional conditions. Associativity conditions apply to $f(\Delta A)$ and could fix it highly uniquely.

3. 2-point correlation function would be given by

$$\langle\Phi_k(s_1)\Phi_l(s_2)\rangle = c_{kl} \int f(\Delta A(s_1,s_2,s)) d\mu_s \tag{108}$$

4. There is a clear difference between $n > 3$ and $n = 3$ cases: for $n > 3$ also non-convex polygons are possible: this means that the interior angle associated with some vertices of the polygon is larger than π. $n = 4$ theory is certainly well-defined, but one can argue that so are also $n > 4$ theories and skeptic would argue that this leads to an inflation of theories. TGD however allows only finite number of preferred points and fusion rules could eliminate the hierarchy of theories.

5. To sum up, the general predictions are following. Quite generally, for $f(0) = 0$ n-point correlation functions vanish if any two arguments coincide which conforms with the spectrum of temperature fluctuations. It also implies that symplectic QFT is free of the usual singularities. For symmetry breaking scenario 3-point functions and thus also 2-point functions vanish also if s_1 and s_2 are at equator. All these are testable predictions using ensemble of CMB spectra.

11.2.4 Generalization to quantum TGD

Since number theoretic braids are the basic objects of quantum TGD, one can hope that the n-point functions assignable to them could code the properties of ground states and that one could separate from n-point functions the parts which correspond to the symplectic degrees of freedom acting as symmetries of vacuum extremals and isometries of the 'world of classical worlds'.

1. This approach indeed seems to generalize also to quantum TGD proper and the n-point functions associated with partonic 2-surfaces can be decomposed in such a manner that one obtains coefficients which are symplectic invariants associated with both S^2 and CP_2 Kähler form.

2. Fusion rules imply that the gauge fluxes of respective Kähler forms over geodesic triangles associated with the S^2 and CP_2 projections of the arguments of 3-point function serve basic building blocks of the correlation functions. The North and South poles of S^2 and three poles of CP_2 can be used to construct symmetry breaking n-point functions as symplectic invariants. Non-trivial 1-point functions vanish also now.

3. The important implication is that n-point functions vanish when some of the arguments co-incide. This might play a crucial role in taming of the singularities: the basic general prediction of TGD is that standard infinities of local field theories should be absent and this mechanism might realize this expectation.

Next some more technical but elementary first guesses about what might be involved.

1. It is natural to introduce the moduli space for n-tuples of points of the symplectic manifold as the space of symplectic equivalence classes of n-tuples. In the case of sphere S^2 convex n-polygon allows $n+1$ 3-sub-polygons and the areas of these provide symplectically invariant coordinates for the moduli space of symplectic equivalence classes of n-polygons (2^n-D space of polygons is reduced to $n+1$-D space). For non-convex polygons the number of 3-sub-polygons is reduced so that they seem to correspond to lower-dimensional sub-space. In the case of CP_2 n-polygon allows besides the areas of 3-polygons also 4-volumes of 5-polygons as fundamental symplectic invariants. The number of independent 5-polygons for n-polygon can be obtained by using induction: once the numbers $N(k,n)$ of independent $k \leq n$-simplices are known for n-simplex, the numbers of $k \leq n+1$-simplices for $n+1$-polygon are obtained by adding one vertex so that by little visual gymnastics the numbers $N(k,n+1)$ are given by $N(k,n+1) = N(k-1,n) + N(k,n)$. In the case of CP_2 the allowance of 3 analogs $\{N,S,T\}$ of North and South poles of S^2 means that besides the areas of polygons (s_1,s_2,s_3), (s_1,s_2,s_3,X), (s_1,s_2,s_3,X,Y), and (s_1,s_2,s_3,N,S,T) also the 4-volumes of 5-polygons (s_1,s_2,s_3,X,Y), and of 6-polygon (s_1,s_2,s_3,N,S,T), $X,Y \in \{N,S,T\}$ can appear as additional arguments in the definition of 3-point function.

2. What one really means with symplectic tensor is not clear since the naive first guess for the n-point function of tensor fields is not manifestly general coordinate invariant. For instance, in the model of CMB, the components of the metric deformation involving S^2 indices would be symplectic tensors. Tensorial n-point functions could be reduced to those for scalars

obtained as inner products of tensors with Killing vector fields of $SO(3)$ at S^2. Again a preferred choice of quantization axis would be introduced and special points would correspond to the singularities of the Killing vector fields.

The decomposition of Hamiltonians of the "world of classical worlds" expressible in terms of Hamiltonians of $S^2 \times CP_2$ to irreps of $SO(3)$ and $SU(3)$ could define the notion of symplectic tensor as the analog of spherical harmonic at the level of configuration space. Spin and gluon color would have natural interpretation as symplectic spin and color. The infinitesimal action of various Hamiltonians on n-point functions defined by Hamiltonians and their super counterparts is well-defined and group theoretical arguments allow to deduce general form of n-point functions in terms of symplectic invariants.

3. The need to unify p-adic and real physics by requiring them to be completions of rational physics, and the notion of finite measurement resolution suggest that discretization of also fusion algebra is necessary. The set of points appearing as arguments of n-point functions could be finite in a given resolution so that the p-adically troublesome integrals in the formulas for the fusion rules would be replaced with sums. Perhaps rational/algebraic variants of $S^2 \times CP_2 = SO(3)/SO(2) \times SU(3)/U(2)$ obtained by replacing these groups with their rational/algebraic variants are involved. Tedrahedra, octahedra, and dodecahedra suggest themselves as simplest candidates for these discretized spaces. Also the symplectic moduli space would be discretized to contain only n-tuples for which the symplectic invariants are numbers in the allowed algebraic extension of rationals. This would provide an abstract looking but actually very concrete operational approach to the discretization involving only areas of n-tuples as internal coordinates of symplectic equivalence classes of n-tuples. The best that one could achieve would be a formulation involving nothing below measurement resolution.

4. This picture based on elementary geometry might make sense also in the case of conformal symmetries. The angles associated with the vertices of the S^2 projection of n-polygon could define conformal invariants appearing in n-point functions and the algebraization of the corresponding phases would be an operational manner to introduce the space-time correlates for the roots of unity introduced at quantum level. In CP_2 degrees of freedom the projections of n-tuples to the homologically trivial geodesic sphere S^2 associated with the particular sector of CH would allow to define similar conformal invariants. This framework gives dimensionless areas (unit sphere is considered). p-Adic length scale hypothesis and hierarchy of Planck constants would bring in the fundamental units of length and time in terms of CP_2 length.

The recent view about M-matrix described in [C2] is something almost unique determined by Connes tensor product providing a formal realization for the statement that complex rays of state space are replaced with $\mathcal{N}$ rays where $\mathcal{N}$ defines the hyper-finite sub-factor of type II_1 defining the measurement resolution. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and need not be unitary. It is identified as square root of density matrix with real expressible as product of of real and positive square root and unitary S-matrix. This S-matrix is what is measured in laboratory. There is also a general vision about how vertices are realized: they correspond to light-like partonic 3-surfaces obtained by gluing incoming and outgoing partonic 3-surfaces along their ends together just like lines of Feynman diagrams. Note that in string models string world sheets are non-singular as 2-manifolds whereas 1-dimensional vertices are singular as 1-manifolds. These ingredients we should be able to fuse together. So we try once again!

1. *Iteration* starting from vertices and propagators is the basic approach in the construction of n-point function in standard QFT. This approach does not work in quantum TGD. Symplectic and conformal field theories suggest that *recursion* replaces iteration in the construction. One starts from an n-point function and reduces it step by step to a vacuum expectation value of a 2-point function using fusion rules. Associativity becomes the fundamental dynamical principle in this process. Associativity in the sense of classical number fields has already shown its power and led to a hyper-octoninic formulation of quantum TGD promising a unification of various visions about quantum TGD [E2].

2. Let us start from the representation of a zero energy state in terms of a causal diamond defined by future and past directed light-cones. Zero energy state corresponds to a quantum superposition of light-like partonic 3-surfaces each of them representing possible particle reaction. These 3-surfaces are very much like generalized Feynman diagrams with lines replaced by light-like 3-surfaces coming from the upper and lower light-cone boundaries and glued together along their ends at smooth 2-dimensional surfaces defining the generalized vertices.

3. It must be emphasized that the generalization of ordinary Feynman diagrammatics arises and conformal and symplectic QFTs appear only in the calculation of single generalized Feynman diagram. Therefore one could still worry about loop corrections. The fact that no integration over loop momenta is involved and there is always finite cutoff due to discretization together with recursive instead of iterative approach gives however good hopes that everything works. Note that this picture is in conflict with one of the earlier approaches based on positive energy ontology in which the hope was that only single generalized Feynman diagram could define the U-matrix thought to correspond to physical S-matrix at that time [E10].

4. One can actually simplify things by identifying generalized Feynman diagrams as maxima of Kähler function with functional integration carried

over perturbations around it. Thus one would have conformal field theory in both fermionic and configuration space degrees of freedom. The light-like time coordinate along light-like 3-surface is analogous to the complex coordinate of conformal field theories restricted to some curve. If it is possible continue the light-like time coordinate to a hyper-complex coordinate in the interior of 4-D space-time sheet, the correspondence with conformal field theories becomes rather concrete. Same applies to the light-like radial coordinates associated with the light-cone boundaries. At light-cone boundaries one can apply fusion rules of a symplectic QFT to the remaining coordinates. Conformal fusion rules are applied only to point pairs which are at different ends of the partonic surface and there are no conformal singularities since arguments of n-point functions do not co-incide. By applying the conformal and symplectic fusion rules one can eventually reduce the n-point function defined by the various fermionic and bosonic operators appearing at the ends of the generalized Feynman diagram to something calculable.

5. Finite measurement resolution defining the Connes tensor product is realized by the discretization applied to the choice of the arguments of n-point functions so that discretion is not only a space-time correlate of finite resolution but actually defines it. No explicit realization of the measurement resolution algebra $\mathcal{N}$ seems to be needed. Everything should boil down to the fusion rules and integration measure over different 3-surfaces defined by exponent of Kähler function and by imaginary exponent of Chern-Simons action. The continuation of the configuration space Clifford algebra for 3-surfaces with cm degrees of freedom fixed to a hyper-octonionic variant of gamma matrix field of super-string models defined in M^8 (hyper-octonionic space) and $M^8 \leftrightarrow M^4 \times CP_2$ duality leads to a unique choice of the points, which can contribute to n-point functions as intersection of M^4 subspace of M^8 with the counterparts of partonic 2-surfaces at the boundaries of light-cones of M^8. Therefore there are hopes that the resulting theory is highly unique. Symplectic fusion algebra reduces to a finite algebra for each space-time surface if this picture is correct.

6. Consider next some of the details of how the light-like 3-surface codes for the fusion rules associated with it. The intermediate partonic 2- surfaces must be involved since otherwise the construction would carry no information about the properties of the light-like 3-surface, and one would not obtain perturbation series in terms of the relevant coupling constants. The natural assumption is that partonic 2-surfaces belong to future/past directed light-cone boundary depending on whether they are on lower/upper half of the causal diamond. Hyper-octonionic conformal field approach fixes the n_{int} points at intermediate partonic two-sphere for a given light-like 3-surface representing generalized Feynman diagram, and this means that the contribution is just N-point function with $N = n_{out} + n_{int} + n_{in}$ calculable by the basic fusion rules. Coupling constant strengths would emerge through the fusion coefficients, and at least in the case of gauge

interactions they must be proportional to Kähler coupling strength since n-point functions are obtained by averaging over small deformations with vacuum functional given by the exponent of Kähler function. The first guess is that one can identify the spheres $S^2 \subset \delta M^4_\pm$ associated with initial, final and, and intermediate states so that symplectic n-points functions could be calculated using single sphere.

These findings raise the hope that quantum TGD is indeed a solvable theory. The coupling constant evolution is based on the same mechanism as in QFT and symplectic invariance replaces ad hoc UV cutoff with a genuine dynamical regulation mechanism. Causal diamond itself defines the physical IR cutoff. p-Adic and real coupling constant evolutions reflect the underlying evolution in powers of two for the temporal distance between the tips of the light-cones of the causal diamond and the association of macroscopic time scale as secondary p-adic time scale to elementary particles (.1 seconds for electron) serves as a first test for the picture. Even if one is not willing to swallow any bit of TGD, the classification of the symplectic QFTs remains a fascinating mathematical challenge in itself. A further challenge is the fusion of conformal QFT and symplectic QFT in the construction of n-point functions. One might hope that conformal and symplectic fusion rules could be treated independently.

11.2.5 More detailed view about the construction of M-matrix elements

After three decades there are excellent hopes of building an explicit recipe for constructing M-matrix elements but the devil is in the details.

1. Elimination of infinities and coupling constant evolution

The elimination of infinities would follow from the symplectic QFT part of the theory. The symplectic contribution to n-point functions vanishes when two arguments co-incide. The UV cancellation mechanism has nothing to do with the finite measurement resolution which corresponds to the size of the causal diamonds inside which the space-time sheets representing radiative corrections are. There is also IR cutoff due to the presence of largest causal diamond.

On can decompose the radiative corrections two two types. First kind of corrections appear both at the level of positive/and negative energy parts of zero energy states. Second kind of corrections appear at the level of interactions between them. This decomposition is standard in quantum field theories and corresponds to the renormalization constants of fields *resp.* renormalization of coupling constants. The corrections due to the increase of measurement resolution in time comes as very specific corrections to positive and negative energy states involving gluing of smaller causal diamonds to the upper and lower boundaries of causal diamonds along any radial light-like ray. The radiative corresponds to the interactions correspond to the addition of smaller causal diamonds in the interior of the larger causal diamond. Scales for the corrections come as scalings in powers of 2 rather than as continuous scaling of measurement resolution.

2. Conformal symmetries

The basic questions are the following ones. How hyper-octonionic/-quaternionic/-complex super-conformal symmetry relates to the super-canonical conformal symmetry at the imbedding space level and the super Kac-Moody symmetry associated with the light-like 3-surfaces? How do the dual $HO = M^8$ and $H = M^4 \times CP_2$ descriptions (number theoretic compactifcation) relate?

Concerning the understanding of these issues, the earlier construction of physical states poses strong constraints [C1].

1. The state construction utilizes both super-canonical and super Kac-Moody algebras. Super-canonical algebra has negative conformal weights and creates tachyonic ground states from which Super Kac-Moody algebra generates states with non-negative conformal weight determining the mass squared value of the state. The commutator of these two algebras annihilates the physical states. This requires that both super conformal algebras must allow continuation to hyper-octonionic algebras, which are independent.

2. The light-like radial coordinate at $\delta M^4_\pm$ can be continued to a hyper-complex coordinate in $M^2_\pm$ defined the preferred commutative plane of non-physical polarizations, and also to a hyper-quaternionic coordinate in $M^4_\pm$. Hence it would seem that super-canonical algebra can be continued to an algebra in $M^2_\pm$ or perhaps in the entire $M^4_\pm$. This would allow to continue also the operators G, L and other super-canonical operators to operators in hyper-quaternionic $M^4_\pm$ needed in stringy perturbation theory.

3. Also the super KM algebra associated with the light-like 3-surfaces should be continueable to hyper-quaternionic $M^4_\pm$. Here $HO-H$ duality comes in rescue. It requires that the preferred hyper-complex plane M^2 is contained in the tangent plane of the space-time sheet at each point, in particular at light-like 3-surfaces. We already know that this allows to assign a unique space-time surface to a given collection of light-like 3-surfaces as hyper-quaternionic 4-surface of HO hypothesized to correspond to (an obviously preferred) extremal of Kähler action. An equally important implication is that the light-like coordinate of X^3 can be continued to hyper-complex coordinate M^2 coordinate and thus also to hyperquaternionic M^4 coordinate.

4. The four-momentum appears in super generators G_n and L_n. It seems that the formal Fourier transform of four-momentum components to gradient operators to $M^4_\pm$ is needed and defines these operators as particular elements of the CH Clifford algebra elements extended to fields in imbedding space.

3. What about stringy perturbation theory?

The analog of stringy perturbation theory does not seems only a highly attractive but also an unavoidable outcome since a generalization of massless fermionic propagator is needed. The inverse for the sum of super Kac-Moody and super-canonical super-Virasoro generators G (L) extended to an operator acting on the difference of the M^4 coordinates of the end points of the propagator line connecting two partonic 2-surfaces should appear as fermionic (bosonic) propagator in stringy perturbation theory. Virasoro conditions imply that only G_0 and L_0 appear as propagators. Momentum eigenstates are not strictly speaking possible since since discretization is present due to the finite measurement resolution. One can however represent these states using Fourier transform as a superposition of momentum eigenstates so that standard formalism can be applied.

Symplectic QFT gives an additional multiplicative contribution to n-point functions and there would be also braiding S-matrices involved with the propagator lines in the case that partonic 2-surface carriers more than 1 point. This leaves still modular degrees of freedom of the partonic 2-surfaces describable in terms of elementary particle vacuum functionals and the proper treatment of these degrees of freedom remains a challenge.

4. What about non-hermiticity of the CH super-generators carrying fermion number?

TGD represents also a rather special challenge, which actually represents the fundamental difference between quantum TGD and super string models. The assignment of fermion number to CH gamma matrices and thus also to the super-generator G is unavoidable. Also M^4 and H gamma matrices carry fermion number. This has been a long-standing interpretational problem in quantum TGD and I have been even ready to give up the interpretation of four-momentum operator appearing in G_n and L_n as actual four-momenta. The manner to get rid of this problem would be the assumption of Majorana property but this would force to give up the interpretation of different imbedding space chiralities in terms of conserved lepton and quark numbers and would also lead to super-string theory with critical dimension 10 or 11. A further problem is how to obtain amplitudes which respect fermion number conservation using string perturbation theory if $1/G = G^\dagger/L_0$ carries fermion number.

The recent picture does not leave many choices so that I was forced to face the truth and see how everything falls down to this single nasty detail! It became as a total surprise that gamma matrices carrying fermion number do not cause any difficulties in zero energy ontology and make sense even in the ordinary Feynman diagrammatics.

1. Non-hermiticity of G means that the center of mass terms CH gamma matrices must be distinguished from their Hermitian conjugates. In particular, one has $\gamma_0 \neq \gamma_0^{d}{}^{agger}$. One can interpret the fermion number carrying M^4 gamma matrices of the complexified quaternion space.

2. One might think that $M^4 \times CP_2$ gamma matrices carrying fermion number

is a catastrophe but this is not the case in massless theory. Massless momentum eigen states can be created by the operator $p^k\gamma_k^\dagger$ from a vacuum annihilated by gamma matrices and satisfying massless Dirac equation. The conserved fermion number defined by the integral of $\overline{\Psi}\gamma^0\Psi$ over 3-space gives just its standard value. A further experimentation shows that Feynman diagrams with non-hermitian gamma matrices give just the standard results since fermionic propagator and boson-emission vertices give compensating fermion numbers.

3. If the theory would contain massive fermions or a coupling to a scalar Higgs, a catastrophe would result. Hence ordinary Higgs mechanism is not possible in this framework. Of course, also the quantization of fermions is totally different. In TGD fermion mass is not a scalar in H. Part of it is given by CP_2 Dirac operator, part by p-adic thermodynamics for L_0, and part by Higgs field which behaves like vector field in CP_2 degrees of freedom, so that the catastrophe is avoided.

4. In zero energy ontology zero energy states are characterized by M-matrix elements constructed by applying the combination of stringy and symplectic Feynman rules and fermionic propagator is replaced with its super-conformal generalization reducing to an ordinary fermionic propagator for massless states. The norm of a single fermion state is given by a propagator connecting positive energy state and its conjugate with the propagator G_0/L_0 and the standard value of the norm is obtained by using Dirac equation and the fact that Dirac operator appears also in G_0.

5. The hermiticity of super-generators G would require Majorana property and one would end up with superstring theory with critical dimension $D = 10$ or $D = 11$ for the imbedding space. Hence the new interpretation of gamma matrices, proposed already years ago, has very profound consequences and convincingly demonstrates that TGD approach is indeed internally consistent.

In this framework coupling constant evolution would have interpretation in terms of addition of intermediate zero energy states corresponding to the generalized Feynman diagrams obtained by the insertion of causal diamonds with a new shorter time scale $T = T_{prev}/2$ to the previous Feynman diagram. p-Adic length scale hypothesis follows naturally. A very close correspondence with ordinary Feynman diagrammatics arises and and ordinary vision about coupling constant evolutions arises. The absence of infinities follows from the symplectic invariance which is genuinely new element. p-Adic and real coupling constant evolutions can be seen as completions of coupling constant evolutions for physics based on rationals and their algebraic extensions.

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